Macroprudential capital requirements and systemic risk*

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Abstract

When regulating banks based on their contribution to the overall risk of the banking system we have to consider that the risk of the banking system as well as each bank's risk contribution changes once bank equity capital gets reallocated. We define macroprudential capital requirements as the fixed point at which each bank's capital requirement equals its contribution to the risk of the system under the proposed capital requirements. This study uses two alternative models, a network based framework and a Merton model, to measure systemic risk and how it changes with bank capital and allocates risk to individual banks based on five risk allocation mechanisms used in the literature. Using a sample of Canadian banks we find that macroprudential capital allocations can differ by as much as 70% from observed capital levels, are not trivially related to bank size or individual bank default probability, increase in interbank assets, and differ substantially from a simple risk attribution analysis. We further find that across both models and all risk allocation mechanisms that macroprudential capital requirements reduce the default probabilities of individual banks as well as the probability of a systemic crisis by about 25%. Macroprudential capital requirements are robust to model risk and are positively correlated to future capital raised by banks as well as future losses in equity value. Our results suggest that financial stability can be substantially enhanced by implementing a systemic perspective on bank regulation.
Keywords: Systemic Risk, Financial Stability, Bank regulation, Risk Management, interbank Market
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Under our plan ... financial firms will be required to follow the example of millions of families across the country that are saving more money as a precaution against bad times. They will be required to keep more capital and liquid assets on hand and, importantly, the biggest, most interconnected firms will be required to keep even bigger cushions.

US Treasury Secretary Geithner (2009)

The recent financial crisis has demonstrated the adverse effects of a large scale breakdown in financial intermediation for banks as well as the overall economy. Government intervention and the bailouts of failed institutions were driven by a concern that the default of a large bank would trigger a chain reaction of insolvencies in the financial sector, causing far greater damage to the banking system than the initial shock. Through contagion, systemic risk is created endogenously within the banking system on top of the risk from the banking sector's outside investments. However, the adverse consequences that a bank's failure or financial distress brings for other banks as well as the economy as a whole are not considered in current bank regulation. All bank regulation is currently aimed at the individual bank level, even though academics, international institutions, and central bankers have argued for some time that bank regulation should be designed from a system, or macroprudential perspective (Borio (2002), Hanson, Kashyap, and Stein (2010)). Macroprudential capital requirements require each bank to hold a buffer of equity capital that corresponds to the bank's contribution to the overall risk of the system. These capital requirements force a bank to internalize some of the externalities that it creates for the banking system and thus reduce the endogenously created component of systemic risk.

When designing regulation based on capital requirements, however, we must bear in mind that any reallocation of bank capital in the system will also have a profound impact on the risk of the system itself. Banks are highly interconnected and levered, and insufficient capital might cause a bank to default and subsequently cause other banks to default. A bank's probability of default is therefore not only driven by its own capitalization but also by the capitalization of the other banks in the system. Changing individual banks' capital requirements will change the banks' default probabilities, their default correlations, and thus change the overall risk of the banking system and the risk contributions of its member banks. In specifying macroprudential capital requirements we therefore have to follow an iterative procedure to solve for a fixed point at which each bank's capital is consistent its contribution to the total risk of the banking system under the proposed capital allocation. To the best of our knowledge, this is the first paper to derive capital requirements with explicitly considering the endogeneity of overall systemic risk. We thus extend a traditional risk attribution analysis that measures systemic risk and risk contributions for the currently observed capital levels by one important step to define consistent macroprudential capital requirements.

In this paper we derive macroprudential capital requirements as a fixed point using five commonly used approaches to measure each bank's contribution to overall risk: component and incremental value-at-risk from the risk management literature (Jorion (2007)), two allocation mechanisms using Shapley values, and the ΔCoVaR measure introduced by Adrian and Brunnermeier (2010). To find the fixed point at which risk contributions equal capital requirements we need a model to analyze how overall risk in the banking system changes when capital requirements change. To ensure that our findings are not driven by model characteristics we use a Merton model, which is estimated from stock prices, as well as a structured network model, which is calibrated using regulatory data on bank loan portfolios and interbank exposures. Using the Merton model we estimate the market value of bank assets and their covariances from stock prices and then draw scenarios from the joint distribution of asset values to measure probabilities of joint default.

The network model can take advantage of our unique data set and explicitly models spillover and contagion effects through network and asset fire sale externalities similar to the models used by many central banks to assess systemic stability. We start with a macro stress scenario under which PDs across all sectors of the economy increase affecting all banks' loan portfolios. Conditional on the macroeconomic shock, we simulate loan losses for each bank using a portfolio credit risk model. Using a model similar to Cifuentes, Shin, and Ferrucci (2005) we assume that banks with an insufficient regulatory capital ratio start selling assets to a market with inelastic demand and the resulting drop in prices forces other banks to sell assets as well. Banks that default either because of loan losses or decreasing asset valuations are not able to fully honor their obligations on the interbank market and can cause the contagious default of other banks. Clearing in the interbank market is modeled using a network model as in Eisenberg and Noe (2001). The spillover effects from asset fire sales and contagious defaults make the correlation of bank defaults dependent on the health of the overall financial system. For a given set of capital requirements, both models allow us to simulate the joint distributions of bank losses and defaults that will then be used to compute each bank's risk contribution. Iterating over capital requirements until they are equal to risk contributions yields the consistent macroprudential capital requirements.

We use a unique data set of the six largest Canadian banks as a representation of the whole Canadian banking system since they hold over 90% of all banking assets. Our sample contains detailed information on the composition of loan books, including the largest loan exposures of individual banks. The dataset includes the full network of exposures from OTC derivatives as well as exposures between banks arising from traditional interbank lending and cross-shareholdings. While derivatives are often blamed for creating systemic risk, the lack of data in many countries (including the U.S.) makes it hard to verify. Our expanded dataset enables us to better capture linkages between banks and contagious bank defaults.

