Inventory Pinch Algorithms for Gasoline Blending

Fields Institute Industrial Seminars

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Topics

- Brief overview of gasoline blending & current solution approaches
- Inventory Pinch concept
- Multiperiod inventory pinch algorithm for blend planning
- Single period inventory pinch algorithm for blend planning
- Extensions to scheduling and general production planning
- Conclusions

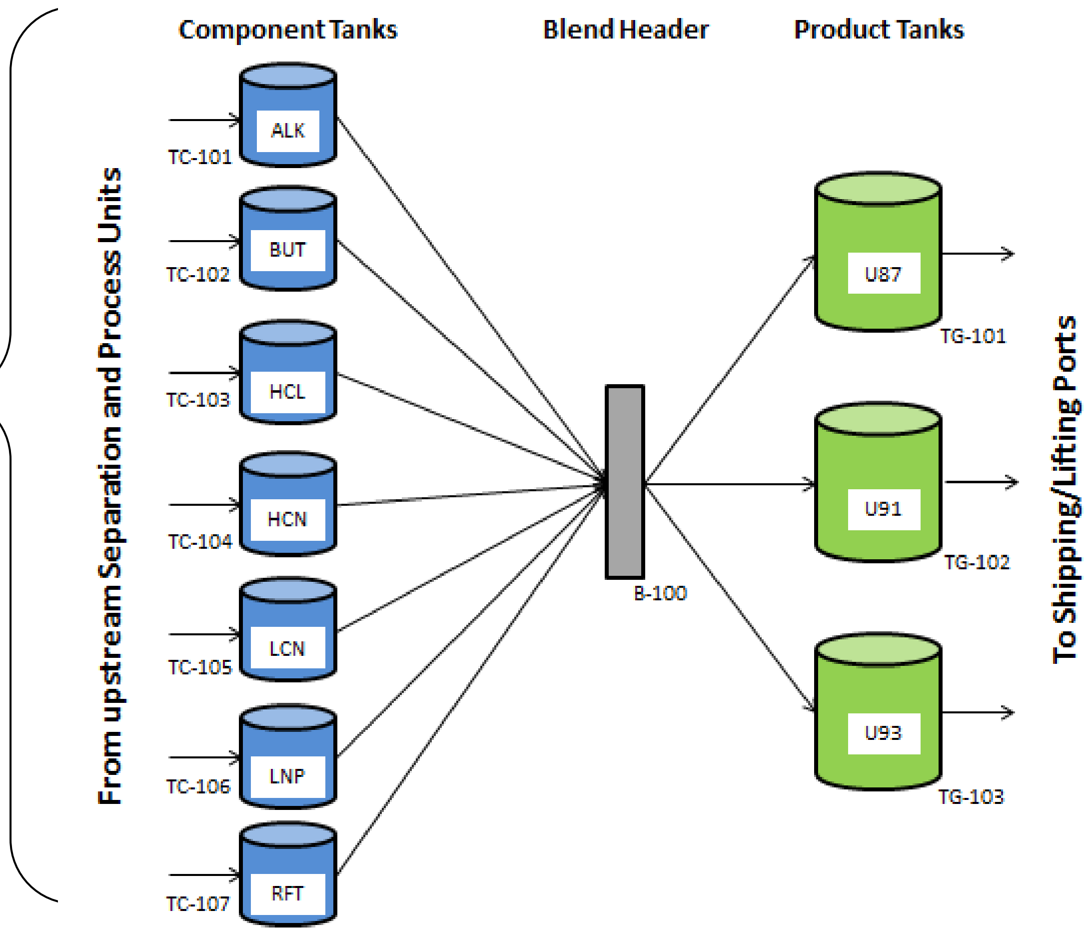
Why gasoline blending?

- From industrial viewpoint:
  - Important component of refinery profits.

- From academic viewpoint:
  - Small, easy to understand model of the physical system.
    - Linear, bilinear, or highly nonlinear
  - Multiple optima
  - Knowledge gained about gasoline blending is often directly applicable to more complex process plants.

Sample Gasoline Blending System

Assumption: Quality constant over time



Assumption: Demand known over time

How Much to Produce and When for Each Product?

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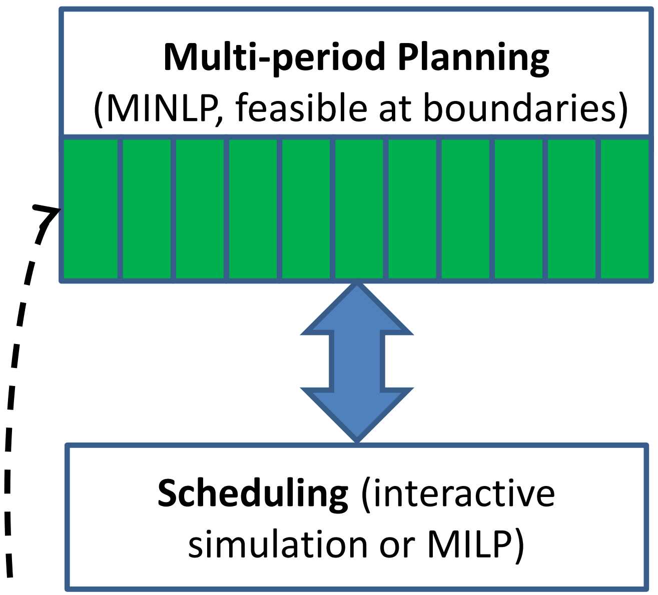
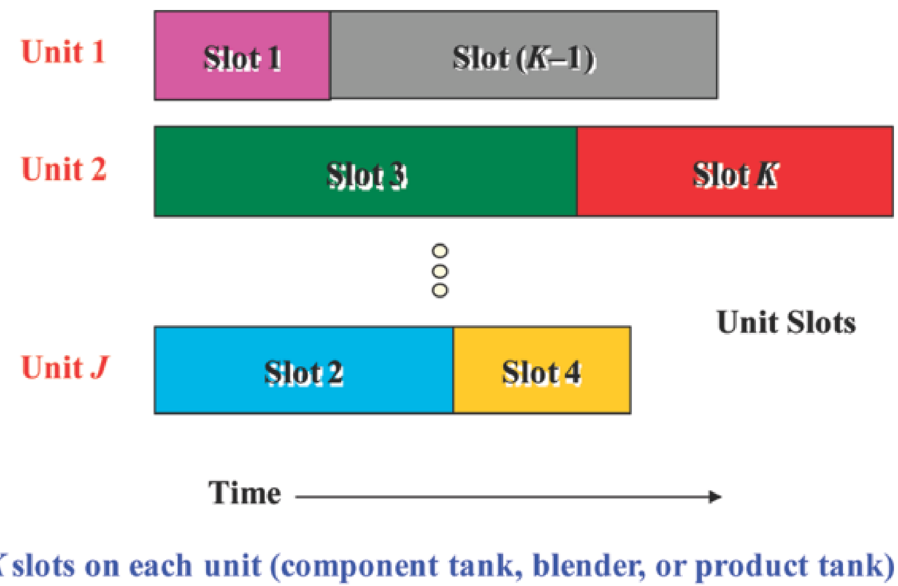
Discrete time approach



The shorter the periods, the more likely is that a feasible schedule can be created.

