Nonlinear valuation under credit gap risk, collateral margins, funding costs and multiple curves

Fields Institute Seminar in Quantitative Finance

October 31th 2014, Toronto

Damiano Brigo
Chair, Dept. of Mathematics, Imperial College London
and Director of the CAPCO Institute

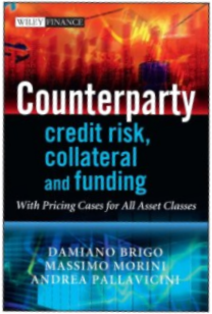
—

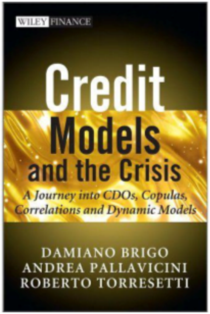
Joint Work with Andrea Pallavicini

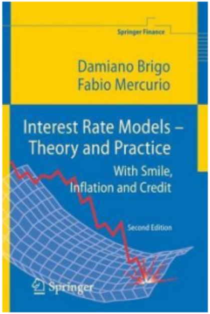
Content I

1. Credit Risk under collateralization
   - CVA, DVA, Collateral and Gap Risk
2. Funding Costs
   - Valuation under Funding Costs
   - The recursive non-decomposable nature of adjusted prices
   - BSDEs, Nonlinear PDEs and an Invariance Theorem
   - Benchmark case: Black Scholes
   - Funding costs, aggregation and nonlinearities
   - NVA
3. Multiple Interest Rate curves
4. CCPs: Initial margins, clearing members defaults, delays...
   - References

See the 2004-2014 papers in the References and Books:







especially the first one (most recent).

CVA, DVA and Collateral

We are a investment bank "I" trading with a counterparty "C".

Credit Valuation Adjustment (CVA)

is the reduction in price we ask to "C" for the fact that "C" may default. See B. and Tarenghi (2004) and B. and Masetti (2005).

Debit Valuation Adjustment (DVA)

is the increase in price we face towards "C" for the fact that *we* may default. See B. and Capponi (2008). In very simple contexts, DVA can also be interpreted as a funding benefit.

CVA/DVA are complex options on netting sets...

containing hundreds of risk factors and with a random maturity given by the first to default between "I" and "C"

CVA, DVA and Collateral

CVA and DVA can be sizeable

Citigroup in its press release on the first quarter revenues of 2009 reported a *positive* mark to market due to its *worsened* credit quality: “Revenues also included [...] a net 2.5$ billion positive CVA on derivative positions, excluding monolines, mainly due to the widening of Citi’s CDS spreads” (DVA)

CVA mark to market losses: BIS

”During the financial crisis, however, roughly two-thirds of losses attributed to counterparty credit risk were due to CVA losses and only about one-third were due to actual defaults.”

Collateral and Gap Risk

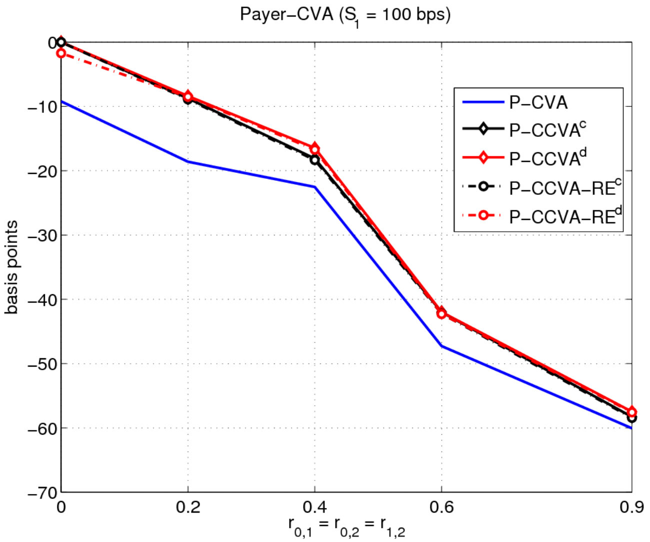
Collateral is a guarantee following mark to market...

and posted from the party that is facing a negative variation of mark to market in favour of the other party. If one party defaults, the other party may use collateral to cover their losses.

However, even under daily collateralization...

there can be large mark to market swings due to contation that make collateral rather ineffective. This is called GAP RISK and is one of the reasons why Central Clearing Counterparties (CCPs) and the new standard CSA have an initial margin as well.

Example of Gap Risk (from B. Capponi Pallavicini (2011)):



Payer–CVA (S_1 = 100 bps)
basis points
0
−10
−20
−30
−40
−50
−60
−70
0
0.2
0.4
0.6
0.9
$r_{0,1} = r_{0,2} = r_{1,2}$
P–CVA
P–CCVA^c
P–CCVA^d
P–CCVA–RE^c
P–CCVA–RE^d

Collateral Management and Gap Risk I

The figure refers to a payer CDS contract as underlying.

See full paper B., Capponi and Pallavicini (2011) for more cases.

Figure: relevant CVA component (part of the bilateral DVA - CVA) starting at 10 and ending up at 60 under high correlation.

Collateral very effective in removing CVA when correlation $= 0$

CVA goes from 10 to 0 basis points.

Collateral not effective as default dependence grows

Collateralized and uncollateralized CVA become closer and for high correlations still get 60 basis points of CVA, even under collateral. Instantaneous contagion $\Rightarrow$ CVA option moneyness jump at default

Inclusion of Funding Cost

When managing a trading position, one needs to obtain cash in order to do a number of operations:

- borrowing / lending cash to implement the replication strategy,
- possibly repo-lending or stock-lending the replication risky asset,
- borrowing cash to post collateral
- receiving interest on posted collateral
- paying interest on received collateral
- using received collateral to reduce borrowing from treasury
- borrowing to pay a closeout cash flow upon default

and so on. Where are such founds obtained from?

Inclusion of Funding Cost

When managing a trading position, one needs to obtain cash in order to do a number of operations:

- borrowing / lending cash to implement the replication strategy,
- possibly repo-lending or stock-lending the replication risky asset,
- borrowing cash to post collateral
- receiving interest on posted collateral
- paying interest on received collateral
- using received collateral to reduce borrowing from treasury
- borrowing to pay a closeout cash flow upon default

and so on. Where are such founds obtained from?

- Obtain cash from her Treasury department or in the market.
- receive cash as a consequence of being in the position.

All such flows need to be remunerated:

- if one is ”borrowing”, this will have a cost,
- and if one is ”lending”, this will provide revenues.

Introduction to Quant. Analysis of Funding Costs I

We now present an introduction to funding costs modeling. Motivation?

Funding Value Adjustment Proves Costly to J.P. Morgan's 4Q Results (Michael Rapoport, Wall St Journal, Jan 14, 2014)

"[...] So what is a funding valuation adjustment, and why did it cost J.P. Morgan Chase $1.5 billion? [...]

J.P. Morgan was persuaded to make the FVA [Funding Valuation Adjustment] change by an industry migration toward such a move [...]