Holding the amount of overall capital in the banking system constant, we find that across both models and all risk allocation mechanisms, macroprudential capital requirements reduce the default risk of the average bank as well as the probability of multiple bank defaults by up to 25%. Macroprudential capital allocations differ from current observed capital levels by up to 70%, and are not trivially related to bank size, bank PD, or risk weighted assets. We also find that setting bank capital requirements based on a risk attribution analysis will lead to substantially different results than computing macrporudential capital requirements using the fixed point. Capital requirements under a risk attribution analysis differ from the fixed point about as much as the fixed point differs from observed capital levels. The ratio of macroprudential capital requirement over currently observed capital is also a useful predictor of future bank risk. For the network model is positively correlated with out of sample future losses in bank stock prices as well future capital raised by banks. We test for model risk by computing bank default probabilities with one model using the macroprudential capital requirements of the other model and still find that bank default probabilities decrease. While different risk allocation mechanisms result in slightly different macroprudential capital requirements, we find that all of them work almost equally well. Our results support efforts by regulators to move the current regulatory regime towards any macroprudential approach.

In the literature we find two main approaches to measure and allocate systemic risk. Most studies use stock market data to get information on banks' correlation structure and potential spillovers. Adrian and Brunnermeier (2010) propose the ΔCoVaR measure, which they compute for a panel of financial institutions and regress on bank characteristics. Acharya, Pedersen, Philippon, and Richardson (2010) use systemic expected shortfall to compute risk attributions for a large sample of US banks.[1] This literature is related to existing studies of contagion in fi-

[1] See also Billio, Getmansky, Lo, and Pelizzon (2010) for a contagion analysis of banks, brokers, and insurance companies, and Jorion and Zhang (2009) for an analysis of contagion risk following bankruptcies.

nancial markets (see among others Forbes and Rigobon (2002), Bae, Karolyi, and Stulz (2003)). Another stream of research builds on a network model in conjunction with an interbank clearing algorithm introduced by Eisenberg and Noe (2001). Elsinger, Lehar, and Summer (2006) and Aikman, Alessandri, Eklund, Gai, Kapadia, Martin, Mora, Sterne, and Willison (2009) use data sets of interbank linkages for the Austrian and British banking system, respectively, and compute measures of systemic risk and systemic importance for individual banks, conditional on a forward-looking stress scenario. Tarashev, Borio, and Tsatsaronis (2009) conduct a simulation study of a stylized banking system and find that the systemic importance of an institution increases in its size as well as its exposure to common risk factors. They use Shapley values to allocate risk measured by value-at-risk as well as expected loss.

These two complementary approaches can be interpreted in light of economic theories of financial amplification mechanisms at work during a financial crisis. For example, the seminal paper by Allen and Gale (1994) shows how asset prices can be optimally determined by cash-in-the-market pricing in a crisis period. Allen and Gale (2000) propose a model of contagion through a network of interbank exposures. Shin (2008) develops a theory of liquidity spillover across a network of financial institutions resulting from expansions and contractions of balance sheets over the credit cycle. Krishnamurthy (2010) reviews the literature on the mechanisms involving balance-sheet, asset prices, and investors' Knightian uncertainty.

We extend previous research in two ways: first we highlight that changing capital requirements change the risk and correlation structure in the banking system and that macroprudential capital requirements have to be seen as a fixed point problem. Second, we provide empirical evidence that macroprudential capital requirements can reduce individual as well as systemic risk using actual data for a whole banking system. The paper is organized as follows. Section 1 describes the approaches to assign macroprudential capital requirements, the models for assessing systemic risk are described in Section 2. We present the results in Section 3 and conclude in Section 4.

1 Macroprudential capital requirements

Setting macroprudential capital requirements raises two fundamental questions. First, what is the total level of capital required in the banking system, which determines the overall magnitude of the shock that a banking system can withstand? Second, how to break down the overall risk of the banking system and set capital requirements equal to each banks' contribution to systemic risk? The first question is a policy decision balancing efficiency of financial intermediation with overall stability of the system which we do not address in this paper. We focus on the second question by comparing alternative mechanisms to allocate a given amount of capital among banks to reduce systemic risk.

Macroprudential capital requirements differ from risk contribution analysis as it is used in portfolio or risk management. In a risk management or portfolio management setting we want to compute risk contributions of assets for a given portfolio with an exogenous level of overall risk. In a banking system both the overall risk and each bank's contribution depend on the capital allocation. As banks hold more capital they are less likely to default through either direct losses or contagion. Reallocating bank capital changes the overall risk of the banking system and thus each bank's risk contribution.[2]

Estimating macroprudential capital requirements is therefore a fixed point problem. We have to reallocate bank capital such that the risk contribution of each of the n banks to total risk equals the allocated capital. We need a model to measure by how much the risk of the system changes once we reallocate capital. Computing macroprudential capital requirements can thus not be done as a purely empirical exercise. Assume that there is a model, like the ones in this paper, that estimates a banking systems' joint loss distribution $\Sigma(C)$ for a given vector of bank capital endowments $C = (C_1, ..., C_n)$. A risk allocation mechanism $f(\Sigma)$ then attributes the overall risk $\Sigma(C)$ to individual banks. A consistent capital allocation C^* must then satisfy

$$C^* = f(\Sigma(C^*)). \tag{1}$$

The difference between performing a risk attribution analysis, which computes $f(\Sigma(C^0))$ for currently observed capital levels C^0, and the fixed point can be substantial. In Section 3.2 we document that capital requirements based on risk attribution analysis can differ from adjustments based on the fixed point by a factor of more than three.