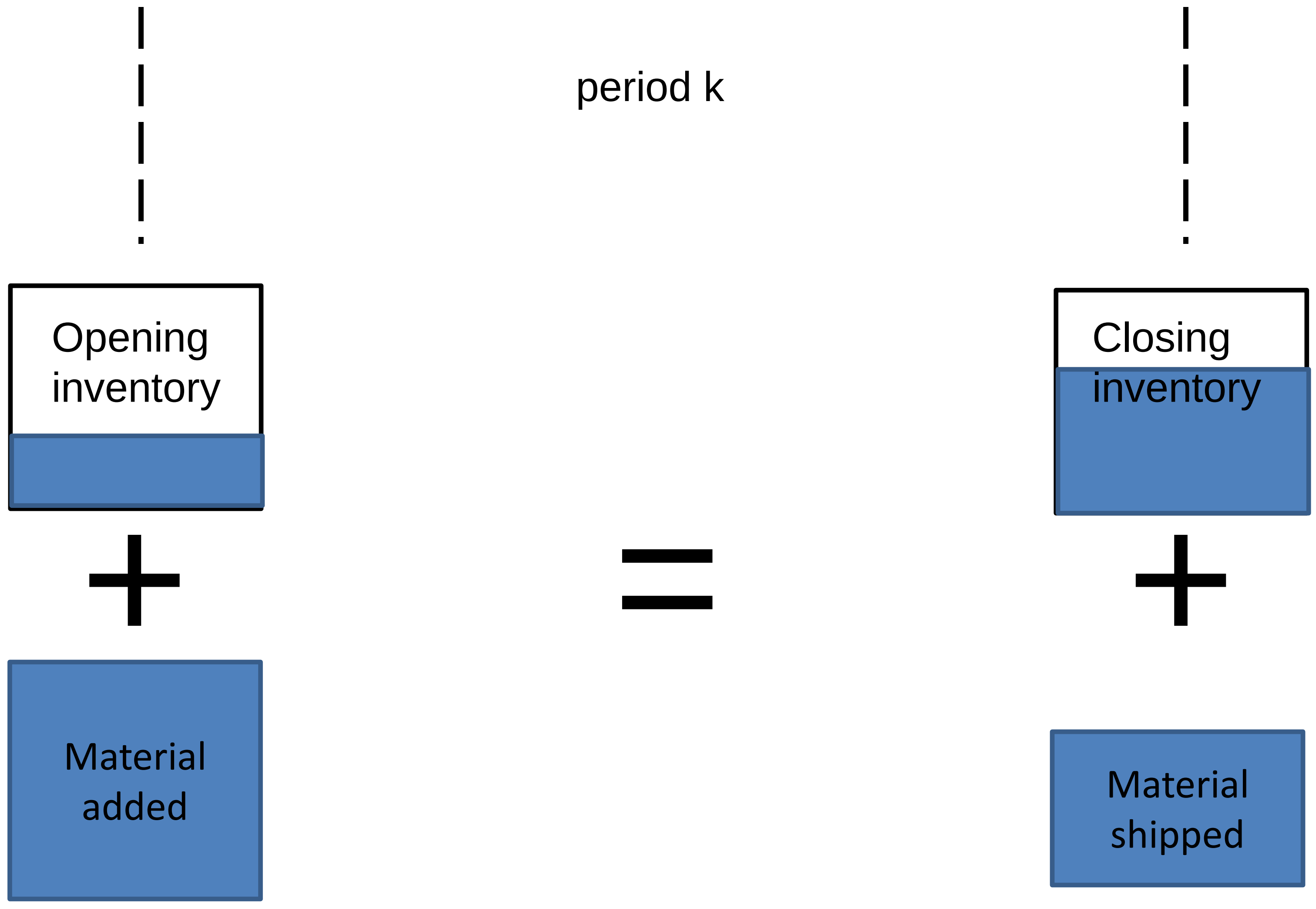
Continuous time approach



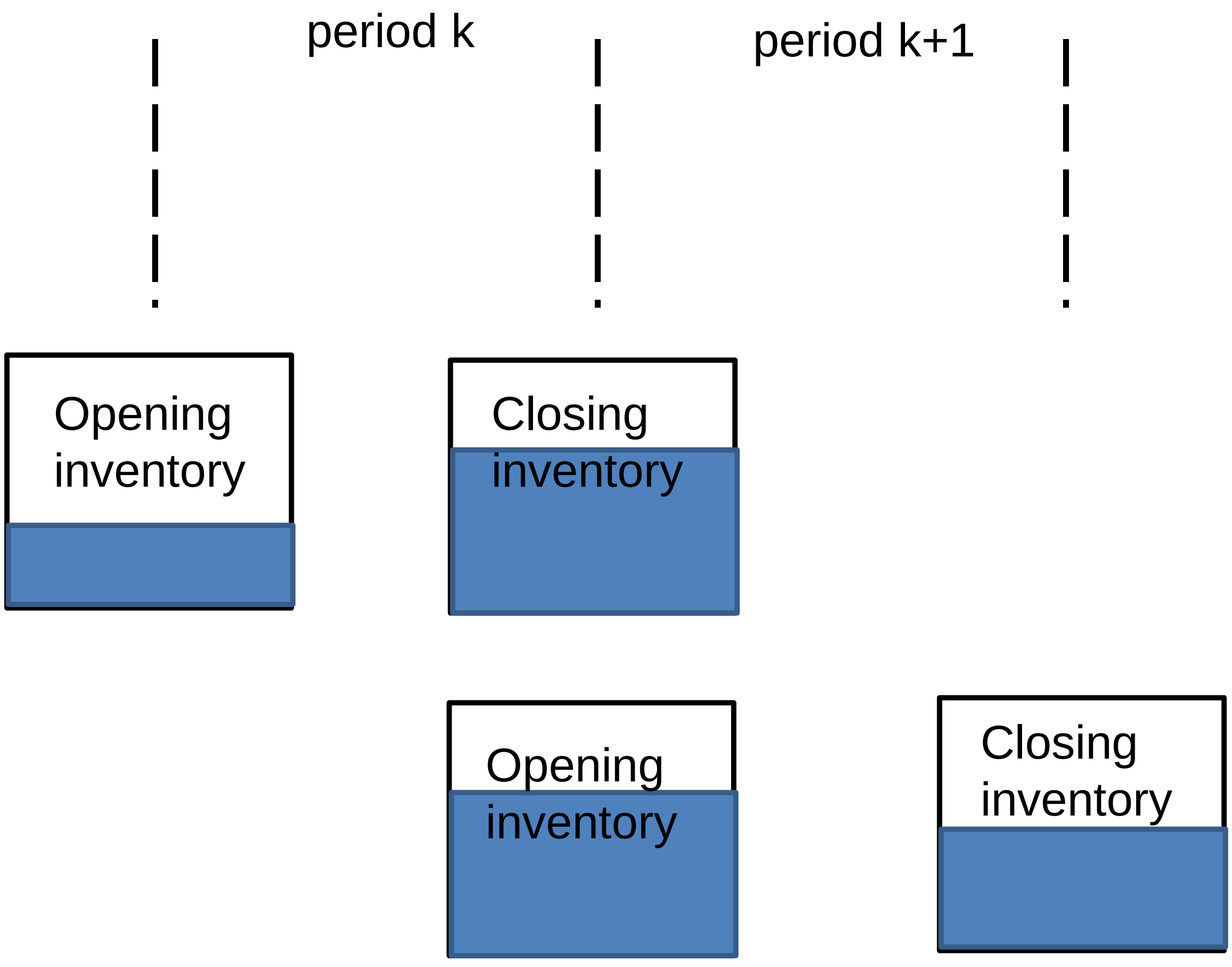
Solve simultaneously for start/end of each blend and for the blend recipe.

Li and Karimi, IEC Research, 2011, pp. 9156-9174

Discrete Time: Opening and Closing Inventory



Discrete Time: Inventory Connects Time Periods



Blending Model: Inventory Constraints

- Volumetric balance - components

$$V_{C,K}^{close}(i) = V_{C,K}^{open}(i) + V_{C,K}^{in}(i) - \sum_{g} V_{C,K}(i,g) + S_{C,K}^{+}(i) - S_{C,K}^{-}(i)$$

- Inventory constraints – components

$$V_{C}^{min}(i) \leq V_{C,K}^{close}(i) \leq V_{C}^{max}(i)$$

- Volumetric balance – products

$$V_{P,K}^{close}(g) = V_{P,K}^{open}(g) + V_{B,K}(g) - D_{P,K}(g) + S_{P,K}^{+}(g) - S_{P,K}^{-}(g)$$

- Inventory constraints - products

$$V_{P}^{min}(g) \leq V_{P,K}^{close}(g) \leq V_{P}^{max}(g)$$

Blending model: Quality constraints

- Quality*volume (for properties blended linearly)

$$\mathrm{Q}_{\mathrm{P}}^{\min}(g,s)\cdot \mathrm{V}_{\mathrm{B,K}}(g,k)\le \sum_{\mathrm{i}} \mathrm{Q}_{\mathrm{C}}(i,s)\cdot \mathrm{V}_{\mathrm{C,K}}(i,g,k)\le \mathrm{Q}_{\mathrm{P}}^{\max}(g,s)\cdot \mathrm{V}_{\mathrm{B,K}}(g,k)$$

- Non-linear quality constraints, e.g. RVP

$$Q_{P,K}(g,s=RVP)=\left[\sum_{i=1}^{I} x_K(i,g)\cdot\left(Q_C(i,s=RVP)\right)^{1.25}\right]^{0.8}$$

- Total blend volumes

$$\mathrm{V}_{\mathrm{C,K}}(i,g)-x_K(i,g)\cdot \mathrm{V}_{B,K}(g)=0$$

$$\sum_{i} \mathrm{x}_{\mathrm{K}}(i,g)=1$$

Blending model: integer constraints

- Threshold production:
  - If grade “g” is blended in period “k” then the amount blended has to be greater than or equal to the “threshold amount”.
  - If grade “g” is blended, then there is a set-up time (lost production capacity) associated with it.

- Not included:
  - Minimize switches (i.e. continue blending “A” in “k+1” if that was the last thing done in “k” and if A needs to be blended in “k+1”)

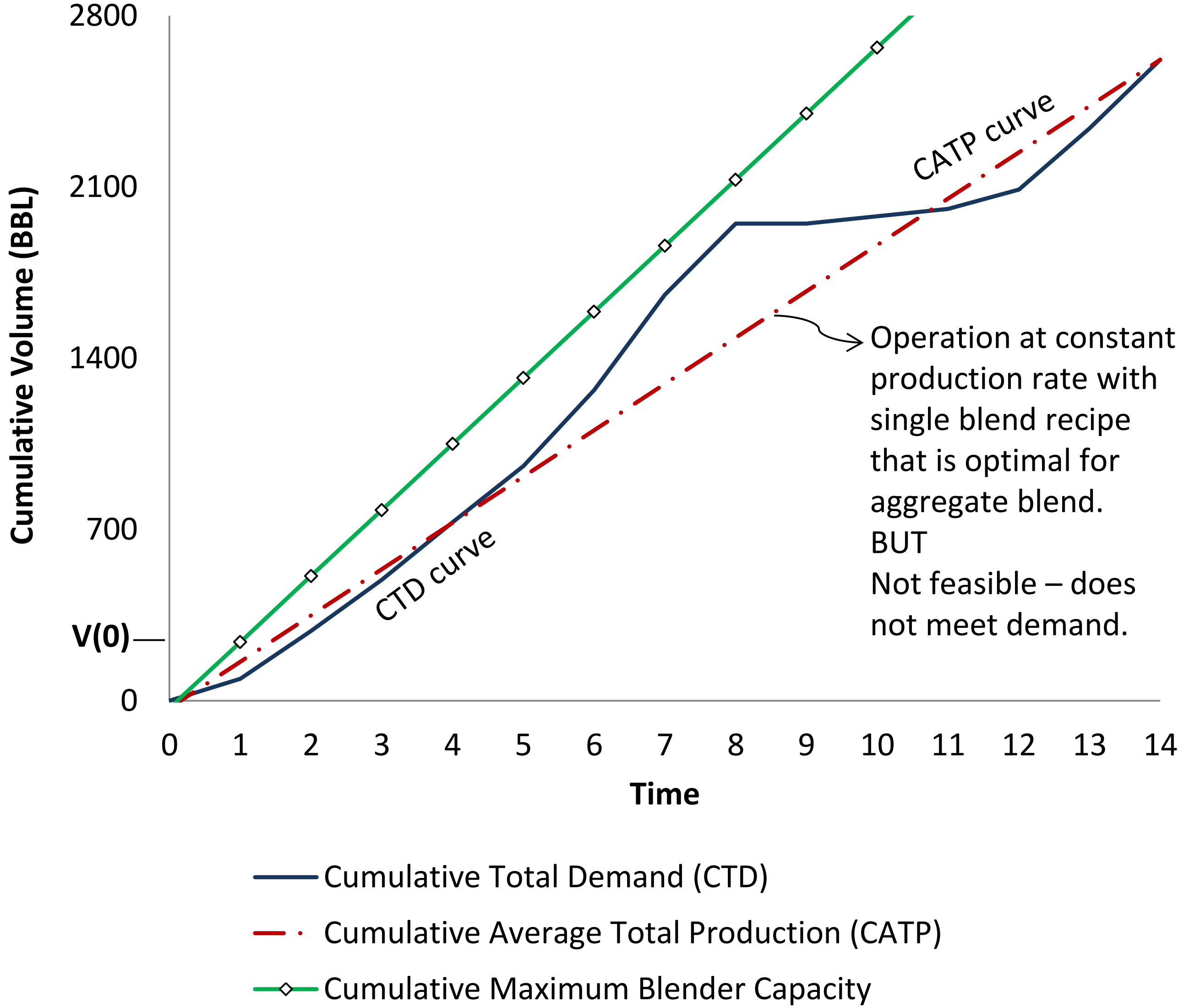
Discrete Time Approach

- Increasing number of periods leads to a rapid increase in MINLP solution times.
- Coarse time periods often lead to solutions that are intra-period infeasible.
- As a rule, each period has blend recipes that are different from the recipes in the adjacent periods.
- There are many optimal solutions (with the same value of the objective function; globally optimal).
- Different solvers arrive at the same value of the objective function but the solution are different.