Some banks already recognize funding valuation adjustments, like Royal Bank of Scotland, which recognized FVA losses of 174 million pounds in 2012 and 493 million pounds in 2011. Goldman Sachs says [...] that its derivatives valuations incorporate FVA

Introduction to Quant. Analysis of Funding Costs II

We now approach funding costs modeling by incorporating funding costs into valuation, adding new cash flows.

We start from scratch from the product cash flows and add collateralization, cost of collateral, CVA and DVA after collateral, and funding costs for collateral and for the replication of the product.

In the following τ_I denotes the default time of the investor / bank doing the calculation of the price. "C" denotes the counterparty.

Basic Payout plus Credit and Collateral: Cash Flows I

- We calculate prices by discounting cash-flows under the pricing measure. Collateral and funding are modeled as additional cashflows (as for CVA and DVA)
- We start from derivative's basic cash flows without credit, collateral of funding risks

$$\bar{V}_t \quad := \quad \mathbb{E}_t[\,\Pi(t, T \wedge \tau) + \dots]$$

where

$\longrightarrow$ $\tau := \tau_C \wedge \tau_I$ is the first default time, and
$\longrightarrow$ $\Pi(t, u)$ is the sum of all payoff terms from t to u, discounted at t

Cash flows are stopped either at the first default or at portfolio's expiry if defaults happen later.

Basic Payout plus Credit and Collateral: Cash Flows II

- As second contribution we consider the collateralization procedure and we add its cash flows.

$$\bar{V}_t := \mathbb{E}_t[\,\Pi(t, T \wedge \tau)\,] + \mathbb{E}_t[\,\gamma(t, T \wedge \tau; C) + \dots]$$

where

$\longrightarrow$ C_t is the collateral account defined by the CSA,

$\longrightarrow$ $\gamma(t, u; C)$ are the collateral margining costs up to time u.

- The second expected value originates what is occasionally called Liquidity Valuation Adjustment (LVA) in simplified versions of this analysis. We will show this in detail later.
- If $C > 0$ collateral has been overall posted by the counterparty to protect us, and we have to pay interest c^+.
- If $C < 0$ we posted collateral for the counterparty (and we are remunerated at interest c^-).

Basic Payout plus Credit and Collateral: Cash Flows III

- The cash flows due to the margining procedure on the time grid $\{t_k\}$ are equal to (Linearization of exponential bond formulas in the continuously compounded rates)

$$\gamma(t,u;C) \approx -\sum_{k=1}^{n-1} 1_{\{t \le t_k < u\}} D(t,t_k) C_{t_k} \alpha_k (\tilde{c}_{t_k}(t_{k+1}) - r_{t_k}(t_{k+1}))$$

where $\alpha_k = t_{k+1} - t_k$ and the collateral accrual rates are given by

$$\tilde{c}_t := c_t^+ 1_{\{C_t > 0\}} + c_t^- 1_{\{C_t < 0\}}$$

Note that if the collateral rates in $\tilde{c}$ are both equal to the risk free rate, then this term is zero.

Close-Out: Trading-CVA/DVA under Collateral – I

- As third contribution we consider the cash flow happening at 1st default, and we have

$$\begin{aligned} \bar{V}_t &:= \mathbb{E}_t[\,\Pi(t, T \wedge \tau)\,] \\ &+ \mathbb{E}_t[\,\gamma(t, T \wedge \tau; C)\,] \\ &+ \mathbb{E}_t\big[\,1_{\{\tau<T\}} D(t,\tau)\theta_\tau(C,\varepsilon) + \dots\big] \end{aligned}$$

  where

  $\longrightarrow$ ε_τ is the close-out amount, or residual value of the deal at default, which we called NPV earlier, and

  $\longrightarrow$ $\theta_\tau(C,\varepsilon)$ is the on-default cash flow.

- θ_τ will contain collateral adjusted CVA and DVA payouts for the instument cash flows

Close-Out: Trading-CVA/DVA under Collateral – II

- We define θ_τ including the pre-default value of the collateral account since it is used by the close-out netting rule to reduce exposure
- In case of no collateral re-hypothecation (see full paper for all cases)

$$
\begin{aligned}
\theta_\tau(C,\varepsilon) &:= \varepsilon_\tau - 1_{\{\tau=\tau_C<\tau_I\}}\Pi_{\text{CVAcoll}} + 1_{\{\tau=\tau_I<\tau_C\}}\Pi_{\text{DVAcoll}} \\
\Pi_{\text{CVAcoll}} &= \mathrm{L}_{\text{GD}_C}(\varepsilon_\tau^+ - C_{\tau^-}^+)^+ \\
\Pi_{\text{DVAcoll}} &= \mathrm{L}_{\text{GD}_I}((-\varepsilon_\tau)^+ - (-C_{\tau^-})^+)^+
\end{aligned}
$$

Funding Costs of the Replication Strategy – I

- As fourth and last contribution we consider the cost of funding for the hedging procedures and we add the relevant cash flows.

$$
\begin{aligned}
\bar{V}_t \quad := \quad & \mathbb{E}_t[\,\Pi(t, T \wedge \tau)\,] + \mathbb{E}_t\Big[\,\gamma(t, T \wedge \tau; C) + 1_{\{\tau < T\}} D(t,\tau)\theta_\tau(C,\varepsilon)\,\Big] \\
+ \quad & \mathbb{E}_t[\,\varphi(t, T \wedge \tau; F, H)\,]
\end{aligned}
$$

  The last term, especially in simplified versions, is related to what is called FVA in the industry. We will point this out once we get rid of the rate r.

  $\longrightarrow$ F_t is the cash account for the replication of the trade,
  $\longrightarrow$ H_t is the risky-asset account in the replication,
  $\longrightarrow$ $\varphi(t,u;F,H)$ are the cash F and hedging H funding costs up to u.

- In classical Black Scholes on Equity, for a call option (no credit risk, no collateral, no funding costs),

$$
\bar{V}_t^{\text{Call}} = \Delta_t S_t + \eta_t B_t =: H_t + F_t, \quad \tau = +\infty, \quad C = \gamma = \varphi = 0.
$$

Funding Costs of the Replication Strategy – II

- Continuously compounding format and linearizing exponentials:

$$
\begin{aligned}
\varphi(t,u) &\approx \sum_{j=1}^{m-1} 1_{\{t \le t_j < u\}} D(t,t_j)(F_{t_j} + H_{t_j})\alpha_k \left(r_{t_j}(t_{j+1}) - \tilde{f}_{t_j}(t_{j+1}) \right) \\
&- \sum_{j=1}^{m-1} 1_{\{t \le t_j < u\}} D(t,t_j) H_{t_j}\alpha_k \left(r_{t_j}(t_{j+1}) - \tilde{h}_{t_j}(t_{j+1}) \right) \\
\tilde{f}_t &:= f_t^+ 1_{\{F_t > 0\}} + f_t^- 1_{\{F_t < 0\}} \quad \tilde{h}_t := h_t^+ 1_{\{H_t > 0\}} + h_t^- 1_{\{H_t < 0\}}
\end{aligned}
$$

- The expected value of φ is related to the so called FVA. If the treasury funding rates $\tilde{f}$ are same as asset lending/borrowing $\tilde{h}$

$$
\varphi(t,u) \approx \sum_{j=1}^{m-1} 1_{\{t \le t_j < u\}} D(t,t_j) F_{t_j}\alpha_k \left(r_{t_j}(t_{j+1}) - \tilde{f}_{t_j}(t_{j+1}) \right)
$$

- If further treasury borrows/lends at risk free $\tilde{f} = r \Rightarrow \varphi = FVA = 0$.