[2]Changing bank capital requirements might also change individual bank risk through another channel: capital requirements might make certain assets more or less attractive and thus can create a long-term incentive to change banks asset portfolios. We cannot control for incentives to change the banks asset portfolio since we cannot observe banks' investment opportunity set. Most empirical papers in the literature face this problem. We consider this channel of second order importance for our analysis for two reasons: First, macroprudential capital requirements can be continuously adjusted as banks' asset portfolios change. Most banks report their asset portfolio composition very frequently to supervisors and a large, well diversified commercial bank's loan portfolio cannot be fundamentally changed quickly. Second, we believe that the direct effect that changes in capital have on bank solvency risk outweigh the indirect incentive effects on banks' optimal asset choice.

Because of the non-linearity in our models, the fixed point in equation (1) can only be found numerically.[3] Our models, which we describe in detail in Section 2, are simulation based. For each simulated scenarios $s, s = 1, \dots, m$, we record the profit or loss $l_{i,s}$ for each bank i to get the joint loss distribution for all banks, i.e. we get an $n \times m$ matrix of losses, which we call L. We then allocate the risk of the whole system to each individual bank using different risk allocation mechanisms $f(.)$.

Next we review the risk allocation mechanisms that we use to compute macroprudential capital requirements.

1.1 Component value-at-risk (beta)

Following Jorion (2007) we compute the contribution of each bank to overall risk as the beta of the losses of each bank with respect to the losses of a portfolio of all banks. Let $l_{i,s}$ be the loss of bank i in scenario s and $l_{p,s} = \sum_i l_{i,s}$, then $\beta_i = \frac{cov(l_i, l_p)}{\sigma^2(l_p)}$. Furthermore let C_i be the current capital allocation of bank i. We reallocate the total capital in the banking system according to the following risk sharing rule

$$C_i^{\beta} = \beta_i \sum_{i=1}^{n} C_i. \tag{2}$$

where C_i^{β} is the reallocated capital of bank i. A nice property of this risk allocation mechanism is that the sum of the betas equals one, which makes the redistribution of total capital amongst the banks straightforward.

1.2 Incremental value-at-risk

We first compute the value-at-risk (VaR) of the joint loss distribution of the whole banking system, which we get by adding the individual losses across banks in each simulated scenario. We chose a confidence level of 99.5% and run 1,000,000 scenarios. The portfolio VaR, *VaR*$_p$, is therefore the 5,000th largest loss of the aggregate losses l_p. Next we compute the VaR of the joint distribution of all banks except bank i, *VaR*$^{-i}$, as the 5,000th largest value of the

[3] We find that it takes on average 20 iterations until the norm of the changes in capital requirements from one iteration to the next is less that $500,000. Due to the non-linearity of the problem we cannot prove that the fixed point is unique, but as detailed in Appendix A we check for robustness by using alternative starting values and find convergence to the same point for our data.

$l_s^{-i} = \sum_{j=1, j\neq i}^{n} l_{j,s}$. The incremental VaR for bank i, $iVaR_i$, is then defined as

$$iVaR_i = VaR_p - VaR^{-i}. \tag{3}$$

The incremental VaR therefore can be interpreted as the increase in risk that is generated by adding bank i to the system.[4]

While component VaR computes the marginal impact of an increase in a bank's size, incremental VaR captures the full difference in risk that one bank will bring to the system. The disadvantage of this risk decomposition is that the sum of the incremental VaRs does not add up to the VaR of the banking system. In our analysis, however, we found that difference to be small (below 5%) and thus scale iVaR capital requirements such that they sum up to the existing total bank capital:

$$C_i^{iVaR} = \frac{iVaR_i}{\sum_i iVar_i} \sum_i C_i. \tag{4}$$

1.3 Shapley values

Shapley values can be seen as efficient outcomes of multi player allocation problems in which each player holds resources that can be combined with others to create value. The Shapley value then allocates a fair amount to each player based on the average marginal value that the player's resource contributes to the total.[5] In a bank regulation context, one can argue that a certain level of capital has to be provided by all banks as a buffer for the banking system and that Shapley values determine how much capital each bank should provide according to its relative contribution to overall risk.[6]

To compute Shapley values we have to define the characteristic function $v(\mathcal{B})$ for a set $\mathcal{B} \subseteq \mathcal{N}$ of banks, which assigns a risk measure to each possible subset of banks. In our analysis

[4] We calculate VaR^{-i} by adding the losses from all banks except bank i. Another way would be to remove bank i from the banking system and then compute the loss distribution of the reduced system. We decided against the latter approach, because removing a bank would leave holes in the remaining banks' balance sheets when claims on bank i do not equal liabilities to bank i as it is the case in our sample.

[5] While Shapley values were originally developed as a concept of cooperative game theory, they are also equilibrium outcomes of noncooperative multi-party bargaining problems (see e.g. Gul (1989)).