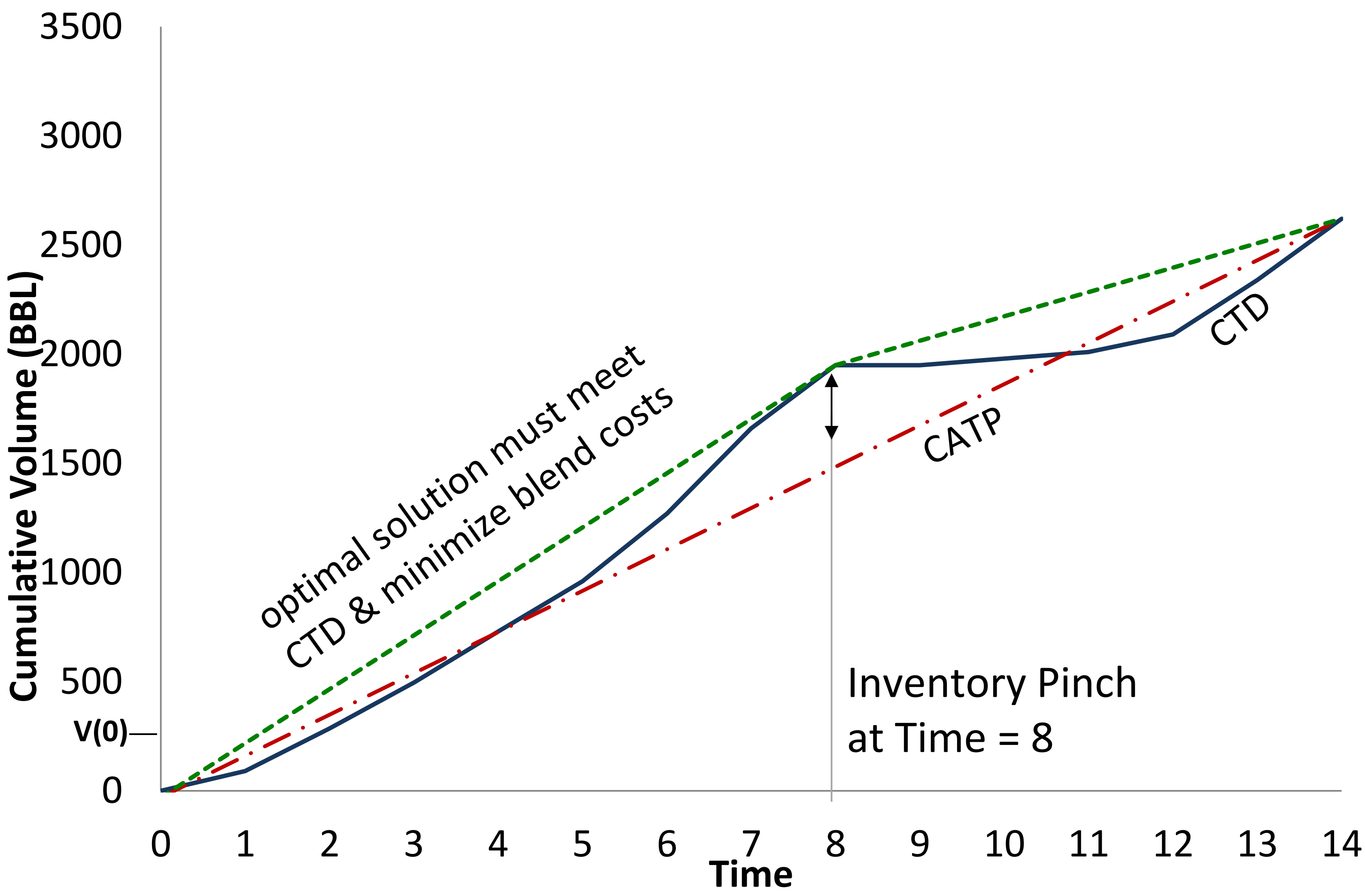
Questions that we want to answer:

- How long can we keep the bend recipe constant along the planning horizon?
  - Does this have anything to do with supply/demand pinch?
- How to exploit existence of intervals with constant blend recipes to reduce computational times at the planning level and compute production plans that are feasible?
- How to exploit such intervals in scheduling?
- Are there wider implications for process plants production planning and scheduling?