Funding Costs of the Replication Strategy – III

Our replica consists in F cash and H risky asset.
Cash is borrowed $F > 0$ from the treasury at an interest f^+ (cost) or is lent $F < 0$ at a rate f^- (revenue)

Risky asset position in the replica is worth H. Cash needed to buy $H > 0$ is borrowed at an interest f from the treasury; in this case H can be used for asset lending (Repo for example) at a rate h^+ (revenue);

Else if risky asset in replica is worth $H < 0$, meaning that we should replicate via a short position in the asset, we may borrow cash from the repo market by posting the asset H as guarantee (rate h^-, cost), and lend the obtained cash to the treasury to be remunerated at a rate f.

It is possible to include the risk of default of the funder and funded, leading to CVA and DVA adjustments for the funding position, see PPB.

Funding rates depend on Treasury policies

- In real applications the funding rate $\tilde{f}_t$ is determined by the party managing the funding account for the investor, eg the bank's treasury:
  - $\longrightarrow$ trading positions may be netted before funding on the mkt
  - $\longrightarrow$ a Funds Transfer Pricing (FTP) process may be implemented to gauge the performances of different business units;
  - $\longrightarrow$ a maturity transformation rule can be used to link portfolios to effective maturity dates;
  - $\longrightarrow$ sources of funding can be mixed into the internal funding curve ...
- In part of the literature the role of the treasury is usually neglected, leading to controversial results particularly when the funding positions are not distinguished from the trading positions.
- See partial claims "funding costs $=$ DVA", or "there are no funding costs", cited in the literature

Recursive non-decomposable Nature of Pricing – I

$$(*) \quad \bar{V}_t = \mathbb{E}_t\left[\, \Pi(t, T \wedge \tau) + \gamma(t, T \wedge \tau) + 1_{\{\tau < T\}} D(t,\tau)\theta_\tau(C,\varepsilon) + \varphi(t, T \wedge \tau) \,\right]$$

Can we interpret:

$$\mathbb{E}_t\left[\, \Pi(t, T \wedge \tau) + 1_{\{\tau < T\}} D(t,\tau)\theta_\tau(C,\varepsilon) \,\right] : \quad \text{RiskFree Price + DVA - CVA?}$$

$$\mathbb{E}_t[\, \gamma(t, T \wedge \tau) + \varphi(t, T \wedge \tau; F, H) \,] : \quad \text{Funding adjustment FVA?}$$

Not really. This is not a decomposition. It is an equation. In fact since

$$\bar{V}_t = F_t + H_t + C_t \quad \text{(re–hypo)}$$

the φ present value depends on future $F_t = \bar{V}_t - H_t + C_t$ and the closeout θ, via ϵ and C, depends on future $\bar{V}$. Terms feed each other: no neat separation. *Recursive pricing: Nonlinear PDE's / BSDEs* for $\bar{V}$

"FinalPrice = RiskFreePrice (+ DVA?) - CVA + FVA" not possible.

See Pallavicini Perini B. (2011, 2012) for $\bar{V}$ equations and algorithms. See also the analysis in Wu (2013).

Recursive non-decomposable Nature of Pricing – II

Write a valuation PDE (or BSDE) by a continuous time limit in the previous equations and by an immersion hypothesis for credit risk.

- We obtain (here $\pi_t\, dt = \Pi(t, t + dt)$)

$$\bar{V}_t = \int_t^T \mathbb{E}\{D(t,u;r+\lambda)[\pi_u + (r_u - \tilde{c}_u)C_u + \lambda_u\theta_u$$

$$+(r_u - \tilde{f}_u)(F_u + H_u) + (\tilde{h}_u - r_u)H_u] | \mathcal{F}_t\}du \quad \text{EQFund1}$$

- We can also write

$$\bar{V}_t = \int_t^T \mathbb{E}\{D(t,u;r+\lambda)[\pi_u + \lambda_u\theta_u + (\tilde{f}_u - \tilde{c}_u)C_u +$$

$$+(r_u - \tilde{f}_u)V_u + (\tilde{h}_u - r_u)H_u] | \mathcal{F}_t\}du \quad \text{EQFund2}$$

Recursive non-decomposable Nature of Pricing – III

- Write this last eq as a BSDEs by completing the martingale term.

$$d\bar{V}_t - (\tilde{f}_t + \lambda_t)\bar{V}_t dt + (\tilde{f}_t - \tilde{c}_t)C_t dt + \pi_t\, dt + \lambda_t \theta(C_t, \bar{V}_t)dt - (r - \tilde{h})H_t dt = dM_t,$$

$$\bar{V}_t = H_t + F_t + C_t, \;\; \varepsilon_t = \bar{V}_t \;\text{ (replacement closeout)}, \quad \bar{V}_T = 0.$$

Recall that $\tilde{f}$ depends on $\bar{V}$ nonlinearly, and so does $\tilde{c}$ on C and $\tilde{h}$ on H. M is a martingale under the pre-default filtration.

BSDEs for valuation under asymmetric borrowing/lending rates

These had been introuced in a short example in El Karoui, Peng and Quenez (1997). We are adding credit gap risk and collateral processes, adding discontinuities and more nonlinearity into the picture.

Recursive non-decomposable Nature of Pricing – IV

- Assume a Markovian vector of underlying assets S (pre- credit and funding) with diffusive generator $\mathcal{L}^{r,\sigma}$ under $\mathbb{Q}$, whose 2nd order part is $\mathcal{L}_2$. Let this be associated with brownian W under $\mathbb{Q}$.

$$dS = rSdt + \sigma(t,S)SdW_t, \quad \mathcal{L}^{r,\sigma}u(t,S) = rS\partial_S u + \frac{1}{2}\sigma(t,S)^2 S^2 \partial_S^2 u$$

- Use Ito's formula on $\bar{V}(t,S)$ and match dt (and dW) terms: obtain PDE (& explicit representation for BSDE term ZdW).
- Assume Delta Hedging: $H_t = S_t \frac{\partial \bar{V}_t}{\partial S}$ This leads to

Recursive non-decomposable Nature of Pricing – V

Nonlinear PDE

$$(\partial_t - \tilde{f}_t - \lambda_t + \mathcal{L}^{\tilde{h},\sigma})\bar{V}_t + (\tilde{f}_t - \tilde{c}_t)C_t + \pi_t + \lambda_t\theta(C_t, \bar{V}_t) = 0, \quad \bar{V}_T = 0.$$

Nonlinearities

This NPDE is NON-LINEAR not only because of θ, but also because $\tilde{f}$ depends on F, and $\tilde{h}$ on H, and hence both on $\bar{V}$ itself.

IMPORTANT INVARIANCE THEOREM:

THIS PDE DOES NOT DEPEND ON r.

This is good, since r is a theoretical rate that does not correspond to any market observable. Only market rates here.