[6] Shapley values are commonly used in the literature on risk allocation. Denault (2001) reviews some of the risk allocation mechanisms used in this paper, including the Shapley value. See also Kalkbrener (2005). In a recent paper, Tarashev, Borio, and Tsatsaronis (2009) propose to use Shapley values to allocate capital requirements to individual banks.

we use expected tail loss (EL) and value-at-risk as risk measures. To compute $v(\mathcal{B})$ we add the profits and losses for all the banks in $\mathcal{B}$ across scenarios to get the loss distribution for $\mathcal{B}$, i.e. $l_{\mathcal{B},s} = \sum_{i\in\mathcal{B}} l_{i,s}$. We assume a confidence level of 99.5% and then assign to $v(\mathcal{B})$ either the corresponding VaR, which is the 5,000th largest loss of $l_{\mathcal{B}}$, or the EL, i.e. the arithmetic average of the 5,000 biggest losses. Furthermore define $v(\emptyset) = 0$, then the Shapley value for bank i, equal to its risk contribution, can be computed as:

$$\phi_i(v) = \sum_{\mathcal{B}\subseteq\mathcal{N}} \frac{|\mathcal{B}|!(|\mathcal{N}| - |\mathcal{B}| - 1)!}{|\mathcal{N}|!} \left(v(\mathcal{B} \cup i) - v(\mathcal{B})\right) \tag{5}$$

Because the sum of the Shapley values will in general not add up to the total capital that is currently employed in the banking system, we scale the Shapley values similar to Equation (4):

$$C_i^{\text{SV}} = \frac{\phi_i}{\sum_i \phi_i} \sum_i C_i. \tag{6}$$

One potential caveat of all macroprudential capital requirements is that capital allocations can be negative, for example if a bank is negatively correlated with the other banks and therefore reduces the risk of the system. This problem also applies to the Shapley value procedure. Unless the characteristic function is monotone, i.e. $v(S \cup T) \geq v(S) + v(T)$, the core can be empty and negative Shapley values can be obtained. For our sample this problem did not occur since bank loss correlations were sufficiently high.

1.4 ΔCoVaR

Following Adrian and Brunnermeier (2010) we define CoVaR if bank i as the value-at-risk of the banking system conditional on bank i realizing a loss corresponding to its VaR. However, since we have to compute the loss distribution by simulation, we observe cases for which a bank realizes a loss exactly equal to the VaR with measure zero and therefore define CoVaR_i for bank i as

$$Pr\left(l_p < \mathit{CoVaR}_i \mid l_i \in [\mathit{VaR}_i(1-\epsilon), \mathit{VaR}_i(1+\epsilon)]\right) = 0.5\% \tag{7}$$

where we set $\epsilon = 0.1$.[7] $\Delta\mathit{CoVaR}$ is then defined as the difference of the *CoVaR* and the

[7] We found that the capital requirements and the overall results are not significatly different for $\epsilon = 0.15$ or $\epsilon = 0.05$.

value-at-risk of the system conditional on bank i realizing a loss equal to its median. We define

$$\Delta CoVaR_i = CoVaR_i - (VaR_p \mid l_i = \text{median}(l_i)) \tag{8}$$

where we approximate the second term with an interval around the median loss analogous to Equation (7).

To get the overall capital requirements we scale the results with total capital

$$C_i^{\Delta CoVaR} = \frac{\Delta CoVaR_i}{\sum_i \Delta CoVaR_i} \sum_i C_i. \tag{9}$$

1.5 Benchmarks

A natural benchmark for macroprudential capital requirements are banks' currently observed capital levels. These might differ from minimum capital requirements as banks want to hold reserves against unexpected losses from risks that are not included in current regulation. Capital levels might also differ due to lumpiness in capital issuance. Most banks have issued new capital before our sample period and individual banks could not have found adequate investment projects for all the funds that they have raised and thus show excessive capital levels. To address the latter problem, we create a second benchmark, for which we redistribute the existing capital such that each bank has the same regulatory capital ratio, which is defined as tier 1 capital over risk weighted assets (RWA).[8] We refer to this benchmark as the ”Basel equal” approach for the rest of the paper:

$$C_i^{Basel\ equal} = \frac{\text{RWA}_i}{\sum_i \text{RWA}_i} \sum_i C_i. \tag{10}$$

We now turn to a description of the two models used to generate the system loss distribution.

[8]Under current Basel capital requirements, banks have to assign a risk weight to each asset that ranges from zero for government backed assets to one for commercial loans. The RWA are the sum of asset values multiplies by their respective risk weight. The Basel Accord requires at least 4% tier 1 capital, but countries are free to set higher limits. Canada requires 7%.

2 Models of the Banking System

2.1 The Network Model

The first model that we use to compute the joint loss distribution of the banking system is built to utilize detailed data that is usually available to bank regulators and explicitly models contagion in the interbank market through fire sale and network externalities. When a bank suffers an adverse shock to its asset portfolio and is not fulfilling its capital requirements it will start selling assets to improve its capital ratio. With inelastic demand asset prices will drop causing mark-to-market losses for other banks that hold the same assets. When these losses are high enough, the other banks will start selling, too, initiating a downward spiral in asset prices. We explicitly model these asset fire sale (AFS) externalities in the spirit of Cifuentes, Shin, and Ferrucci (2005). When banks default and are therefore not able to pay their obligations in the interbank market they can create contagion through the network of interbank obligations and cause other banks to default as well. We model these network externalities explicitly through a clearing mechanism in the interbank market that identifies banks that are in contagious default.

To model the network of interbank obligations we extend the model of Eisenberg and Noe (2001) to include bankruptcy costs and uncertainty as in Elsinger, Lehar, and Summer (2006). Consider a set $\mathcal{N} = \{1, ..., N\}$ of banks. Each bank $i \in \mathcal{N}$ has a claim on specific assets A_i outside of the banking system, which we can interpret as the bank's portfolio of non-bank loans and securities. Each bank is partially funded by issuing senior debt or deposits D_i to outside investors. Bank i's obligations against other banks $j \in \mathcal{N}$ are characterized by nominal liabilities x_{ij}. In our numerical analysis we expose the bank's loan portfolio A_i to shocks ε_i, which we interpret as loan losses and describe in more detail in Section 2.2, and examine how these shocks propagate through the banking system, potentially triggering AFS and contagion in the interbank network.