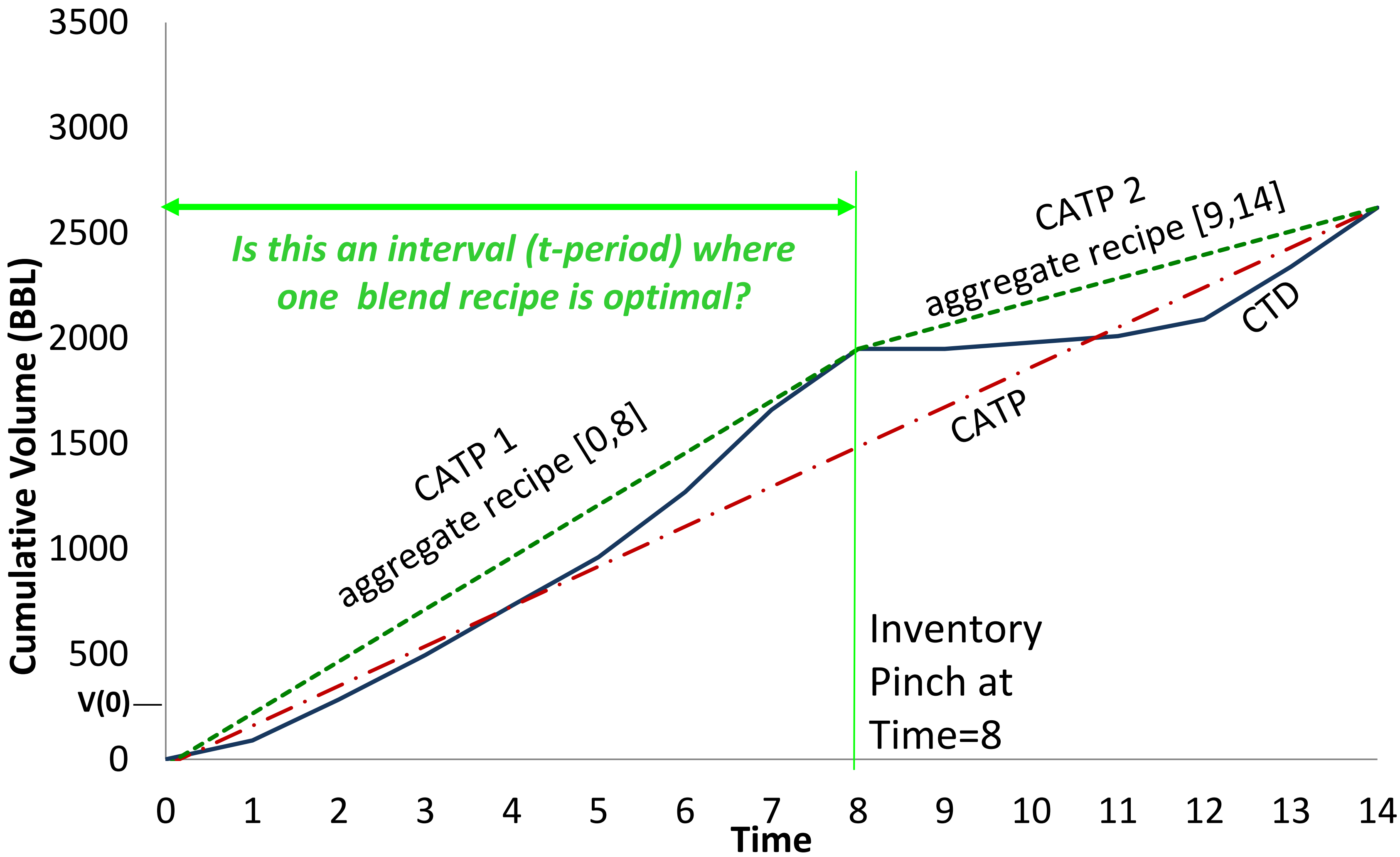
Total Demand vs. Production Capacity



Optimal Solution

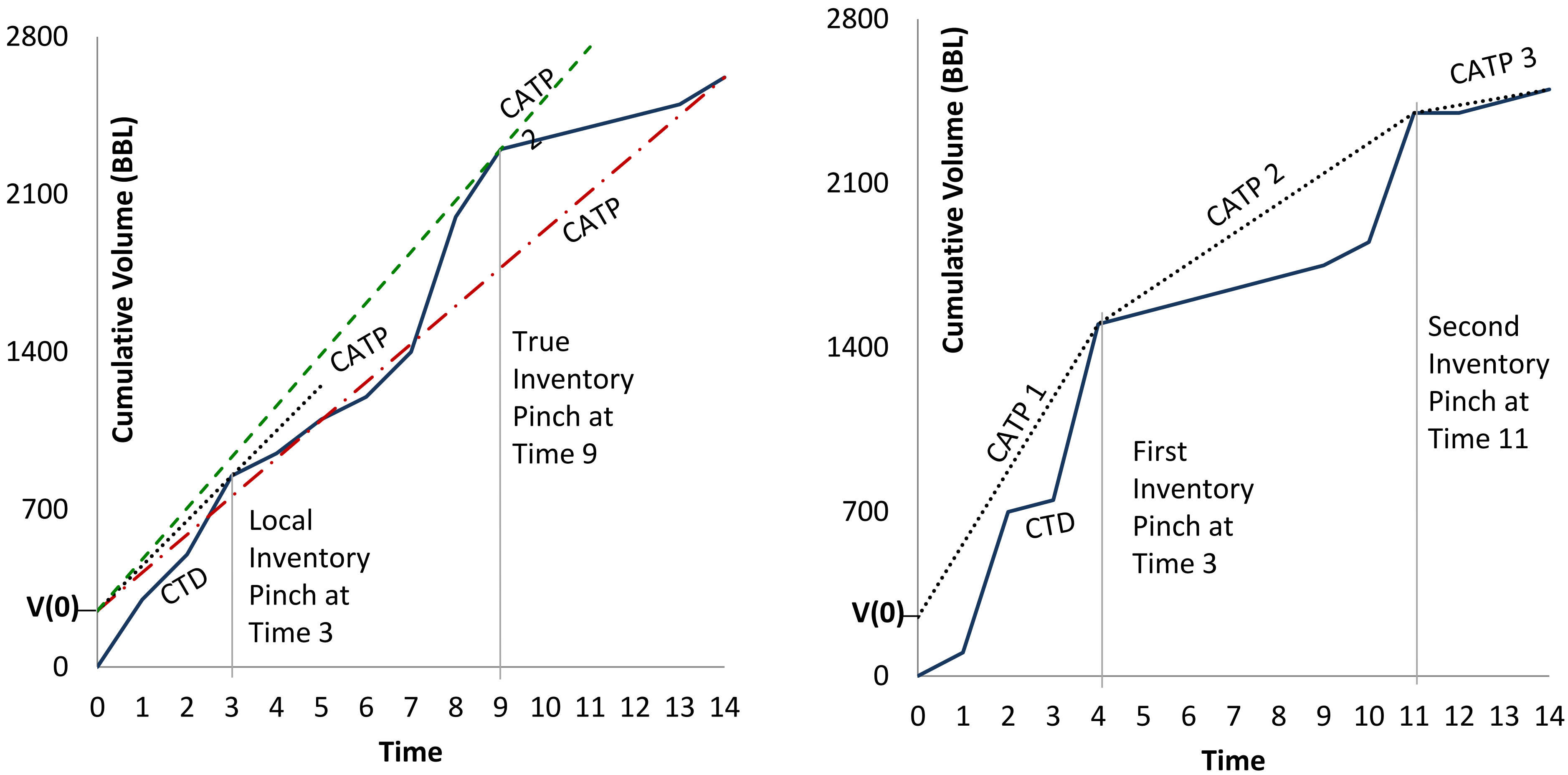


Hypothesis



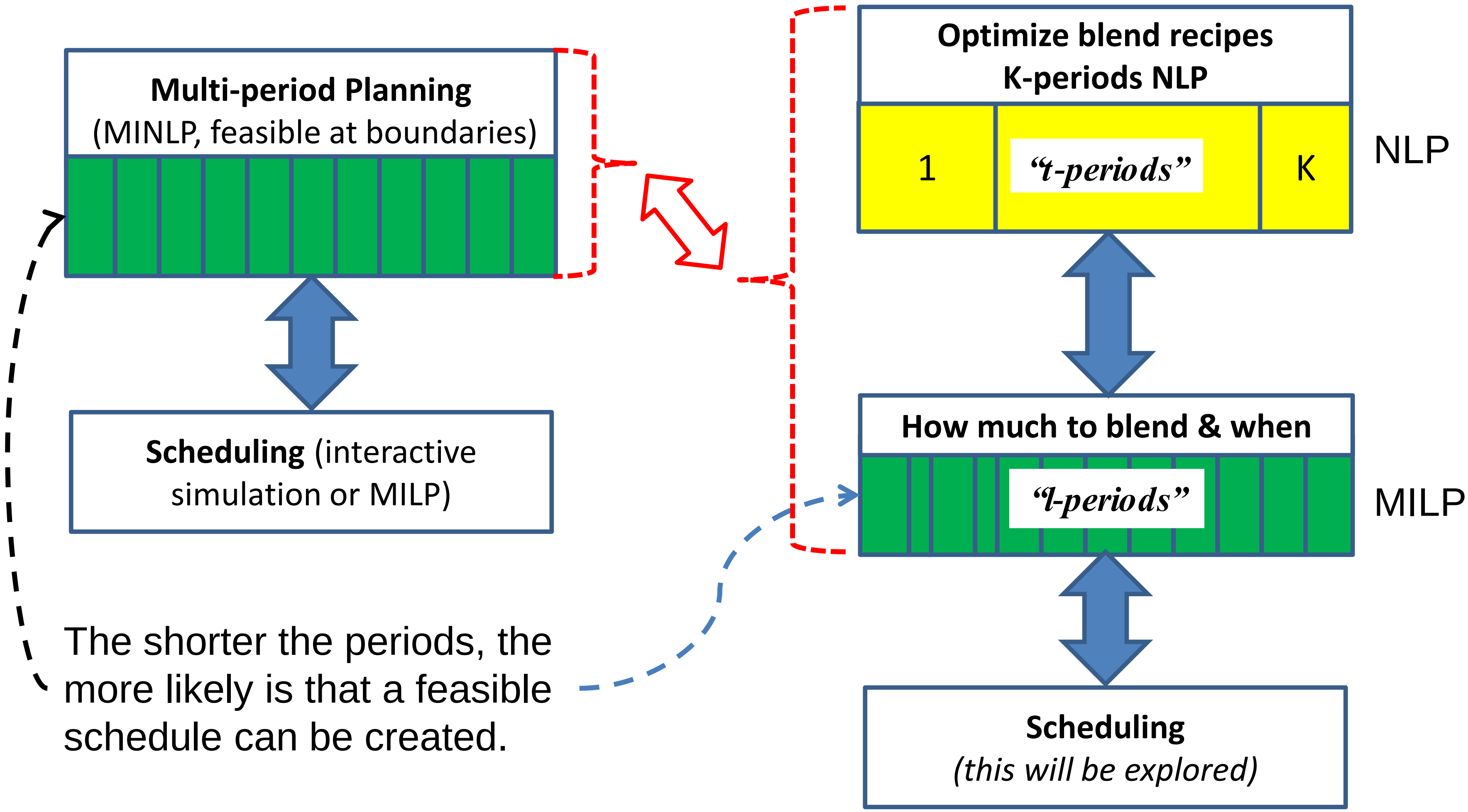
3500
3000
2500
2000
1500
1000
500
V(0)
0
Cumulative Volume (BBL)
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
Time
Is this an interval (t-period) where one blend recipe is optimal?
CATP 1
aggregate recipe [0,8]
CATP 2
aggregate recipe [9,14]
CTD
CATP
Inventory Pinch at Time=8

Inventory Pinch Point Definition



Multi-Period Inventory Pinch Algorithm for Production Planning

Current discrete time approach | **Inventory pinch multiperiod model**



Multi-Period Inventory Pinch Algorithm for Production Planning

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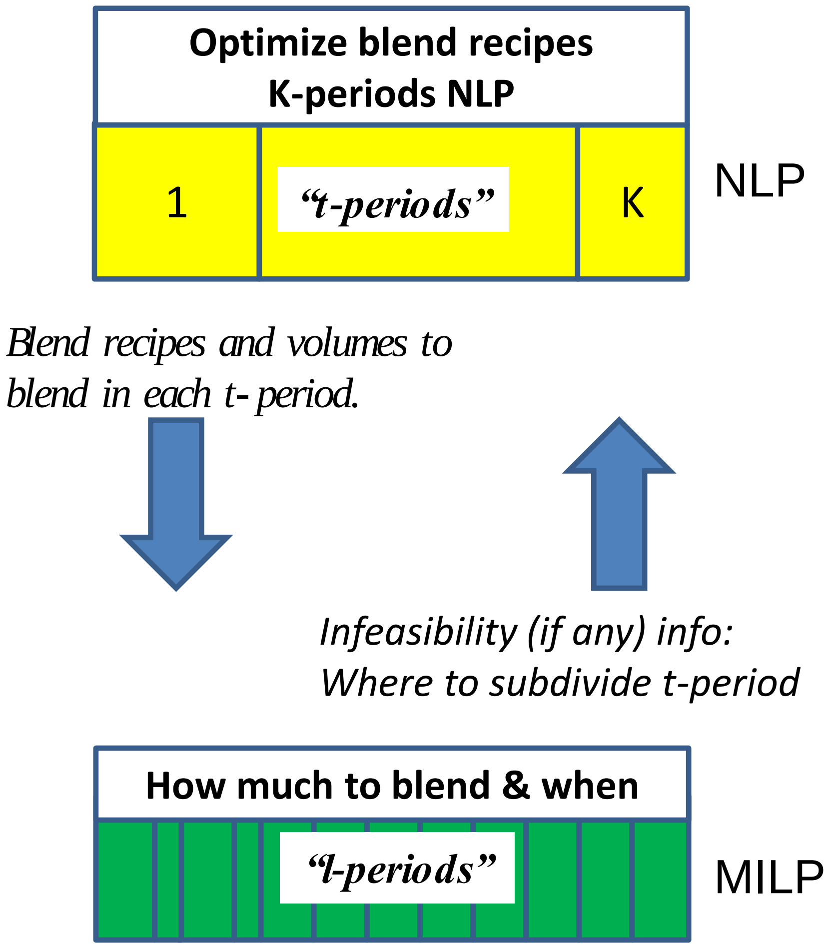
Inventory pinch multiperiod model



Pinch points determine period boundaries. If operation is infeasible, pinch- delimited period is subdivided.

Constraints:

- *Minimum blend size threshold .*
- *Inventory constraints.*

If operation is infeasible, identify the l-period where infeasible. Subdivide t-period at that point.

Lower Level: Determine best feasible solution

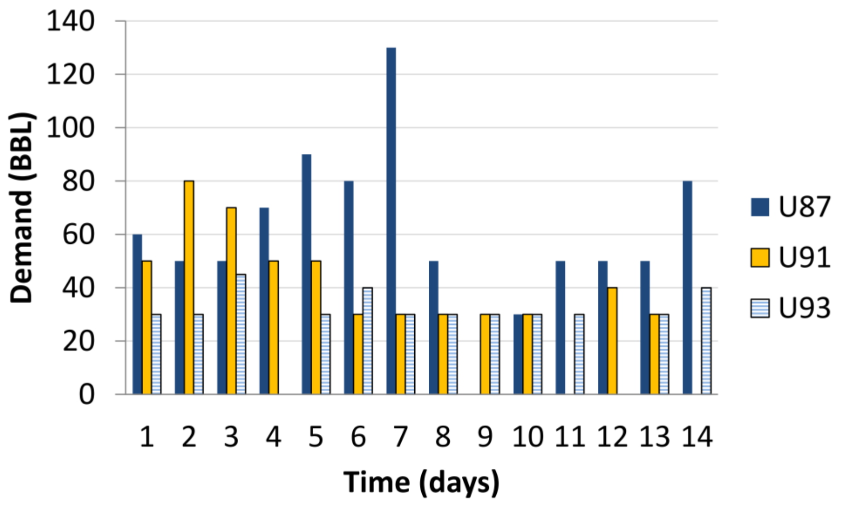
"Push" product inventory infeasibilities as far forward as possible, i.e.

$$\min\left\{\sum_{n}\left(\sum_{g}\left(S_P^{+}(g,n)+S_P^{-}(g,n)\right)\times Penalty_P(g,n)\right)\right\}$$