Recursive non-decomposable Nature of Pricing – VI

We may now use nonlinear Feynman Kac to rewrite this last PDE, free from r, as an expected value. We obtain

EQFund3: Deal-dependent probability measure for valuation

$$\bar{V}_t = \int_t^T \mathbb{E}^{\tilde{h}}\{D(t,u;\tilde{f}+\lambda)[\pi_u + \lambda_u\theta_u + (\tilde{f}_u - \tilde{c}_u)C_u]|\mathcal{F}_t\}du$$

Here $\mathbb{E}^h$ is the expected value under a probability measure where the underlying assets evolve with a drift rate (return) of $\tilde{h}$.
Remember: $\tilde{h}$ depends on H, and hence on V.
PRICING MEASURE DEPENDS ON FUTURE VALUES OF THE VERY PRICE V WE ARE COMPUTING. Nonlinear expectation
EVERY DEAL/PORTFOLIO HAS A SEPARATE PRICING MEASURE

By rearranging terms in the previous EQFund1, it is tempting...

Recursive non-decomposable Nature of Pricing – VII

... it is tempting to set $\bar{V}$ = RiskFreePrice + LVA + FVA - CVA + DVA

$$RiskFreePrice = \int_t^T \mathbb{E}\left\{ D(t,u;r) 1_{\{\tau > u\}} \left[\pi_u + \delta_\tau(u) \varepsilon_u \right] | \mathcal{G}_t \right\} du$$

$$LVA = \int_t^T \mathbb{E}\left\{ D(t,u;r) 1_{\{\tau > u\}} (r_u - \tilde{c}_u) C_u | \mathcal{G}_t \right\} du$$

$$FVA = \int_t^T \mathbb{E}\left\{ D(t,u;r) 1_{\{\tau > u\}} \left[(r_u - \tilde{f}_u)(F_u + H_u) + (\tilde{h}_u - r_u) H_u \right] | \mathcal{G}_t \right\} du$$

$$-CVA = \int_t^T \mathbb{E}\left\{ D(t,u;r) 1_{\{\tau > u\}} \left[- 1_{\{u = \tau_C < \tau_I\}} \Pi_{\text{CVAcoll}}(u) \right] | \mathcal{G}_t \right\} du$$

$$DVA = \int_t^T \mathbb{E}\left\{ D(t,u;r) 1_{\{\tau > u\}} \left[1_{\{u = \tau_I < \tau_C\}} \Pi_{\text{DVAcoll}}(u) \right] | \mathcal{G}_t \right\} du$$

Recursive non-decomposable Nature of Pricing – VIII

If we insist in applying these equations, rather than the r-independent NPDE or EQFund3, then we need to find a proxy for r.

$r \approx$ OIS. Further, if we assume $\tilde{h} = \tilde{f}$ then

$$FVA = \int_t^T \mathbb{E}\left\{ D(t,u;r) 1_{\{\tau > u\}} \left[(r_u - \tilde{f}_u) F_u \right] | \mathcal{G}_t \right\} du$$

Notice that when we are borrowing cash F, since usually $f > r$, FVA is negative and is a cost. Also LVA can be negative.

The above decomposition however, as pointed out earlier, only makes sense a posteriori and is not a real decomposition.

Black Scholes PDE + credit, collateral and funding I

NPDE can be further specified by assuming for example $C_t = \alpha_t \bar{V}_t$, with α being $\mathcal{F}_t$ adapted and positive. Assume $\tilde{h} = \tilde{f}$ and

$$\tilde{f}_t = f_+ 1_{F \geq 0} + f_- 1_{F \leq 0}, \;\; \tilde{c}_t = c_+ 1_{\bar{V}_t \geq 0} + c_- 1_{\bar{V}_t \leq 0}, \;\;\; f_{+,-} \text{ and } c_{+,-} \text{ constants.}$$

NONLINEAR PDE (SEMILINEAR)

$$\partial_t V - f_+(V - S_t \partial_S V_t - \alpha V)^+ + f_-(-V + S_t \partial_S V_t + \alpha V)^+ - \lambda_t V +$$

$$+ \frac{1}{2}\sigma^2 S^2 \partial_S^2 V - c_+ \alpha_t (V_t)^+ + c_- \alpha_t (-V_t)^+ + \pi_t + \lambda_t \theta_t(V_t) = 0$$

λ is the first to default intensity, π is the ongoing dividend cash flow process of the payout, θ are the complex optional contractual cash flows at default including CVA and DVA payouts after collateral. c_+ and c_- are the borrowing and lending rates for collateral.

Black Scholes PDE + credit, collateral and funding II

We can use Lipschitz coefficients results to investigate $\exists!$ of viscosity solutions. Classical soultions may also be found but require much stronger assumptions and regularizations.

The equation is consistent with Black Scholes: If

$$f_+ = f_- = r, \ \alpha = 0, \ \lambda = 0$$

we get back

$$\partial_t \ V(t,S) + rS \ \partial_S \ V(t,S) + \frac{1}{2} \ \sigma^2 S^2 \ \partial_S^2 \ V(t,S) = rV(t,S),$$

Nonlinearities due to funding I

NONLINEAR PDEs cannot be solved as Feynman Kac expectations.

Backward Stochastic Differential Equations (BSDEs)

For NPDEs, the correct translation in stochastic terms are BSDEs. The equations have a recursive nature and simulation is quite complicated.

Aggregation–dependent and asymmetric valuation

Worse, the valuation of a portfolio is aggregation dependent and is different for the two parties in a deal. In the classical pricing theory a la Black Scholes, if we have 2 or more derivatives in a portfolio we can price each separately and then add up. Not so with funding. Without funding, the price to one entity is minus the price to the other one. Not so with funding.

Nonlinearities due to funding II

Consistent global modeling across asset classes and risks

Once aggregation is set, funding valuation is non–separable. Holistic consistent modeling across trading desks & asset classes needed

Value depends on specific trading entities and their policies. Often one forces symmetries and linearization to have funding included as discounting and avoid organizational/implementation problems.

NVA

In the recent paper `http://ssrn.com/abstract=2430696` we introduce a Nonlinearity Valuation Adjustment (NVA), which analyzes the double counting involved in forcing linearization. Our numerical examples for simple call options show that NVA can easily reach 2 or 3% of the deal value even in relatively standard settings.

Nonlinearities due to funding III

Equity call option (long or short), $r = 0.01$, $\sigma = 0.25$, $S_0 = 100$, $K = 80$, $T = 3y$, $V_0 = 28.9$ (no credit risk or funding/collateral costs). Precise credit curves are given in the paper.

$$NVA = \bar{V}_0(nonlinear) - \bar{V}_0(\text{linearized})$$

Table: NVA with default risk and collateralization

Funding Rates bps			Default risk, low[a] Long	Default risk, low[a] Short	Default risk, high[b] Long	Default risk, high[b] Short
f^+	f^-	$\hat{f}$				
300	100	200	-3.27 (11.9%)	-3.60 (10.5%)	-3.16 (11.4%)	-3.50 (10.1%)
100	300	200	3.63 (10.6%)	3.25 (11.8%)	3.52 (10.2%)	3.13 (11.3%)

The percentage of the total call price corresponding to NVA is reported in parentheses.