The total value of a bank is the value of its assets minus the outside liabilities, $A_i - \varepsilon_i - D_i$, plus the value of all payments to and from counterparties in the banking system. If the total value of a given bank becomes negative, the bank is insolvent. In this case we assume that its outside assets are reduced by a proportional bankruptcy cost Φ. After outside debtholders are paid off, any remaining value is distributed proportionally to creditor banks. We denote by d_i the total obligations of bank i toward the rest of the system, i.e. $d_i = \sum_{j \in \mathcal{N}} x_{ij}$ and define a new matrix $\Pi \in [0, 1]^{N \times N}$ with elements π_{ij} which is derived by normalizing x_{ij} by total obligations.

$$\pi_{ij} = \begin{cases} \frac{x_{ij}}{d_i} & \text{if } d_i > 0 \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

Following Cifuentes, Shin, and Ferrucci (2005) we divide each bank's stock of outside assets, A_i, into liquid and illiquid assets. Bank i's stock of liquid assets is given by λ_i and includes cash, government's securities and government insured mortgages. Exposures between banks are also assumed liquid for simplicity. The remainder of the bank's assets, e_i, are considered illiquid. The price of the illiquid asset of bank i, p_i, is determined in equilibrium, and the liquid asset has a constant price of 1.

Payments in the interbank market are defined by a *clearing payment vector* X^*. The clearing payment vector consists of the aggregate payments of each bank to the interbank market and has to respect limited liability of banks and proportional sharing in case of default. It denotes the total payments made by the banks under the clearing mechanism. We follow the formulation in David and Lehar (2010) to incorporate liquidation costs and define each component of X^* as

$$x_i^* = \min\left[d_i, \max\left((p_i e_i + \lambda_i - \varepsilon_i)\left(1 - \Phi \mathbf{1}_{[p_i e_i + \lambda_i - \varepsilon_i + \sum_j \pi_{ji} x_j^* - D_i < d_i]}\right) + \sum_j \pi_{ji} x_j^* - D_i, 0\right)\right] \tag{12}$$

A bank's aggregate payment x_i^* to the interbank market is always between zero and the face value of its obligations d_i. It also cannot exceed a bank's net wealth, which consists of the market value of its assets, $p_i e_i + \lambda_i - \varepsilon_i$, minus potential liquidation costs, should the bank be in default, plus payments from other banks, $\sum \pi_{ji} x_j^*$, minus the bank's senior deposits D_i. To find a clearing payment vector, we employ a variant of the fictitious default algorithm developed by Eisenberg and Noe (2001).

Banks sell assets to comply with regulatory capital requirements, which we model in the spirit of the Basel II capital accord. Since all liquid assets are backed by the government, they carry a zero risk-weight. Illiquid assets of bank i are assumed to attract a risk-weight equal to the average risk-weight of the bank's balance-sheet, w_i, and we define the average risk-weight across banks as $\overline{w}$. Banks must satisfy a minimum capital ratio which stipulates that the ratio of the bank's tier 1 capital to the mark-to-market risk-weighted value of its assets must be above some prespecified minimum r^*.[9] When a bank violates this constraint, we assume that it has to sell assets to improve its regulatory capital ratio.[10] Our minimum capital requirement is therefore given by

$$\frac{p_i e_i + \lambda_i - \varepsilon_i + \sum_j \pi_{ji} x_j - x_i - D_i}{w_i p_i (e_i - s_i) - \varepsilon_i} \geq r^*. \tag{13}$$

The numerator is the equity value of the bank where the interbank claims and liabilities are calculated in terms of the realized payments. The denominator is the marked-to-market risk-weighted value of the bank's assets after the sale of s_i units of the illiquid assets. Assets are sold for cash and cash does not have a capital requirement. Thus if the bank sells s_i units of the illiquid assets, the value of the numerator is unchanged since this involves only a transformation of assets into cash, while the denominator is decreased since cash has zero risk-weight. Thus, by selling some illiquid assets, the bank can increase the regulatory capital ratio.[11]

For the mark-to-market value of the banks' illiquid assets to reflect their riskiness, we assume that the price of bank i's assets, p_i, is a linear function of the equilibrium average price p, and the deviation of the bank risk-weight from the banking sector mean,

$$p_i = \min(1, p + (\overline{w} - w_i)\kappa) \tag{14}$$

where $\kappa > 0$ to ensure that assets sold by a riskier bank have lower mark-to-market value. Average prices p are determined by the inverse demand curve for the illiquid asset that is assumed to be

$$p = e^{-\alpha(\sum_i s_i)} \tag{15}$$

where is α a positive constant. We define $p_{min} = p(\sum_i e_i)$ as the lowest average price for the illiquid assets when all assets are sold.[12]

[9] For the remainder of the paper we follow the Canadian regulation and assume a minimum tier 1 capital ratio of 7%.

[10] We do not consider the possibility of raising fresh capital nor the need to sell assets because of a loss of funding. The consequences of the latter would be similar to those described here, assuming that the new securities would have to be sold at a discount.

[11] A decrease in price should be seen as the average price decrease of all the illiquid assets on the balance-sheet, some assets' price potentially being unaffected while others suffering from huge mark-to-market losses. We assume that banks cannot short-sell assets, i.e. $s_i \in [0, e_i]$.

[12] The demand curve (parameter α) and the asset price function (parameter κ) need to be calibrated such that an equilibrium price exists for all potential positive levels of aggregate supply. We assume an exogenously fixed lower bound on the asset price p_{min} and then calibrate α accordingly. Default probabilities are more sensitive with respect to p_{min} than macroprudential capital requirements as relative risk contributions stay similar. We set

In each scenario, which is defined by a set of loan losses ε for each bank, we find for each bank i the smallest sale of illiquid assets s_i^* that ensures that the capital adequacy condition (13) is satisfied.[13] Clearing payments in the interbank market x_i^* are determined according to Equation (12) and the average price of the illiquid asset p^* is given by (15).