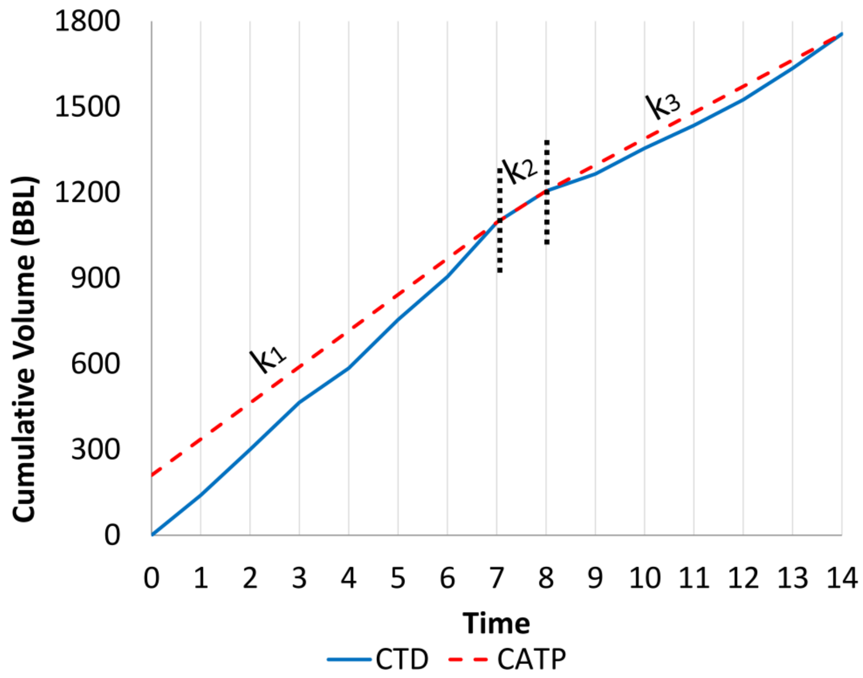
$$Penalty_P(g,n) \gg Penalty_P(g,n+1)$$

$$V_P^{close}(g,n) = V_P^{open}(g,n) + V_B(g,n) - D_P(g,n) + S_P^{+}(g,n) - S_P^{-}(g,n)$$

Case Study

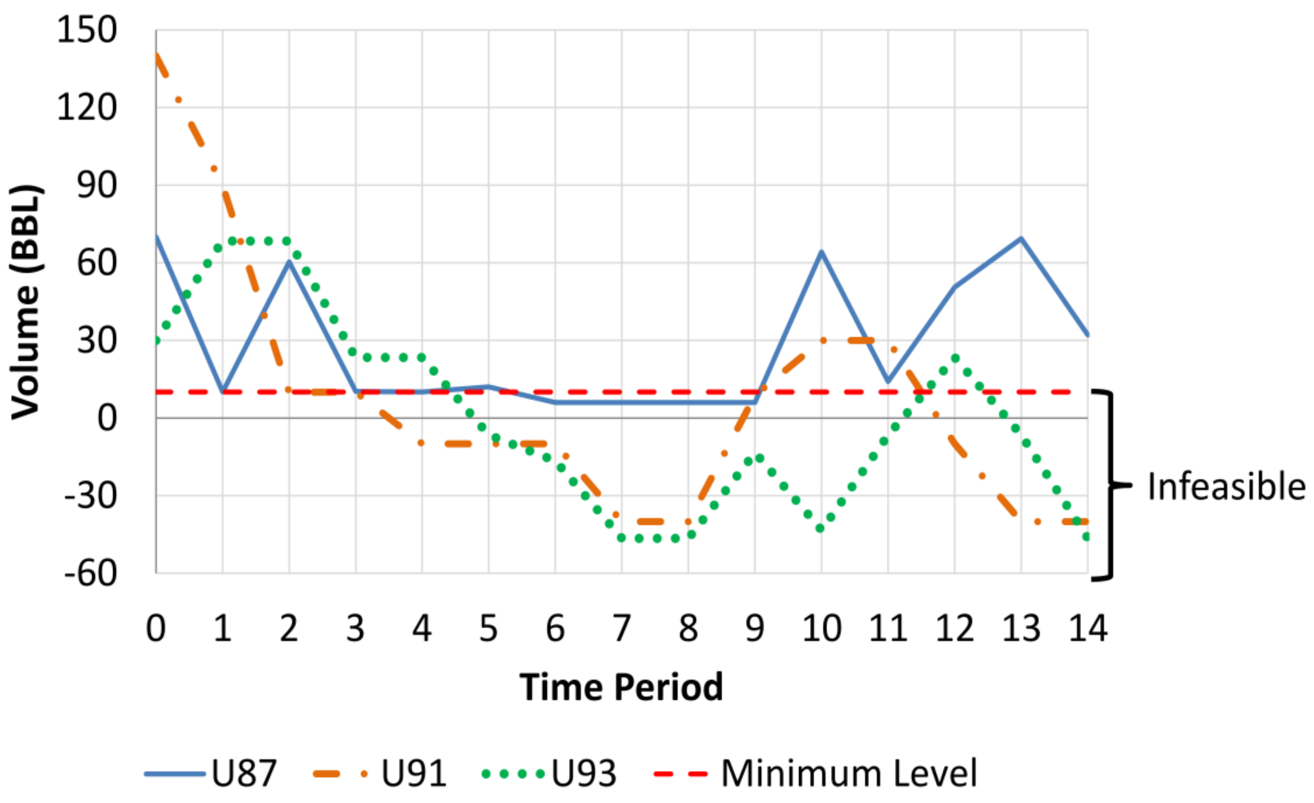


140
120
100
80
60
40
20
0
Demand (BBL)
1 2 3 4 5 6 7 8 9 10 11 12 13 14
Time (days)
U87
U91
U93

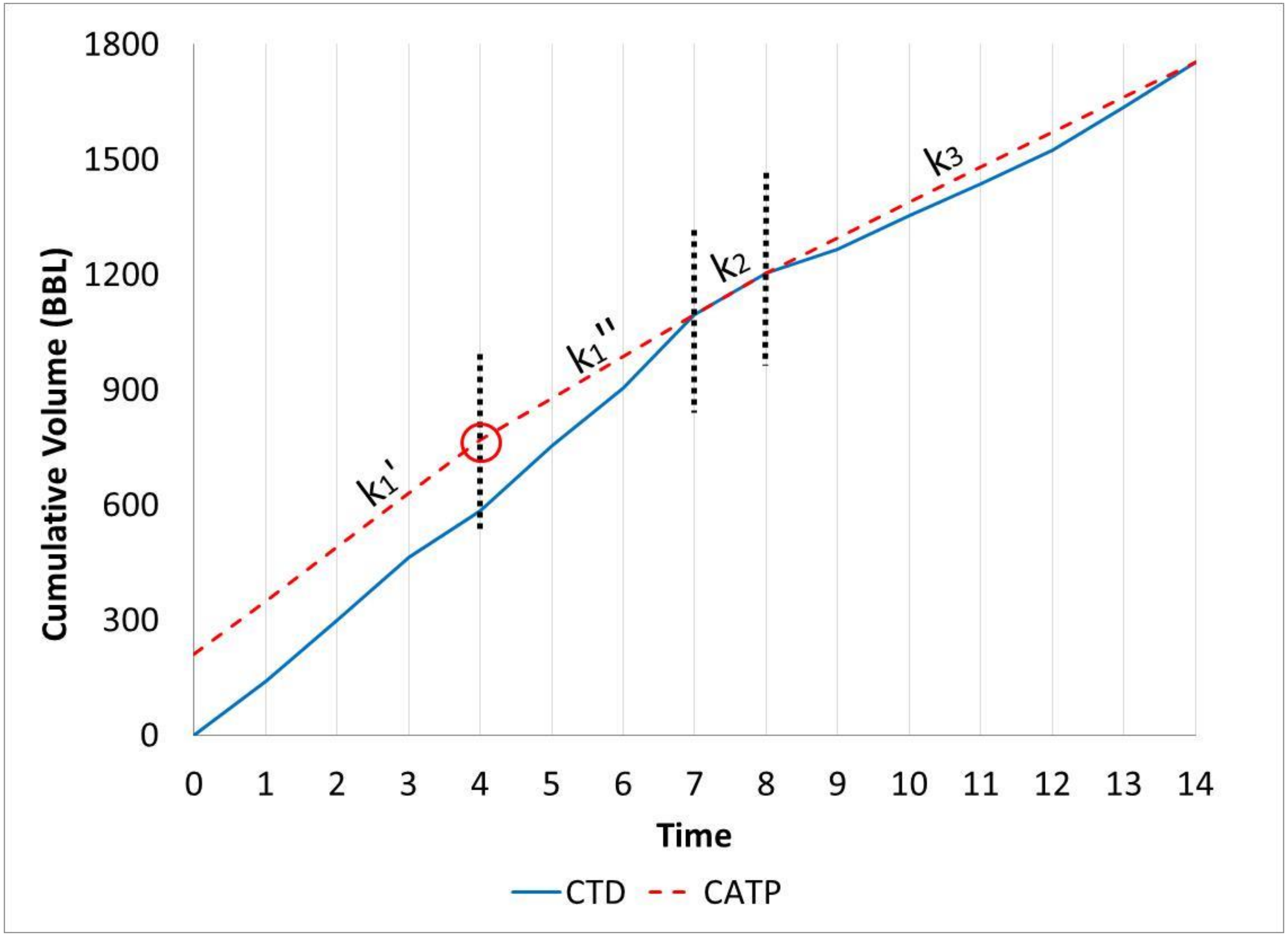


1800
1500
1200
900
600
300
0
Cumulative Volume (BBL)
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14
Time
k1
k2
k3
CTD
CATP

