Aberration and phase corrections for High Intensity Focused Ultrasound (HIFU)

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Outline

Introduction

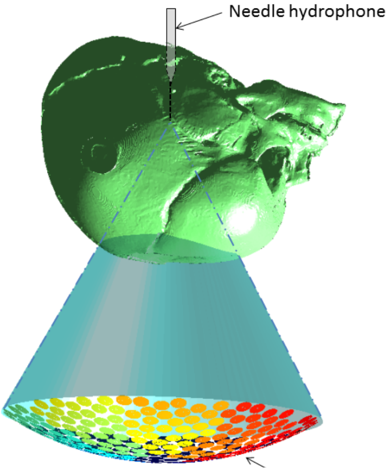


Image courtesy of SickKids hospital.

- Array of ultrasound elements producing a focused beam.
- Tissue heats up at the focal point.
- Propagation of waves through the tissue defocuses the beam.

- Change of phase for each element moves the focal point.
- Compensate for tissue absorbtion.
- Effect of tissue on each element

$$z_j = m_j e^{i\theta_j}, \quad j = 1, \cdots, J \tag{1}$$

- Phase and amplitude of each element

$$\alpha_j = a_j e^{i\phi_j}, \quad j = 1, \cdots, J \tag{2}$$

- Intensity at the focal point

$$I = \left| \sum_j m_j a_j e^{i(\theta_j + \phi_j)} \right|^2 \tag{3}$$

- How do you focus the beam given a finite number of measurements?
- Need the properties of the tissue (parameter estimation).
- Design a set of experiments $\{\mathbf{a}^n, \boldsymbol{\phi}^n\}$

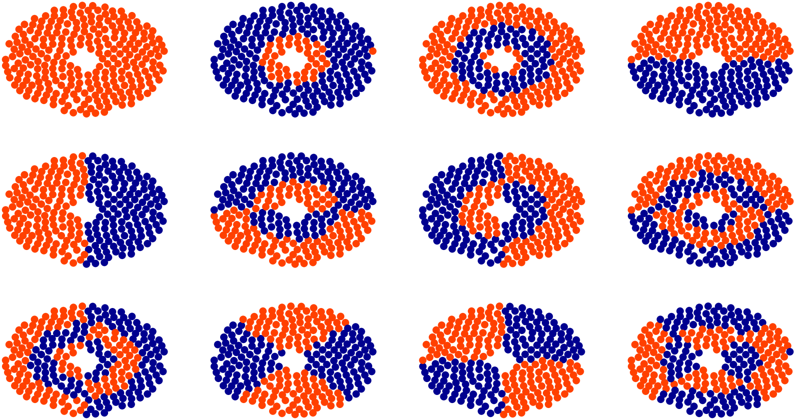
$$\begin{aligned} \mathbf{a}^n &= (a_1^n, a_2^n, \cdots, a_J^n)', \\ \boldsymbol{\phi}^n &= (\phi_1^n, \phi_2^n, \cdots, \phi_J^n)'. \end{aligned} \tag{4}$$

  and measure $\mathbf{d} = I(\mathbf{a}^n, \boldsymbol{\phi}^n)$.
- Estimate the parameters

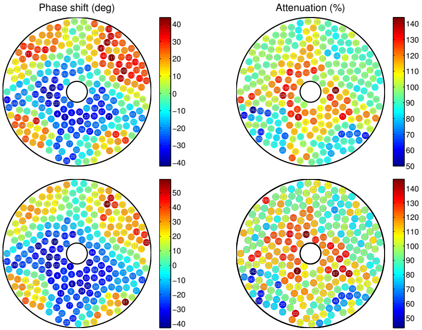
$$\begin{aligned} \mathbf{m} &= (m_1, m_2, \cdots, m_J)', \\ \boldsymbol{\theta} &= (\theta_1, \theta_2, \cdots, \theta_J)'. \end{aligned} \tag{5}$$

Design of experiments

- What is a good choice of $\{\mathbf{a}^n, \boldsymbol{\phi}^n\}$?
- Choosing random experiments gives noisy data.
- Group the elements at different scales.