We define the loss $l_{i,s}$ of bank i in scenario s which is used to derive the macroprudential capital requirements as detailed in Section 1 as

$$l_{i,s} = (p_i^* e_i + \lambda_i - \varepsilon_{i,s}) \left(1 - \Phi \mathbf{1}_{[x_i^* < d_i]}\right) + \sum_j \pi_{ji} x_j^* - D_i - x_i^* - v_i^0 \tag{16}$$

where

$$v_i^0 = A_i + \sum_j \pi_{ji} d_j - D_i - d_i \tag{17}$$

is the net worth of the bank without any shocks and AFS. A scenario s is defined by a particular draw $\varepsilon_{i,s}$ of the bank specific shock. The loss is then the net worth of the bank in this scenario minus the net worth of the bank without any shocks and AFS, v_i^0. With our data no bank defaults in the latter case and thus they can all pay the promised payments d in the interbank market. Since the network model relies on book values, initial bank capital C^0 is identical to v^0. As we search for the fixed point and bank equity capital requirements change we assume that the banks hold their asset portfolios constant and substitute outside debt D for equity.

2.2 Simulation of credit losses

For the modeling of credit losses, we first generate a macro stress scenario in which average default rates in each sector are specified. Depending on composition of their loan portfolio, all banks are affected by this shock to a certain extent. We then use an extended CreditRisk+ model to simulate individual loan losses for each bank.

The macro stress scenario generates sectoral default rates that capture systematic factors affecting all banks' loans simultaneously and is based on Canada's Financial Sector Assessment Program (FSAP) update with the IMF in 2007. It relates the default rates of bank loans in different sectors to the overall performance of the economy as captured by a selected set of macroe-

κ equal to 0.5 and find that our findings are qualitatively unaffected by different choices for κ.

[13]If there is no value of $s_i \in [0, e_i]$ for which the capital condition is satisfied then $s_i^* = e_i$ and the bank gets liquidated.

Table 1. Summary statistics of simulated default rate distributions. Columns two to four show the minimum, maximum and average default rates generated for each sector. Column five gives the historic peak over the 1988-2006 period.

	Minimum	Average	Maximum	Historic Peaks
Accommodation	3.0	11.7	21.0	7.6
Agriculture	1.0	1.7	2.0	0.8
Construction	2.0	6.4	10.0	3.3
Manufacturing	5.0	12.2	20.0	8.3
Retail	0.0	4.3	8.0	5.3
Wholesale	2.0	7.0	12.0	4.6
Mortgage	0.0	0.6	1.0	0.6

conomic variables.[14] The included macroeconomic variables are GDP growth, unemployment rate, interest rate (medium-term business loan rate), and the credit/GDP ratio. We simulate sectoral distributions of 10,000 default rates for 2009Q2 under a severe recession macro scenario. Descriptive statistics of these distributions as well as historic peaks over the 1988-2006 period are presented in Table 1. Consistent with the severity of the macro scenario, mean default rates are much higher than historic peaks.[15]

In order to capture idiosyncratic risk factors arising from the granularity of banks' exposures, we use an extended CreditRisk+ model as in Elsinger, Lehar, and Summer (2006).[16] The

[14]The sectoral classification used in constructing the default rates is the one used by banks in reporting their balance sheet loan exposures to the Bank of Canada. The seven sectors included were accommodation, agriculture, construction, manufacturing, retail, wholesale, and mortgages in the household sector. For more details on the construction of historical default rates, see Misina and Tessier (2007). The FSAP scenario assumes a recession that is about one-third larger than experienced in the early 1990s. See Lalonde, Misina, Muir, St-Amant, and Tessier (2008) for a detailed description of the scenario. Sectoral distributions of default rates are centered on fitted values from sectoral regressions, and are generated using the correlation structure of historical default rates. See Misina, Tessier, and Dey (2006) for more details on the simulation of default rates.

[15]A key component in modeling credit losses is banks' sectoral Exposure-at default (EAD). Since the sectoral classification used for reporting EADs by banks under Basel II is more aggregated than the one used in constructing default rates, we simulated for each Basel II sector a distribution of weighted average default rate (PD_w) according to:

$$PD_w = \sum_{i=1}^{k} \frac{BSE_i}{\sum_{i=1}^{k} BSE_i} PD_i$$

where k represents the number of balance sheet sub-sectors that can be subsumed in one of the Basel II sectors, PD_i represents our model's default rates of sector i, and $\mathrm{BSE_i}$ the corresponding balance sheet exposure.

[16]CreditRisk+ is a trademark of Credit Suisse Financial Products (CSFP). It is described in detail in Credit Suisse (1997).

CreditRisk+ model specifies a loss distribution of a loan portfolio given the number of loans in predefined size buckets and the average PD of the loans in each bucket. We obtain each bank's loan portfolio composition by sector from the Bank of Canada Banking and Financial Statistics and get banks' largest exposures towards non-banks from the Office of the Superintendent of Financial Institutions (OSFI). For each bank, we draw 100 independent loan loss scenarios for each of the $10,000$ sectoral default rates simulated previously, yielding a total of 1 million loan loss scenarios. We assume a loss-given-default (LGD) of 50%[17] and take sectoral exposure-at-default (EAD) as reported by banks to the Bank of Canada.[18]

Table 2 shows the importance of considering both sources of uncertainty. When considering only systematic factors, i.e. the rise in the expected loss due to the increase in PDs in the macro stress test scenarios, aggregate expected losses of the 6 big banks average \$45.7 billion or 47.7% of aggregate tier 1 capital, with a standard deviation of \$7.9 billion. Taking both systematic and idiosyncratic factors into account, the expected losses are approximately the same (\$46.4 billion on average), while the standard deviation and tail losses increase (the 99% VaR is \$68.7 billion as compared to \$63.7 billion in the first distribution).