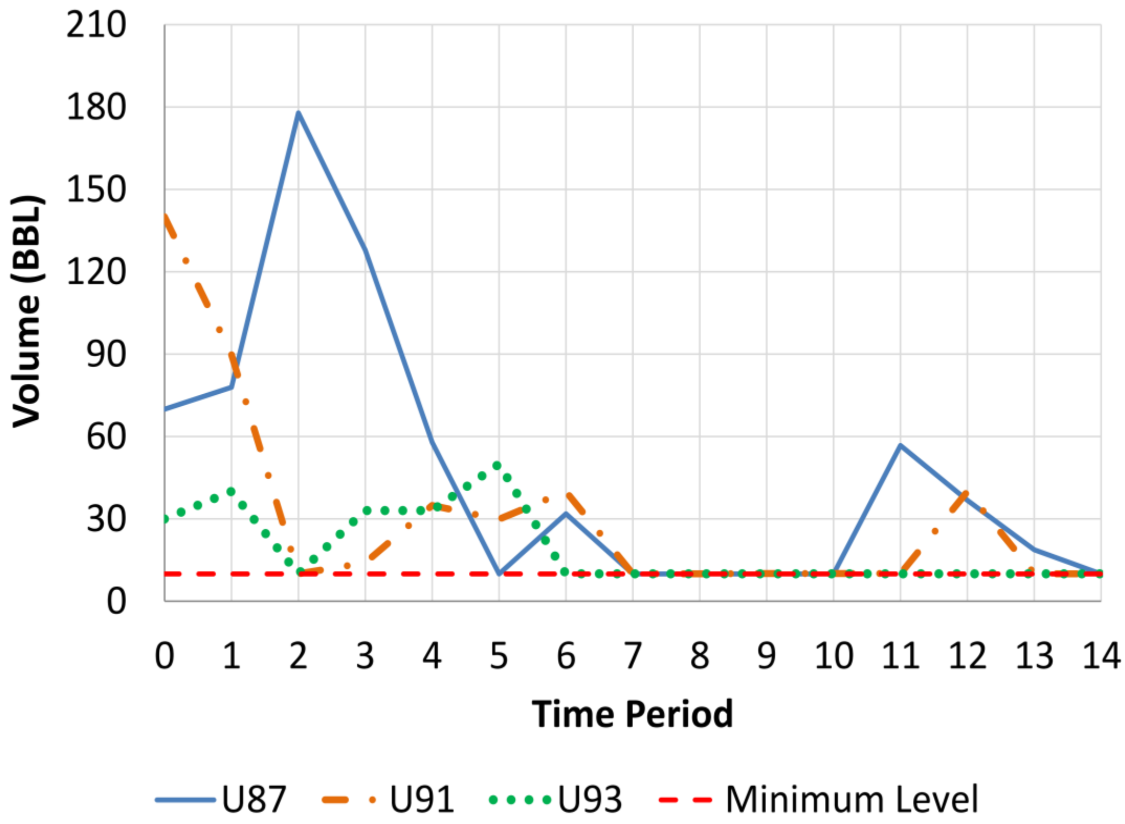
Case Study / Product Inventory - Iteration 1



Case Study /t-periods for iteration 2



Case Study / Product Inventory – Iteration 2



Case Study / Top Level: Optimal Blend Recipes

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1st Iteration										
Time period		k1			k2			k3		
Gasoline Grade		U87	U91	U93	U87	U91	U93	U87	U91	U93
Blending Comp.	ALK	0.1568	0.2467	0.1846	0.1493	0.2461	0.1848	0.0079	0.1418	0.0975
	BUT	0.0262	0.0363	0.0438	0.0255	0.0361	0.0434	0.0204	0.0345	0.0407
	HCL	0.0258	0.0389	0.028	0.0265	0.0351	0.0243	0	0	0
	HCN	0.0437	0.0651	0.0514	0.052	0.065	0.0475	0.0007	0.0033	0.0013
	LCN	0.2573	0.192	0.1378	0.2925	0.2081	0.1517	0.3933	0.1849	0.1856
	LNP	0.18	0.0694	0.0242	0.1678	0.0674	0.0244	0.2126	0.1472	0.0712
	RFT	0.3102	0.3515	0.5302	0.2863	0.3421	0.5238	0.3651	0.4883	0.6037

2nd Iteration													
Time period		k1'			k1''			k2			k3		
Gasoline Grade		U87	U91	U93	U87	U91	U93	U87	U91	U93	U87	U91	U93
Blending Comp.	ALK	0.1498	0.2446	0.1902	0.1644	0.2432	0.1795	0.1492	0.2463	0.1849	0.008	0.1415	0.0976
	BUT	0.0257	0.0351	0.043	0.0272	0.0373	0.0443	0.0256	0.0361	0.0435	0.0204	0.0344	0.0407
	HCL	0.0208	0.0277	0.0243	0.0332	0.0514	0.031	0.0259	0.0345	0.024	0	0	0
	HCN	0.0255	0.0359	0.0367	0.0711	0.0898	0.0612	0.0522	0.0652	0.0476	0.0008	0.0027	0.0016
	LCN	0.2513	0.2117	0.1541	0.2548	0.1705	0.1265	0.2925	0.2082	0.1519	0.3917	0.1873	0.1863
	LNP	0.1947	0.0849	0.0275	0.1617	0.0572	0.0221	0.1681	0.0676	0.0244	0.213	0.147	0.0708
	RFT	0.3321	0.3601	0.524	0.2876	0.3506	0.5354	0.2865	0.342	0.5237	0.3661	0.4872	0.6029

Volumes to Blend at the Top Level

		Volumes to blend (BBL)		
1st Iteration (infeasible)				
Gasoline Grade		**U87**	**U91**	**U93**
Time period	**k1**	470	230	185
	k2	50	30	30
	k3	260	130	160
	Total	**780**	**390**	**375**
2nd Iteration (feasible)				
Gasoline Grade		**U87**	**U91**	**U93**
Time period	**k1**	217.89	144.97	107.98
	k2	252.11	85.03	77.02
	k3	50	30	30
	k4	260	130	160
	Total	**780**	**390**	**375**

Summary of Case Studies – Multiperiod Inventory Pinch

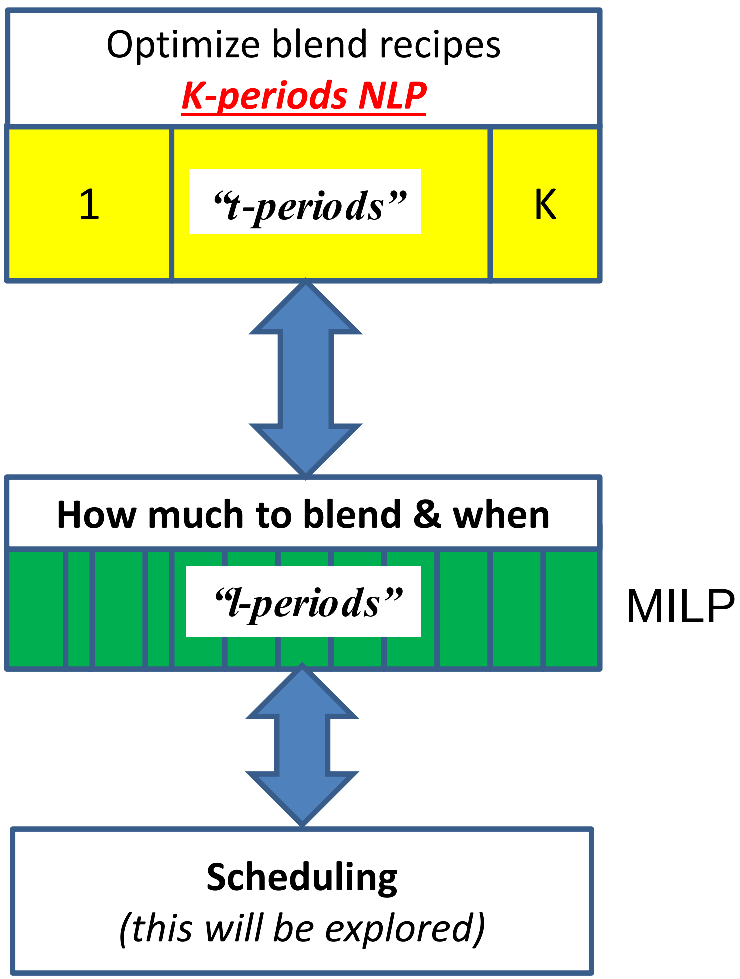
Case Study	Supply rate	DICOPT Solution (MINLP model)		Multi-Period Inventory Pinch Algorithm Solution (IPOPT, CPLEX)		
		Objective Function ($\times 10^3$\$)	Total CPU time (s)	Objective Function ($\times 10^3$\$)	Total CPU time (s)	Iterations
1	Regular	37,542.2	13.879	37,542.5	2.200	1
2	Regular	38,120.9	99.444	38,121.2	1.913	1
3	Regular	38,309.6	11.582	38,309.9	0.956	1
4	Regular	37,990.9	12.495	37,991.1	1.089	1
5	Regular	37,863.9	7.041	37,864.2	1.005	1
6	Regular	37,680.3	17.208	37,680.6	0.990	1
7	Regular	37,324.0	17.228	37,324.5	7.825	4
8	Regular	37,761.5	7.679	37,761.8	1.548	1
9	Regular	37,377.2	14.520	37,377.5	1.203	1
10	Irregular	37,943.1	14.387	37,943.4	0.863	1
11	Irregular	38,518.2	17.738	38,518.5	1.019	1
12	Irregular	38,753.9	14.635	38,754.2	2.531	2
13	Irregular	38,405.0	22.266	38,405.3	1.076	1
14	Irregular	38,195.7	14.161	38,196.0	1.331	1
15	Irregular	38,073.1	18.715	38,073.4	1.156	1
16	Irregular	37,784.2	21.872	37,784.5	2.497	2
17	Irregular	38,192.3	15.586	38,192.6	1.344	1
18	Irregular	37,796.2	14.432	37,796.5	1.521	1

Single Period Inventory Pinch Algorithm

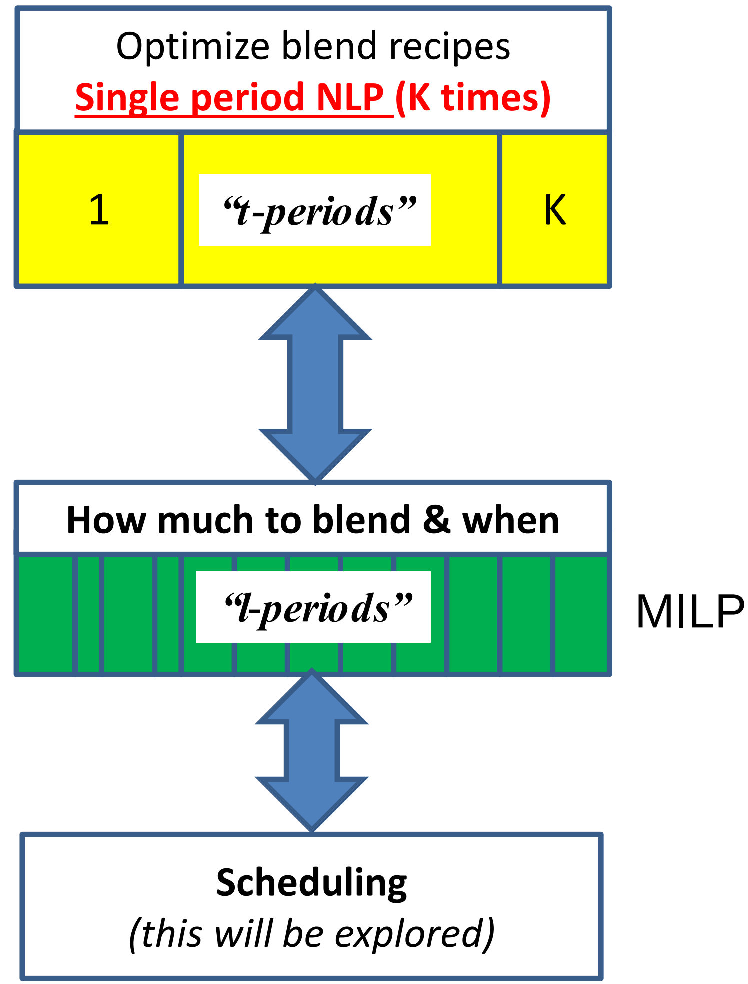
- Can we solve a series of single period NLPs at the top level and still get the optimal solution?