[a] Based on the joint default distribution D_{low} with low dependence.

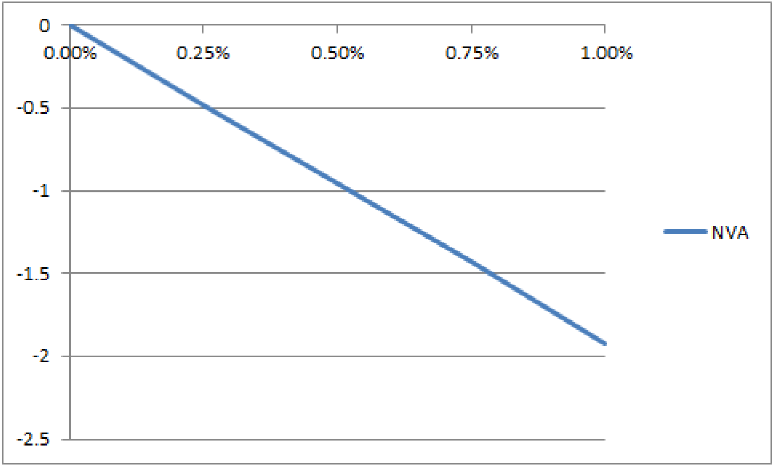
[b] Based on the joint default distribution D_{high} with high dependence.

Nonlinearities due to funding IV

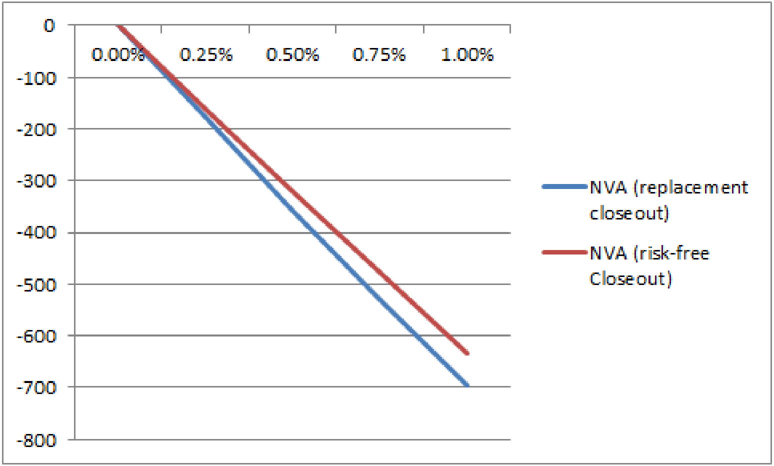
Table: NVA with default risk, collateralization and rehypothecation

Funding Rates bps			Default risk, low[a] Long	Default risk, low[a] Short	Default risk, high[b] Long	Default risk, high[b] Short
f^+	f^-	$\hat{f}$				
300	100	200	-4.02 (14.7%)	-4.45 (12.4%)	-3.91 (14.0%)	-4.35 (12.0%)
100	300	200	4.50 (12.5%)	4.03 (14.7%)	4.40 (12.2%)	3.92 (14.0%)

The percentage of the total call price corresponding to NVA is reported in parentheses.
[a] Based on the joint default distribution D_{low} with low dependence.
[b] Based on the joint default distribution D_{high} with high dependence.



NVA for long call as a function of $f^+ - f^-$, with $f^- = 1\%$, $f+$ increasing over 1% and $\hat{f}$ increasing accordingly. NVA expressed as an additive price component on a notional of 100, risk free option price 29. Risk free closeout. For example, $f + -f- = 25 bps$ results in NVA=-0.5 circa, 50 bps ⇒ NVA = -1



NVA for long call as a function of $f^+ - f^-$, with $f^- = 1\%$, $f+$ increasing over 1% and $\hat{f}$ increasing accordingly. NVA expressed as a percentage (in bps) of the linearized $\hat{f}$ price. For example, $f^+ - f^- = 25$bps results in NVA=-100bps = -1% circa, replacement closeout relevant (red/blue) for large $f^+ - f^-$

Multiple Interest Rate Curves

Derive interest rate dynamics consistently with credit, collateral and funding costs as per the above master valuation equations.

- We use our maket based (no r_t) master equation to price OIS & find OIS equilibrium rates. Collateral fees will be relevant here, driving forward OIS rates.
- Use master equation to price also one period swaps based on LIBOR market rates. LIBORs are market given and not modeled from first principles from bonds etc. Forward LIBOR rates obtained by zeroing one period swap and driven both from primitive market LIBOR rates and by collateral fees.
- We'll model OIS rates and forward LIBOR/SWAP jointly, using a mixed HJM/LMM setup
- In the paper we look at non-perfectly collateralized deals too, where we need to model treasury funding rates.
- See `http://ssrn.com/abstract=2244580`

Pricing under Initial Margins: SCSA and CCPs I

CCPs: Default of Clearing Members, Delays, Initial Margins...

Our general theory can be adapted to price under Initial Margins, both under CCPs and SCSA.

The type of equations is slightly different but quantitative problems are quite similar.

See B. and Pallavicini (2014) for details [35], JFE 1, pp 1-60. Here we give a summary.

Pricing under Initial Margins: SCSA and CCPs II

So far all the accounts that need funding have been included within the funding netting set defining F_t.

If additional accounts needed, for example segregated initial margins, as with CCP or SCSA, their funding costs must be added.

Initial margins kept into a segregated account, one posted by the investor ($N_t^I \leq 0$) and one by the counterparty ($N_t^C \geq 0$):

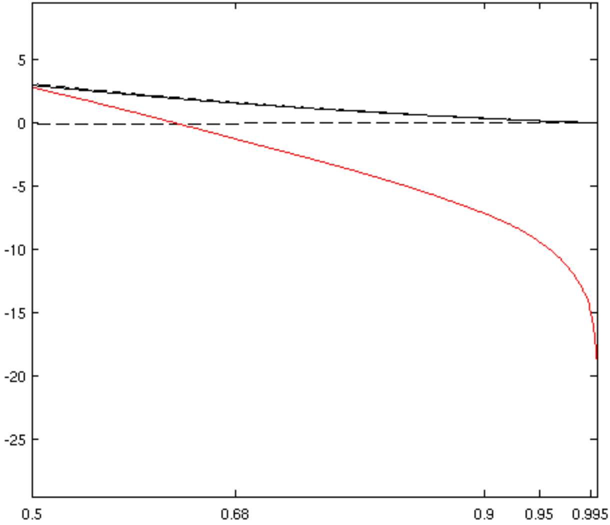
$$\begin{aligned}\varphi(t,u) \;:=\; & \int_t^u dv\,(r_v - f_v)F_v D(t,v) - \int_t^u dv\,(f_v - h_v)H_v D(t,v) \quad (1)\\ + & \int_t^u dv(f_v^{N^C} - r_v)N_v^C + \int_t^u dv(f_v^{N^I} - r_v)N_v^I\,,\end{aligned}$$

with $f_t^{N^C}$ & $f_t^{N^I}$ assigned by the Treasury to the initial margin accounts. $f^N \neq f$ as initial margins not in funding netting set of the derivative.