Table 2. Aggregate losses due to credit risk from non-bank loans. Descriptive statistics of expected losses considering systematic factors only (Columns 1 and 2) and both systematic and idiosyncratic factors (Columns 3 and 4).

	Systematic factors		Systematic and idiosyncratic factors	
	\$Billion	%of Tier1 capital	\$Billion	%of Tier1capital
Mean	-45.7	47.7	-46.4	48.5
Standard Deviation	7.9	8.4	9.5	9.9
Quantiles:				
99%	-27.3	28.5	-25.7	26.9
10%	-55.8	58.4	-58.8	61.4
1%	-63.7	66.6	-68.7	71.8

[17]There is little information on loss-given-default in Canada. Based on available information from the Office of the Superintendent of Bankruptcy, Misina, Tessier, and Dey (2006) estimated an average loss-given-bankruptcy over the 1988-2006 period of 65%. This overstates losses in case of default because bankruptcy is the last stage of distress, and includes more than losses related to missed interest payments.

[18]Under Basel II, banks are required to provide an estimate of the credit exposure of a facility, should that facility go into default at the risk horizon (typically one year).

2.3 Data on exposures between banks

We extend previous studies of systemic risk in banking systems (see among others Sheldon and Maurer (1998), Wells (2002) and Upper and Worms (2004)) that cover exposures between banks that arise from traditional lending (unsecured loans and deposits) by including cross-shareholdings and off-balance sheet instruments such as OTC derivatives. While derivatives are often blamed for creating systemic risk, the lack of data in many countries (including the U.S.) makes it hard to verify. Our expanded dataset enables us to better capture linkages among banks and contagious bank defaults.[19]

We collect data for end of May 2008 with the exception of exposures related to derivatives which are recorded as of April 2008. We present descriptive statistics in Table 3. Data on interbank deposits and unsecured loans come from the banks' monthly balance-sheet reports to OSFI. These monthly reports reflect the aggregate asset and liability exposures of a bank for deposits, and only aggregate asset exposures for unsecured loans. Data on exposures related to derivatives come from a survey conducted by OSFI, in which banks are asked to report their 100 largest mark-to-market counterparty exposures from OTC derivatives larger than $25 million. They are reported after netting and before collateral and guarantees.[20] Data on cross-shareholdings exposures were collected from Bank of Canada's quarterly securities returns.[21]

The aggregate size of interbank exposures is $21.6 billion for the six major Canadian banks. As summarized in Table 3, total exposures between banks accounted on average for around 25% of bank capital. Exposures related to traditional lending (deposits and unsecured loans) were the largest ones compared with mark-to-market derivatives and cross-shareholdings exposures. In May 2008, exposures related to traditional lending represented around $12.7 billion on aggregate or 16.3% of banks' tier 1 capital. Together, mark-to-market derivatives and cross-shareholdings represented 10% of banks' tier 1 capital.

[19]Zero-risk exposures, mainly repo style transactions, were excluded despite their large size. They account for more than 80% of total exposures between the Big Six Canadian banks in our sample.

[20]Anecdotal evidence suggests that the major Canadian banks often rely on collateral to mitigate their exposures to OTC derivatives. However, as Stulz (2010) points out, even full collateralization can leave a bank with counterparty risk.

[21]These returns provide for each bank aggregate holdings of all domestic financial institutions' shares. Due to data limitations, cross-shareholdings among the Big Six banks were estimated by (i) distributing the aggregate holdings of a given bank according to the ratio of its assets to total assets of domestic financial institutions, and (ii) excluding shares that were held for trading (assuming that they are hedged). Relaxing either of these assumptions does not change our main findings.

Table 3. Summary statistics on exposures between Banks. Panel A gives the aggregate size of interbank exposures related to traditional lending, derivatives and cross-shareholdings (reported in $billion and as percentage of banks' Tier 1 capital). Panel B gives banks' bilateral exposures as percentage of Tier 1 capital under two assumptions: entropy maximization and relationship banking.

Panel A: Aggregate exposures between banks				
	Agregate exposure ($Billion)	As percentage of Tier1 capital		
		Minimum	Average	Maximum
Traditional lending	12.7	5.25	16.3	38.6
Derivatives exposures	5.4	0.0	5.9	21.1
Cross-shareholdings	3.5	0.3	4.1	8.8
Total exposures	21.6	5.5	26.4	51.2
Panel B: Banks' bilateral exposures as percentage of Tier 1 capital				
Assumption:		Minimum	Average	Maximum
Entropy maximization		0.6	4.4	15.6
Proportional to asset size		0.5	4.4	16.2

A description of linkages between banks requires a complete matrix of the bilateral exposures. Such a complete matrix was available only for exposures related to derivatives. For bilateral exposures from interbank lending and cross shareholdings we only had aggregate information, i.e. how much each bank has lent to other banks and how much each bank has borrowed from other banks. We estimate the matrix assuming that banks spread their lending and borrowing as widely as possible across all other banks using an entropy maximization algorithm (see e.g. Blien and Graef (1997)).[22] This approach might underestimate contagion as it assumes that all lending and borrowing activities between banks are completely diversified. As a robustness check, banks' bilateral exposures were also estimated under the assumption that concentrations of exposures between banks are broadly consistent with their asset sizes. As shown in panel B of Table 3, banks' bilateral exposures are comparable under these two assumptions and we find that our results are robust with respect to the method of estimating of bilateral exposures from interbank lending.