Single-Period vs. Multi-Period Inventory Pinch Algorithms for Production Planning

Inv. pinch multiperiod algorithm



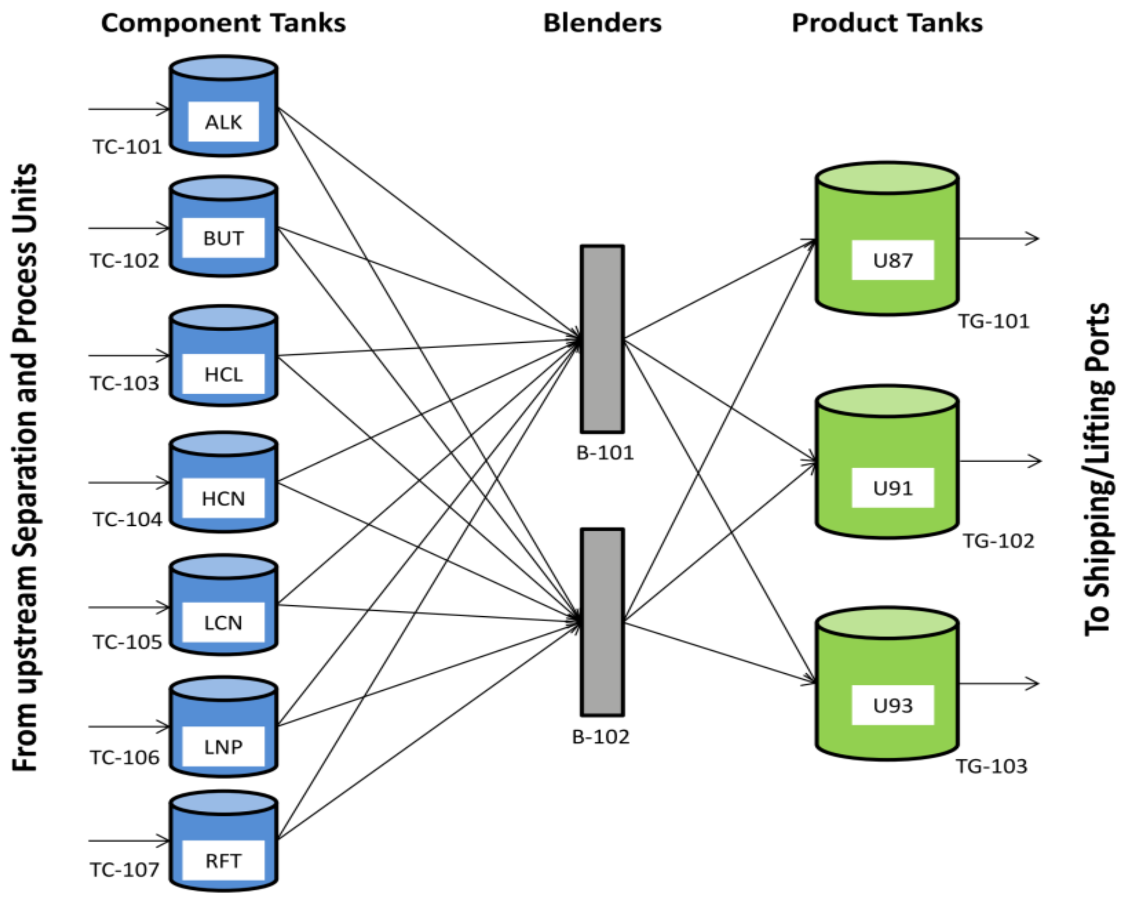
Inv. pinch single period algorithm



Single-Period Inventory Pinch Algorithm

1. Solve at the top level a separate NLP for each t-period.
2. Solve at the lower level a MINLP for the entire planning horizon.
3. If feasible, STOP. Otherwise:
4. Positive slacks on the product inventory shows how much more product needs to be produced in the previous period:
   - Increase the amount to be produced in the previous t-period by that increment.
   - Decrease amount to be produced in the current t-period by the same amount.
   - Subdivide t-period.
   - Go to 1.

Example: Two Blenders System

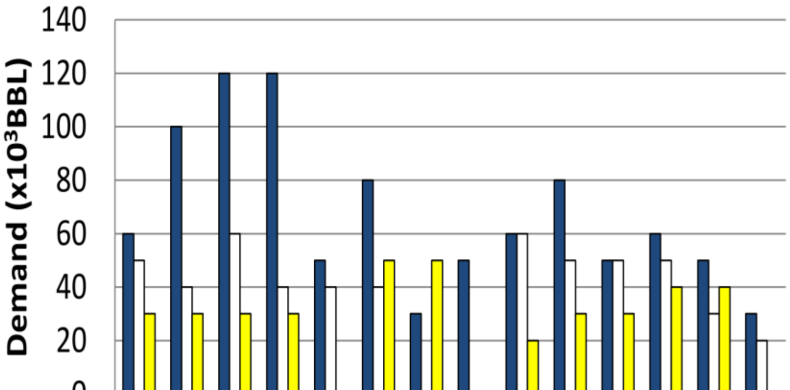


System structure is represented at the lower level (MILP).

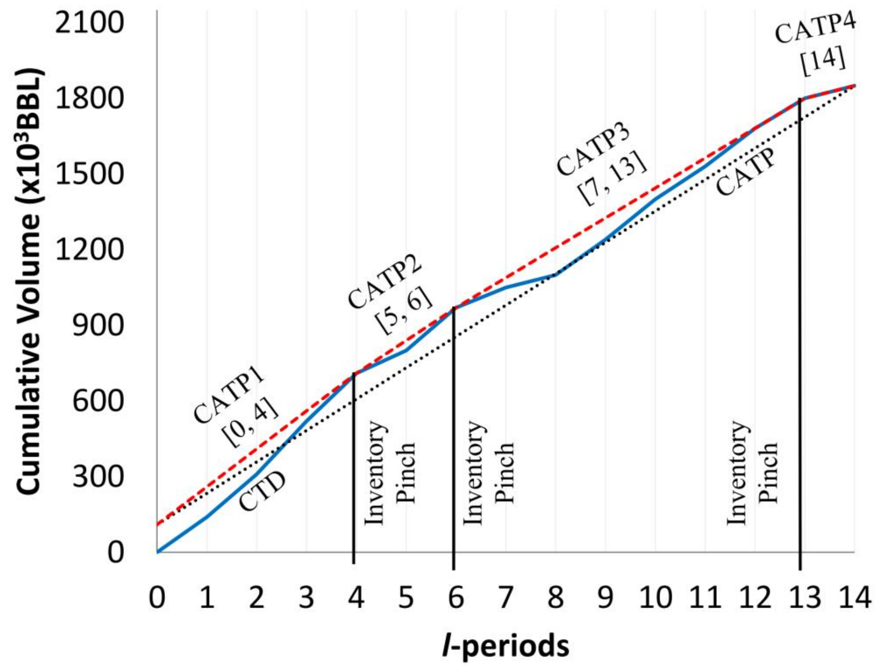
Problem Size

Model	# Equations	# Continuous Variables	# Discrete Variables	# Non-zeros
MINLP model (1 blender, 14 time periods)	1,723	1,275	42	7,187
MINLP model (2 blenders, 14 time periods)	2,885	1,989	84	12,983
NLP model (MPIP algorithm, 2 time periods)	231	171	0	869
NLP model (SPIP algorithm)	106	76	0	403
MILP model (1 blender, 14 time periods, 2 fixed recipes)	1,267	939	42	3,203
MILP model (2 blenders, 14 time periods, 2 fixed recipes)	1,967	1,317	84	5,009

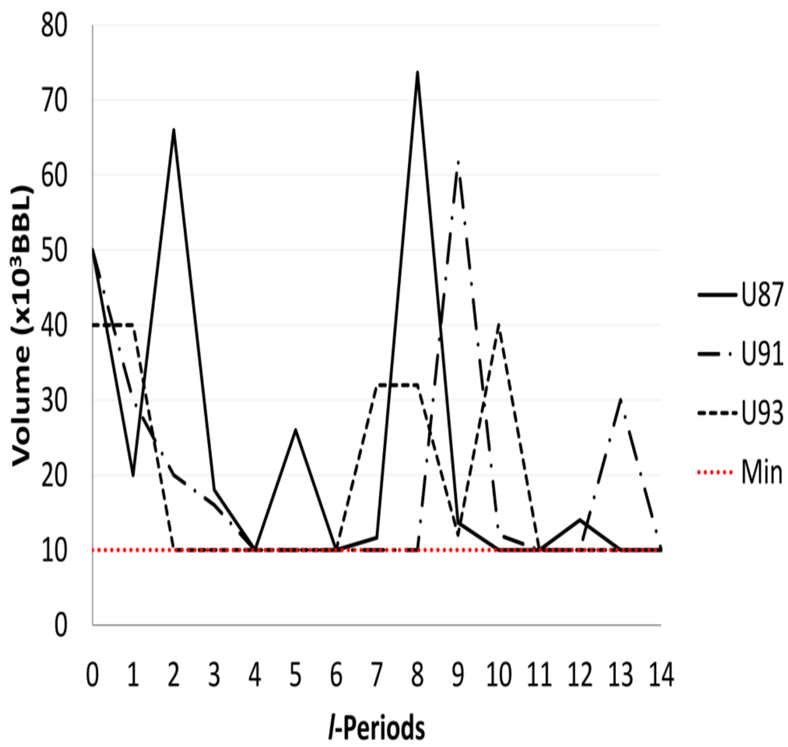
Case Study / 2 Blenders



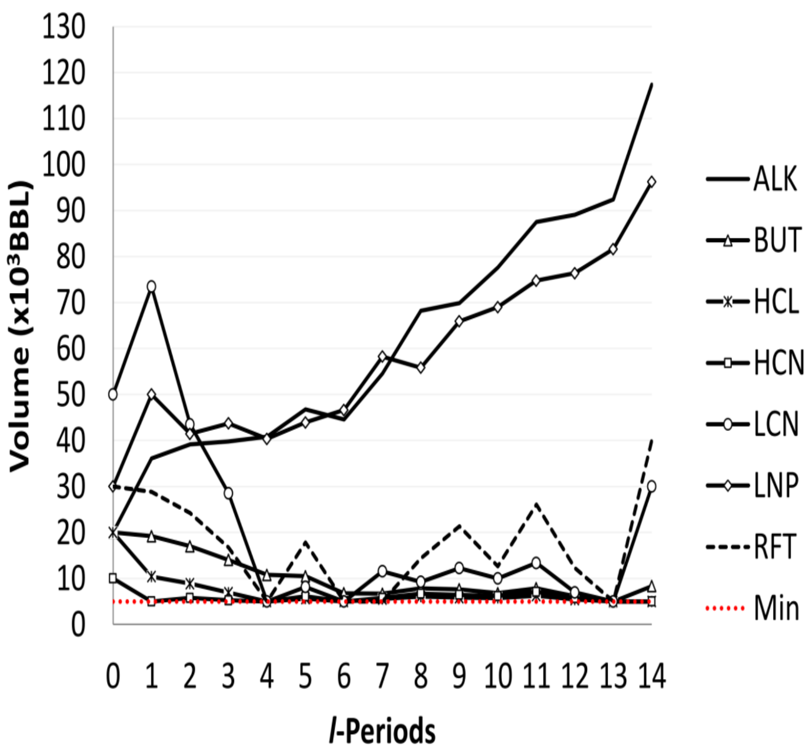
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
U87	60	100	120	120	50	80	30	50	60	80	50	60	50	30
U91	50	40	60	40	40	40	0	0	60	50	50	50	30	20
U93	30	30	30	30	0	50	50	0	20	30	30	40	40	0