Pricing under Initial Margins: SCSA and CCPs III

$$\ldots + \int_t^u dv(f_v^{N^C} - r_v)N_v^C + \int_t^u dv(f_v^{N^I} - r_v)N_v^I$$

Assume for example $f > r$. The party that is posting the initial margin has a penalty given by the cost of funding this extra collateral, while the party which is receiving it reports a funding benefit, but only if the contractual rules allow to invest the collateral in low-risk activity, otherwise $f = r$ and there are no price adjustments.
In the paper we also deal with delays in the closeout payments.


5
0
-5
-10
-15
-20
-25
0.5
0.68
0.9
0.95
0.995

CCP Pricing: Figure explanation

Ten-year receiver IRS traded with a CCP

Prices are calculated from the point of view of the CCP client

Mid-credit-risk for CCP clearing member, high for CCP client.

Initial margin posted at various confidence levels q.

Black continuous line: price inclusive of residual CVA and DVA after margining but not funding costs

Dashed black lines represent CVA and the DVA contributions.

red line is the price inclusive both of credit & funding costs. Symmetric funding policy. No wrong way correlation overnight/credit.

Prices in basis points with a notional of one Euro.

CCP Pricing: Tables (see paper for WWR etc)

Table: Prices of a ten-year receiver IRS traded with a CCP (or bilaterally) with a mid-risk parameter set for the clearing member (investor) and a high-risk parameter set for the client (counterparty) for initial margin posted at various confidence levels q. Prices are calculated from the point of view of the client (counterparty). Symmetric funding policy. WWR correlation $\bar{\rho}$ is zero. Prices in basis points with a notional of one Euro.

	Receiver, CCP, $\beta^- = \beta^+ = 1$				Receiver, Bilateral, $\beta^- = \beta^+ = 1$			
q	**CVA**	**DVA**	**MVA**	**FVA**	**CVA**	**DVA**	**MVA**	**FVA**
50.0	-0.126	3.080	0.000	-0.1574	-2.1317	4.3477	0.0000	-0.0842
68.0	-0.066	1.605	-2.933	0.1251	-1.1176	2.2613	-4.1389	0.2491
90.0	-0.015	0.357	-8.037	0.5492	-0.2578	0.4997	-11.3410	0.7924
95.0	-0.007	0.154	-10.316	0.7205	-0.1149	0.2151	-14.5561	1.0250
99.0	-0.001	0.025	-14.590	1.0290	-0.0204	0.0346	-20.5869	1.4544
99.5	-0.001	0.013	-16.154	1.1402	-0.0107	0.0176	-22.7947	1.6107
99.7	-0.000	0.008	-17.233	1.2165	-0.0070	0.0114	-24.3164	1.7184
99.9	-0.000	0.004	-19.381	1.3684	-0.0035	0.0056	-27.3469	1.9326

Thank you for your attention!

Questions?

References I

[1] Albanese, C., Brigo, D., and Oertel, F. (2011). Restructuring Counterparty Credit Risk. Deutsche Bundesbank Working Paper. Forthcoming.

[2] Assefa, S., Bielecki, T. R., Crèpey, S., and Jeanblanc, M. (2009). CVA computation for counterparty risk assessment in credit portfolios. In Recent advancements in theory and practice of credit derivatives, T. Bielecki, D. Brigo and F. Patras, eds, Bloomberg Press.

[3] Basel Committee on Banking Supervision "International Convergence of Capital Measurement and Capital Standards A Revised Framework Comprehensive Version" (2006), "Strengthening the resilience of the banking sector" (2009). Available at `http://www.bis.org`.

References II

[4] Bergman, Y. Z. (1995). Option Pricing with Differential Interest Rates. The Review of Financial Studies, Vol. 8, No. 2 (Summer, 1995), pp. 475-500.

[5] Bianchetti, M. (2010). Two Curves, One Price. Risk, August 2010.

[6] Bianchetti, M. (2012). The Zeeman Effect in Finance: Libor Spectroscopy and Basis Risk Management. Available at arXiv.com

[7] Bielecki, T., and Crepey, S. (2010). Dynamic Hedging of Counterparty Exposure. Preprint.

[8] Bielecki, T., Rutkowski, M. (2001). *Credit risk: Modeling, Valuation and Hedging*. Springer Verlag .

References III

[9] Biffis, E., Blake, D. P., Pitotti L., and Sun, A. (2011). The Cost of Counterparty Risk and Collateralization in Longevity Swaps. Available at SSRN.

[10] Blanchet-Scalliet, C., and Patras, F. (2008). Counterparty Risk Valuation for CDS. Available at defaultrisk.com.

[11] Blundell-Wignall, A., and P. Atkinson (2010). Thinking beyond Basel III. Necessary Solutions for Capital and Liquidity. OECD Journal: Financial Market Trends, No. 2, Volume 2010, Issue 1, Pages 9-33.
Available at
`http://www.oecd.org/dataoecd/42/58/45314422.pdf`

[12] Brigo, D. (2005). Market Models for CDS Options and Callable Floaters. Risk Magazine, January issue

References IV

[13] D. Brigo, Constant Maturity CDS valuation with market models (2006). Risk Magazine, June issue. Earlier extended version available at http://ssrn.com/abstract=639022

[14] D. Brigo (2012). Counterparty Risk Q&A: Credit VaR, CVA, DVA, Closeout, Netting, Collateral, Re-hypothecation, Wrong Way Risk, Basel, Funding, and Margin Lending. Circulated Internally, Banco Popolare (Claudio Nordio)

[15] Brigo, D., and Alfonsi, A. (2005) Credit Default Swaps Calibration and Derivatives Pricing with the SSRD Stochastic Intensity Model, Finance and Stochastic, Vol. 9, N. 1.

[16] Brigo, D., and Bakkar I. (2009). Accurate counterparty risk valuation for energy-commodities swaps. *Energy Risk*, March issue.

References V

[17] Brigo, D., and Buescu, C., and Morini, M. (2011). Impact of the first to default time on Bilateral CVA. Available at arXiv.org

[18] Brigo, D., Buescu, C., Pallavicini, A., and Liu, Q. (2012). Illustrating a problem in the self-financing condition in two 2010-2011 papers on funding, collateral and discounting. Available at ssrn.com and arXiv.org

[19] Brigo, D., and Capponi, A. (2008). Bilateral counterparty risk valuation with stochastic dynamical models and application to CDSs. Working paper available at `http://arxiv.org/abs/0812.3705`. Short updated version in *Risk*, March 2010 issue.

References VI

[20] Brigo, D., Capponi, A., and Pallavicini, A. (2011). Arbitrage-free bilateral counterparty risk valuation under collateralization and application to Credit Default Swaps. To appear in *Mathematical Finance*.

[21] Brigo, D., Capponi, A., Pallavicini, A., and Papatheodorou, V. (2011). Collateral Margining in Arbitrage-Free Counterparty Valuation Adjustment including Re-Hypotecation and Netting. Working paper available at `http://arxiv.org/abs/1101.3926`

[22] Brigo, D., and Chourdakis, K. (2008). Counterparty Risk for Credit Default Swaps: Impact of spread volatility and default correlation. *International Journal of Theoretical and Applied Finance*, **12**, 7.