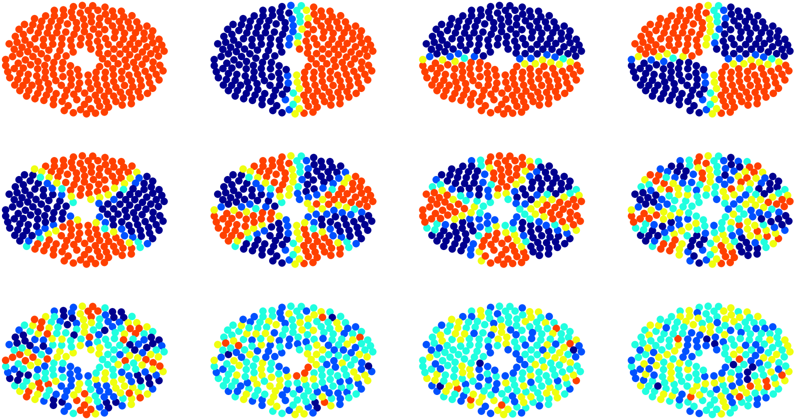


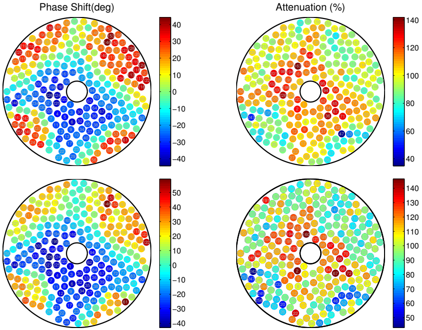
Bessel basis



error (%)	1024 data points	153 Bessel basis
ϕ	5.8	13
$\mathbf{m}$	9	7.6

Ring and sector

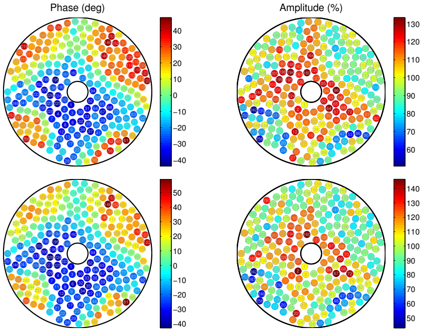




error (%)	1024 data points	169 ring and sectors
ϕ	5.8	9.6
$\mathbf{m}$	9	8.7

Improving previous methods

- The method of Herbet et al. [1] uses four measurements per group of elements (1024).
- We can solve the problem with three measurements (768) with the same total amount of energy.



error (%)	4 meas.	3 meas.
ϕ	5.8	5
$\mathbf{m}$	9	5.5

Optimization algorithm

- Aiming for even less measurements.
- Borrow ideas from convex optimization.
- We want the best match of the data

$$\underset{(\mathbf{m},\boldsymbol{\phi})}{\arg\min}\,\mathcal{J} := \|I(\mathbf{m},\boldsymbol{\phi}) - \mathbf{d}\|_2^2 \tag{6}$$

Optimization algorithm

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$$\underset{(\mathbf{m},\boldsymbol{\phi})}{\arg\min}\, \mathcal{J} := \|I(\mathbf{m},\boldsymbol{\phi}) - \mathbf{d}\|_2^2 + \beta\mathcal{R}(\mathbf{m},\boldsymbol{\phi}) \tag{6}$$