Case Study / Product & Component Inventories



Product Inventories



Component Inventories

Summary of Case Studies – Single Period Inventory Pinch

Case Study	Number of blenders	Initial Product Inventory	RVP Blend Property	DICOPT Solution (MINLP model)		Single-Period Inventory Pinch Algorithm Solution (IPOPT, CPLEX)		
				Objective Function ($\times 10^3$\$)	Total CPU time (s)	Objective Function ($\times 10^3$\$)	Total CPU time (s)	Iterations
19	1	On-Spec	Linear	43420.2	12.26	43421.74	4.2	2
20	1	On-Spec	Linear	41349.62	29.08	41350.01	1.23	1
21	1	On-Spec	Linear	43161.03	37.69	43161.34	1.4	1
21	2	On-Spec	Linear	43161.05	63.52	43161.33	1.5	1
22	1	On-Spec	Linear	41874.17	20.53	41874.44	1.36	1
22	2	On-Spec	Linear	41874.45	115.15	41874.45	2.75	1
23	1	On-Spec	Nonlinear	43657.71	13.84	43658	0.98	1
24	1	On-Spec	Nonlinear	43611.69	12.81	43611.97	2.19	2
25	1	On-Spec	Nonlinear	43611.69	33.2	43611.98	1.08	1
25	2	On-Spec	Nonlinear	43611.69	34.64	43611.97	1.19	1
26	1	On-Spec	Nonlinear	43934.67	18.51	43934.95	1.71	1
26	2	On-Spec	Nonlinear	43934.95	82.32	43934.95	2.04	1
27	1	On-Spec	Nonlinear	43627.27	372.59	43627.45	8.77	5
27	1	On-Spec	Linear	43142.09	424.95	43142.25	7.8	5
28	1	On-Spec	Nonlinear	43611.69	641.76	43667.68	5.02	3
28	1	On-Spec	Linear	43101.12	235.61	43126.28	4.33	3
29	1	Off-Spec	Linear	43424.87	15.17	43425.15	2.87	1
30	1	Off-Spec	Linear	41470.45	7.73	41470.74	3.05	1
30	2	Off-Spec	Linear	41470.45	56.87	41470.74	3.57	1

Comparison: Multi-period MINLP vs. Multi-Period & Single Period Inventory Pinch Algorithms

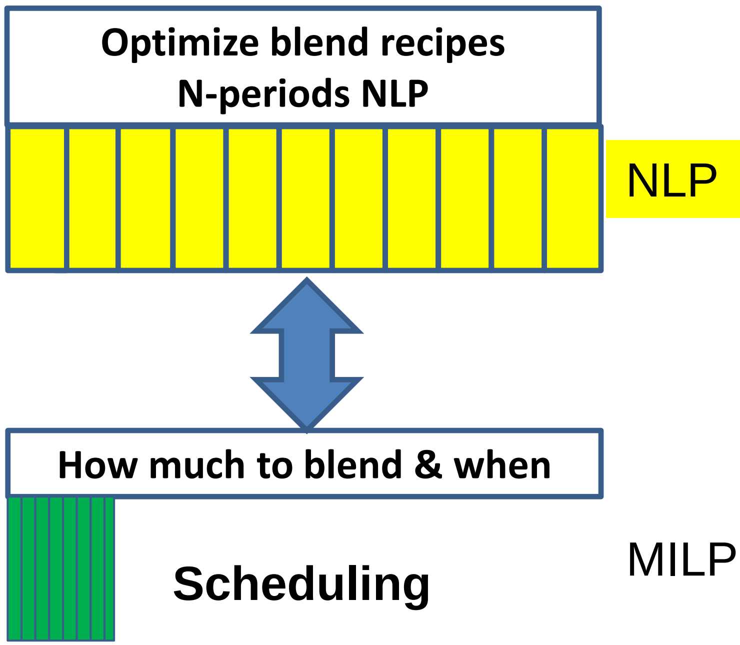
McMaster University

Case Study	Number of blenders	Initial Product Inventory	RVP Blend Property	DICOPT Solution (MINLP model)		Single-Period Inventory Pinch Algorithm Solution (IPOPT, CPLEX)			Multi-Period Inventory Pinch Algorithm Solution (IPOPT, CPLEX)		
				Objective Function ($\times 10^3$\$)	Total CPU time (s)	Objective Function ($\times 10^3$\$)	Total CPU time (s)	Iterations	Objective Function ($\times 10^3$\$)	Total CPU time (s)	Iterations
22	1	On-Spec	Linear	41874.17	20.53	41874.44	1.36	1	41874.45	0.864	1
22	2	On-Spec	Linear	41874.45	115.15	41874.45	2.75	1	41874.45	1.178	1
26	1	On-Spec	Nonlinear	43934.67	18.51	43934.95	1.71	1	43934.95	1.777	1
26	2	On-Spec	Nonlinear	43934.95	82.32	43934.95	2.04	1	43934.95	2.117	1
27	1	On-Spec	Nonlinear	43627.27	372.59	43627.45	8.77	5	43627.56	2.058	2
27	1	On-Spec	Linear	43142.09	424.95	43142.25	7.8	5	43142.37	2.23	2
28	1	On-Spec	Nonlinear	43611.69	641.76	43667.68	5.02	3	43611.97	0.852	1
28	1	On-Spec	Linear	43101.12	235.61	43126.28	4.33	3	43101.4	0.86	1

Preliminary conclusion: Increase from 1 to 2 blenders leads to 4+ times higher execution times with DICOPT. MPIP increase is less than 1.5.

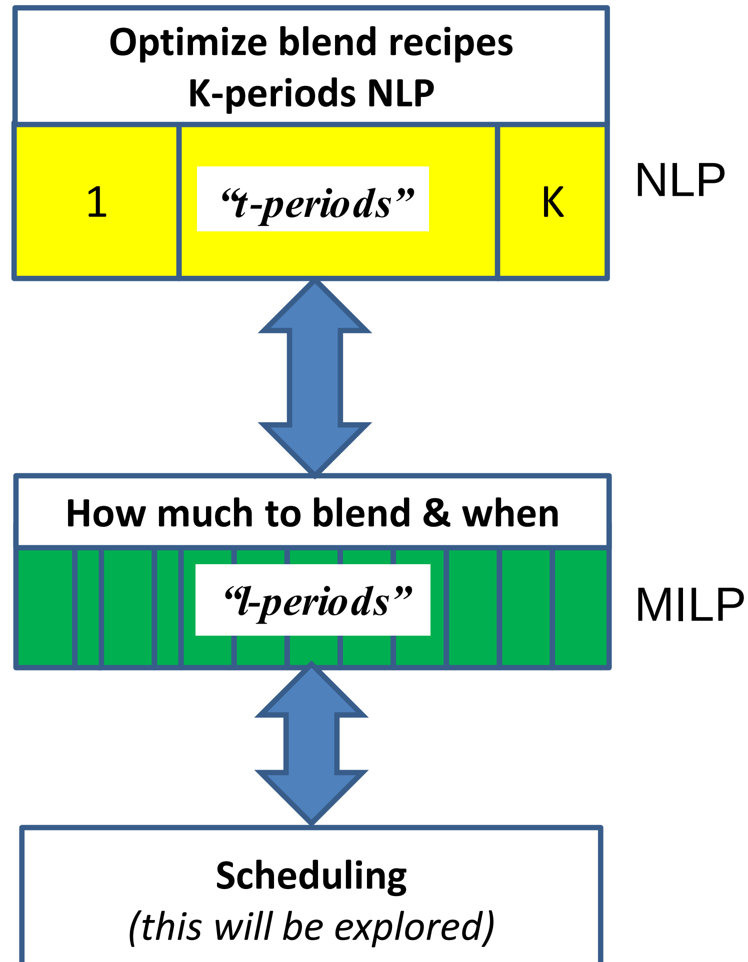
Previous related work

Glismann and Gruhn



- Calendar based periods
- Many different blend recipes in MINLP
- Scheduling based on fixed duration (2hr) periods. Multiple choices of fixed recipes if infeasibility encountered.

Inventory pinch multiperiod model



Scheduling

- We do not have completed the studies.
- Possibility:
  - Can we combine the best of both worlds:
    - Discreet-time inventory pinch delimited planning
    - Continuous time scheduling (with fixed recipes)

  AND have very short execution times.

Does it work for process plants (e.g. refinery) planning?

- Expectation: YES
- Work remains to be completed.
- Impact on practice:
  - Non-linear, computationally intensive models of the plants to be used for production planning with execution times that are much shorter than with the “calendar set” time periods.

Conclusions / 1

- Inventory pinch enables a new decomposition of linear and non-linear (gasoline) production planning problems.
- Multi-period inventory pinch algorithm:
  - Computes the same optimal value of the objective function as MINLP.
  - Compared to multi-period MINLP, the algorithm substantially reduces number of different operating conditions (blend recipes) at which the system needs to operate.
  - Computational times are substantially lower than multiperiod MINLP.
    - Will this lead to more elaborate (more detailed) non-linear refinery production planning (or multi-refinery planning) models?
- Extension to scheduling is yet to be explored.
  - Will this enable us to combine the best of discrete-time and continuous-time approaches?

Conclusions / 2

- Single period inventory pinch algorithm computes objective function optimums that are in most cases identical to multiperiod MINLP.
  - If not optimal, the difference is very small.
  - Is there a way to modify the algorithm to guarantee optimality?
- Is there potential to use existing rigorous simulation and optimization (single period) software for production planning?

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End