References VII

[23] D. Brigo, L. Cousot (2006). A Comparison between the SSRD Model and the Market Model for CDS Options Pricing. International Journal of Theoretical and Applied Finance, Vol 9, n. 3

[24] Brigo, D., and El–Bachir, N. (2010). An exact formula for default swaptions pricing in the SSRJD stochastic intensity model. Mathematical Finance, Volume 20, Issue 3, Pages 365-382.

[25] Brigo, D., and Masetti, M. (2005). Risk Neutral Pricing of Counterparty Risk. In Counterparty Credit Risk Modelling: Risk Management, Pricing and Regulation, *Risk Books*, Pykhtin, M. editor, London.

[26] Brigo, D., Mercurio, F. (2001). Interest Rate Models: Theory and Practice with Smile, Inflation and Credit, Second Edition 2006, Springer Verlag, Heidelberg.

References VIII

[27] Brigo, D., Morini, M. (2006) Structural credit calibration, *Risk*, April issue.

[28] Brigo, D., and Morini, M. (2010). Dangers of Bilateral Counterparty Risk: the fundamental impact of closeout conventions. Preprint available at `ssrn.com` or at `arxiv.org`.

[29] Brigo, D., and Morini, M. (2010). Rethinking Counterparty Default, Credit flux, Vol 114, pages 18–19

[30] Brigo D., Morini M., and Tarenghi M. (2011). Equity Return Swap valuation under Counterparty Risk. In: Bielecki, Brigo and Patras (Editors), Credit Risk Frontiers: Sub- prime crisis, Pricing and Hedging, CVA, MBS, Ratings and Liquidity, Wiley, pp 457–484

References IX

[31] Brigo, D., and Nordio, C. (2010). Liquidity Adjusted Market Risk Measures with Random Holding Period. Available at SSRN.com, arXiv.org, and sent to BIS, Basel.

[32] Brigo, D., and Pallavicini, A. (2007). Counterparty Risk under Correlation between Default and Interest Rates. In: Miller, J., Edelman, D., and Appleby, J. (Editors), Numercial Methods for Finance, Chapman Hall.

[33] D. Brigo, A. Pallavicini (2008). Counterparty Risk and Contingent CDS under correlation, Risk Magazine, February issue.

[34] Brigo, D., and Pallavicini, A. (2014). CCP Cleared or Bilateral CSA Trades with Initial/Variation Margins under credit, funding and wrong-way risks: A Unified Valuation Approach. SSRN.com and arXiv.org

References X

[35] Brigo, D. and A. Pallavicini (2014). Nonlinear consistent valuation of CCP cleared or CSA bilateral trades with initial margins under credit, funding and wrong-way risks. Journal of Financial Engineering 1 (1), 1-60.

[36] Brigo, D., Pallavicini, A., and Papatheodorou, V. (2011). Arbitrage-free valuation of bilateral counterparty risk for interest-rate products: impact of volatilities and correlations, International Journal of Theoretical and Applied Finance, 14 (6), pp 773–802

[37] Brigo, D., Pallavicini, A., and Torresetti, R. (2010). Credit Models and the Crisis: A journey into CDOs, Copulas, Correlations and Dynamic Models. Wiley, Chichester.

References XI

[38] Brigo, D. and Tarenghi, M. (2004). Credit Default Swap Calibration and Equity Swap Valuation under Counterparty risk with a Tractable Structural Model. Working Paper, available at `www.damianobrigo.it/cdsstructural.pdf`. Reduced version in *Proceedings of the FEA 2004 Conference at MIT, Cambridge, Massachusetts, November 8-10* and in *Proceedings of the Counterparty Credit Risk 2005 C.R.E.D.I.T. conference*, Venice, Sept 22-23, Vol 1.

[39] Brigo, D. and Tarenghi, M. (2005). Credit Default Swap Calibration and Counterparty Risk Valuation with a Scenario based First Passage Model. Working Paper, available at `www.damianobrigo.it/cdsscenario1p.pdf` Also in: *Proceedings of the Counterparty Credit Risk 2005 C.R.E.D.I.T. conference*, Venice, Sept 22-23, Vol 1.

References XII

[40] Burgard, C., Kjaer, M. (2010). PDE Representations of Options with Bilateral Counterparty Risk and Funding Costs Available at ssrn.com.

[41] Burgard, C., Kjaer, M. (2011). In the Balance Available at ssrn.com.

[42] Canabarro, E., and Duffie, D.(2004). Measuring and Marking Counterparty Risk. In Proceedings of the Counterparty Credit Risk 2005 C.R.E.D.I.T. conference, Venice, Sept 22–23, Vol 1.

[43] Canabarro, E., Picoult, E., and Wilde, T. (2005). Counterparty Risk. *Energy Risk*, May issue.

[44] Castagna, A. (2011). Funding, Liquidity, Credit and Counterparty Risk: Links and Implications, Available at http://ssrn.com/abstract=1855028

References XIII

[45] G. Cesari, J. Aquilina , N. Charpillon, Z. Filipovic, G. Lee and I. Manda (2010). Modelling, Pricing, and Hedging Counterparty Credit Exposure: A Technical Guide, Springer Verlag, Heidelberg.

[46] Cherubini, U. (2005). Counterparty Risk in Derivatives and Collateral Policies: The Replicating Portfolio Approach. In: *ALM of Financial Institutions* (Editor: Tilman, L.), Institutional Investor Books.

[47] Collin-Dufresne, P., Goldstein, R., and Hugonnier, J. (2002). A general formula for pricing defaultable securities. Econometrica 72: 1377-1407.

[48] Crépey, S. (2011). A BSDE approach to counterparty risk under funding constraints. Available at `grozny.maths.univ-evry.fr/pages_perso/crepey`

References XIV

[49] Crépey S., Gerboud, R., Grbac, Z., Ngor, N. (2012a). Counterparty Risk and Funding: The Four Wings of the TVA. Available at arxiv.org.

[50] Crouhy M., Galai, D., and Mark, R. (2000). A comparative analysis of current credit risk models. *Journal of Banking and Finance* 24, 59-117

[51] Danziger, J. (2010). Pricing and Hedging Self Counterparty Risk. Presented at the conference "Global Derivatives", Paris, May 18, 2010.

[52] Duffie, D., and Huang, M. (1996). Swap Rates and Credit Quality. Journal of Finance 51, 921–950.

References XV

[53] Duffie, D., and Zhu H. (2010). Does a Central Clearing Counterparty Reduce Counterparty Risk? Working Paper, Stanford University.

[54] Ehlers, P. and Schoenbucher, P. (2006). The Influence of FX Risk on Credit Spreads, ETH working paper, available at defaultrisk.com

[55] Fries, C. (2010). Discounting Revisited: Valuation Under Funding, Counterparty Risk and Collateralization. Available at SSRN.com

[56] Fujii, M., Shimada, Y., and Takahashi, A. (2010). Collateral Posting and Choice of Collateral Currency. Available at `ssrn.com`.

[57] J. Gregory (2009). "Being two faced over counterparty credit risk", *Risk Magazine* 22 (2), pages 86-90.