Optimization algorithm

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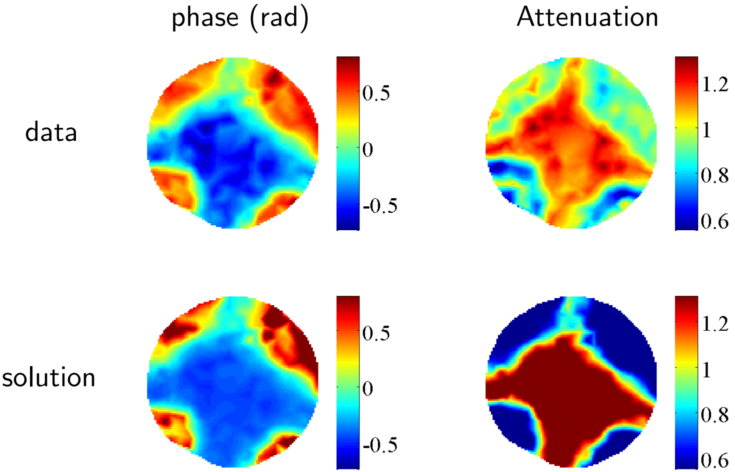
- We want the best match of the data

$$\arg\min_{(\mathbf{m},\boldsymbol{\phi})} \mathcal{J} := \|I(\mathbf{m},\boldsymbol{\phi}) - \mathbf{d}\|_2^2 + \beta\mathcal{R}(\mathbf{m},\boldsymbol{\phi}) \tag{6}$$

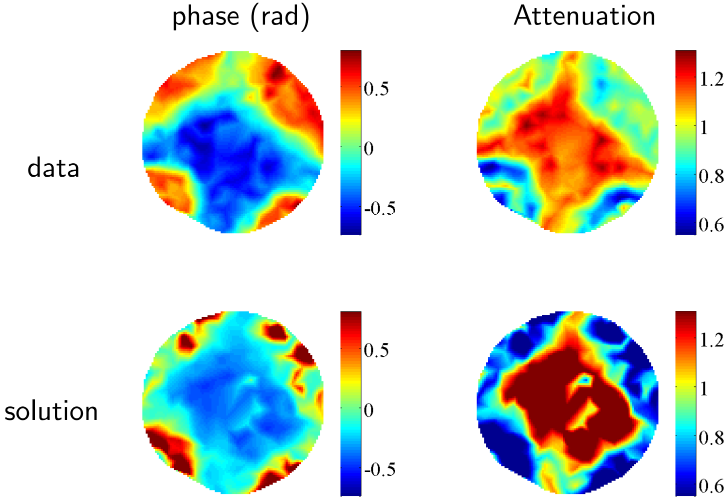
- most simple choice $\mathcal{R} := \|\mathbf{m}\|_2^2 + \|\boldsymbol{\phi}\|_2^2$.
- Good choices still works with insufficient data.

- Apply your favorite optimization algorithm.
- Line search methods (Wright and Nocedal [2]).
    - initial guess $(\mathbf{m}_0, \boldsymbol{\phi}_0)$
    - until convergence repeat:
        - compute gradient $\nabla \mathcal{J}(\mathbf{m}_k, \boldsymbol{\phi}_k)$.
        - choose a search direction $\mathbf{p}_k = B^{-1} \nabla \mathcal{J}(\mathbf{m}_k, \boldsymbol{\phi}_k)$
        - choose step size γ_k.
        - update solution $(\mathbf{m}_{k+1}, \boldsymbol{\phi}_{k+1}) \leftarrow (\mathbf{m}_k, \boldsymbol{\phi}_k) + \gamma_k \mathbf{p}_k$.
- We use steepest descent.

Full data set



Half data set



- More a proof of concept.
- Works with other improvements (ex. grouped measurements, basis reduction, etc).
- less sensitive to noise.
- Can handle insufficient data.

Future work

- Optimal recipe for experiments.
- Different choice of the regularization operator.
- Better optimizer.
- Approximation in smooth basis.

Summary

- Dealing with a parameter estimation problem.
- Proposed measurements have a huge impact.
- Optimal choice of experiments is not trivial.
- An optimization framework shows potential.

References

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