References XVI

[58] J. Gregory (2010). Counterparty Credit Risk: The New Challenge for Global Financial Markets, Wiley, Chichester.

[59] Hull, J., White, A. (2012). The FVA debate *Risk Magazine*, 8

[60] ISDA. "Credit Support Annex" (1992), 'Guidelines for Collateral Practitioners"(1998), "Credit Support Protocol" (2002), "Close-Out Amount Protocol" (2009), "Margin Survey" (2010), "Market Review of OTC Derivative Bilateral Collateralization Practices" (2010). Available at `http://www.isda.org`.

[61] Jamshidian, F. (2002). Valuation of credit default swap and swaptions, FINANCE AND STOCHASTICS, 8, pp 343–371

[62] Jones, E. P., Mason, S.P., and Rosenfeld, E. (1984). Contingent Claims Analysis of Corporate Capital Structure: An Empirical Investigation. *Journal of Finance* 39, 611-625

References XVII

[63] Jorion, P. (2007). Value at Risk, 3-d edition, McGraw Hill.

[64] Keenan, J. (2009). Spotlight on exposure. *Risk* October issue.

[65] Kenyon, C. (2010). Completing CVA and Liquidity: Firm-Level Positions and Collateralized Trades. Available at `arXiv.org`.

[66] McNeil, A. J., Frey, R., and P. Embrechts (2005). Quantitative Risk Management: Concepts, Techniques, and Tools, Princeton university press.

[67] Leung, S.Y., and Kwok, Y. K. (2005). Credit Default Swap Valuation with Counterparty Risk. The Kyoto Economic Review 74 (1), 25–45.

References XVIII

[68] Lipton A, Sepp A, Credit value adjustment for credit default swaps via the structural default model, The Journal of Credit Risk, 2009, Vol:5, Pages:123-146

[69] Lo, C. F., Lee H.C. and Hui, C.H. (2003). A Simple Approach for Pricing Barrier Options with Time-Dependent Parameters. *Quantitative Finance* 3, 98-107

[70] Merton R. (1974) On the Pricing of Corporate Debt: The Risk Structure of Interest Rates. *The Journal of Finance* 29, 449-470.

[71] Moreni, N. and Pallavicini, A. (2010). Parsimonious HJM Modelling for Multiple Yield-Curve Dynamics. Accepted for publication in Quantitative Finance.

References XIX

[72] Moreni, N. and Pallavicini, A. (2012). Parsimonious Multi-Curve HJM Modelling with Stochastic Volatility, in: Bianchetti and Morini (Editors), Interest Rate Modelling After The Financial Crisis, Risk Books.

[73] Morini, M. (2011). Understanding and Managing Model Risk. Wiley.

[74] Morini, M. and Brigo, D. (2011). No-Armageddon Measure for Arbitrage-Free Pricing of Index Options in a Credit Crisis, Mathematical Finance, 21 (4), pp 573–593

[75] Morini, M. and Prampolini, A. (2011). Risky Funding: A Unified Framework for Counterparty and Liquidity Charges, Risk Magazine, March 2011 issue.

References XX

[76] Pallavicini, A. (2010). Modelling Wrong-Way Risk for Interest-Rate Products. Presented at the conference “6th Fixed Income Conference”, Madrid, 23-24 September 2010.

[77] A. Pallavicini, and D. Brigo (2013). Interest-Rate Modelling in Collateralized Markets: Multiple curves, credit-liquidity effects, CCPs. SSRN.com and arXiv.org

[78] Pallavicini, A., Perini, D., and Brigo, D. (2011). Funding Valuation Adjustment consistent with CVA and DVA, wrong way risk, collateral, netting and re-hypotecation. Available at SSRN.com and arXiv.org

References XXI

[79] Pallavicini, A., Perini, D., and Brigo, D. (2012). Funding, Collateral and Hedging: uncovering the mechanics and the subtleties of funding valuation adjustments. Available at SSRN.com and arXiv.org

[80] Parker E., and McGarry A. (2009) The ISDA Master Agreement and CSA: Close-Out Weaknesses Exposed in the Banking Crisis and Suggestions for Change. Butterworths Journal of International Banking Law, 1.

[81] Picoult, E. (2005). Calculating and Hedging Exposure, Credit Value Adjustment and Economic Capital for Counterparty Credit Risk. In Counterparty Credit Risk Modelling (M. Pykhtin, ed.), Risk Books.

References XXII

[82] Piterbarg, V., (2010). Funding beyond discounting: collateral agreements and derivatives pricing. Risk Magazine, February 2010.

[83] Pollack, Lisa (2012). Barclays visits the securitisation BISTRO. Financial Times Alphaville Blog, Posted by Lisa Pollack on Jan 17, 11:20.

[84] Pollack, Lisa (2012b). The latest in regulation-induced innovation Part 2. Financial Times Alphaville Blog, Posted by Lisa Pollack on Apr 11 16:50.

[85] Pollack, Lisa (2012c). Big banks seek regulatory capital trades. Financial Times Alphaville Blog, Posted by Lisa Pollack on April 29, 7:27 pm.

References XXIII

[86] Rosen, D., and Pykhtin, M. (2010). Pricing Counterparty Risk at the Trade Level and CVA Allocations. Journal of Credit Risk, vol. 6 (Winter 2010), pp. 3-38.

[87] Sorensen, E.H., and Bollier, T. F. (1994). Pricing Swap Default Risk. *Financial Analysts Journal*, 50. 23–33.

[88] Watt, M. (2011). Corporates fear CVA charge will make hedging too expensive. Risk Magazine, October issue.

[89] Weeber, P., and Robson E. S. (2009) Market Practices for Settling Derivatives in Bankruptcy. ABI Journal, 9, 34–35, 76–78.

References XXIV

[90] F. M. Ametrano and M. Bianchetti.
Bootstrapping the illiquidity: Multiple yield curves construction for market coherent forward rates estimation.
In F. Mercurio, editor, *Modeling Interest Rates: Latest Advances for Derivatives Pricing*. Risk Books, 2009.

[91] M. Brunnermeier and L. Pedersen.
Market liquidity and funding liquidity.
The Review of Financial Studies, 22 (6), 2009.

[92] J. Eisenschmidt and J. Tapking.
Liquidity risk premia in unsecured interbank money markets.
Working Paper Series European Central Bank, 2009.

References XXV

[93] M. Fujii and A. Takahashi.
Clean valuation framework for the usd silo.
Working Paper, 2011a.
URL ssrn.com.

[94] M. Fujii and A. Takahashi.
Collateralized cds and default dependence.
Working Paper, 2011b.
URL ssrn.com.

[95] D. Heller and N. Vause.
From turmoil to crisis: Dislocations in the fx swap market.
BIS Working Paper, 373, 2012.

References XXVI

[96] M. Henrard.
The irony in the derivatives discounting part ii: The crisis.
ssrn, 2009.

[97] A. Pallavicini and M. Tarenghi.
Interest-rate modelling with multiple yield curves.
2010.

[98] C. Pirrong.
The economics of central clearing: Theory and practice.
ISDA Discussion Papers Series, 2011.