[22]For a system of six banks we need to estimate a 6x6 matrix of bilateral exposures. we do know, however that the diagonal is zero (as no bank lends to itself) and we know the row and column sums that represent banks total borrowing and lending in the interbank market. Thus we need to estimate 30 exposures given 12 constraints.

2.4 A Merton Model of the Banking System

Following Merton (1973) we assume that the market value of the banks' assets V follows a geometric Brownian motion and bank equity E is interpreted as a call option on bank assets with a strike price equal to total outside and interbank liabilities $D+d$ and an assumed maturity T of one year.[23] We assume that all bank debt is insured and will therefore grow at the risk-free rate.[24] The value of bank equity at a point in time t is then given by:

$$E_t = V_t\ N(h_t) - (D_t + d_t)\ N(h_t - \sigma\sqrt{T}) \tag{18}$$

where

$$h_t = \frac{\ln(V_t/(D_t + d_t)) + (\sigma^2/2)T}{\sigma\sqrt{T}} \tag{19}$$

We use the maximum likelihood estimator developed by Duan (1994, 2000) to estimate the market values of banks' assets and their volatilities from stock price data. Given a sequence $\mathbf{E} = (E_t), t \in \{1 \dots m\}$ of equity values, the mean and standard deviation (μ, σ) of the increments in the asset value process can be estimated by maximizing the following likelihood function:

$$\begin{aligned} L(\mathbf{E}, \mu, \sigma) \quad = \quad & -\frac{m-1}{2}\ln(2\pi) - \frac{m-1}{2}\ln\sigma^2 - \sum_{t=2}^{m}\ln\hat{V}_t(\sigma) \\ & -\sum_{t=2}^{m}\ln\left(N(\hat{h}_t)\right) - \frac{1}{2\sigma^2}\sum_{t=2}^{m}\left[\ln\left(\frac{\hat{V}_t(\sigma)}{\hat{V}_{t-1}(\sigma)}\right) - \mu\right]^2 \end{aligned} \tag{20}$$

where $\hat{V}_t(\sigma)$ is the solution of Equation (18) with respect to V and $\hat{h}_t$ corresponds to h_t in Equation (19) with V_t replaced by $\hat{V}_t(\sigma)$. To estimate the parameters of the model we use daily bank stock prices from the beginning of June 2006 to the end of May 2008. From the estimation we get a time series of market values of banks' asset portfolios.

Analogous to the to the macro-stress test assumption of the network model (Section 2.2) and to get comparable default probabilities between the two models we reduce all bank asset values by 5%. We then simulate bank asset values over a one year horizon using a Cholesky

[23]The maturity of debt can also be seen as the time until the next audit of the bank, because then the regulator can observe V and close the bank, if it is undercapitalized.

[24]Relaxing this assumption will not dramatically change the results, since the paper's focus is not on deposit insurance pricing. From the available data, we cannot determine the amount of uninsured deposits for every bank. Because of this assumption, the strike price of the option is $D_t + d_T = (D_t + d_t)e^{rT}$ and Equation (19) is slightly different than in the classical Black and Scholes (1973) formula.

decomposition of the covariance matrix, which we estimate from the last 4 months of bank asset returns. Denote the draw of the asset value for bank i in scenario s as $V_{i,s}$. We then define the loss from the Merton model as

$$l_{i,s} = \min(V_{i,s} - (D_i + d_i), 0) \tag{21}$$

Analogous to the network model we draw one million loss scenarios which we use to estimate macroprudential capital requirements. Regulatory capital requirements are always based on book values of equity which do relate to the Merton model directly. Analogous to the network model we assume that as banks adjust to changing capital requirements by swapping equity for debt. Specifically we assume that when bank capital requirements for bank i change from C_i^0 to C_i^1 the outside debt changes accordingly such that $D_i^1 = D_i^0 - (C_i^1 - C_i^0)$.

3 Results

3.1 Macroprudential capital requirements

Macroprudential capital requirements partially internalize the externalities in the financial system. Table 4 presents the change in capital requirements to reach the fixed point of the five capital allocation mechanisms presented in Section 1 in percent of actual observed capital requirements, i.e. $(C^* - C^0)/C^0$. All risk allocation rules both under the network and the Merton model suggest that banks 6 is undercapitalzied from a macroprudential perspective and that bank 5 holds more capital than its contribution to the overall risk of the system would require. Results are mixed for he other banks: all capital allocation mechanisms under the Merton model as well as three out of five under the network model show that bank 1 holds to little capital. Four out of five mechanisms under the Merton model would allow bank 2 to decrease its capital while most mechanisms under the network model require an increase.

Table 10 shows correlations between macroprudential capital ratio, defined as capital over total assets, and selected bank characteristics. Across both models and all risk allocation mechanisms macroprudential capital requirements are not positively related to bank size measured by total assets. Recent regulatory proposals that demand a higher capitalization rate from large

Table 4. Change in capital requirements for macroprudential capital allocation mechanisms in percent of observed tier 1 capital (in %): Capital requirements are computed such that they match the risk contributions under the five risk allocation mechanisms. Loss distributions are computed for the macro stress scenario. Panel A shows the results for the network model including asset fire sales and a minimum price of the illiquid asset $P_{min} = 0.98$, Panel B is for the Merton model.

Bank	Component VaR	Incremental VaR	Shapley value: Expected loss	Shapley value: VaR	$\Delta CoVaR$
Panel A: Network model					
1	-3.60	3.93	4.18	4.23	-6.84
2	3.83	1.55	1.93	1.81	-1.53
3	-4.00	-5.76	-5.97	-5.97	2.32
4	9.92	11.28	10.71	10.97	7.38
5	-7.94	-8.83	-8.79	-8.87	-8.13
6	5.65	3.83	3.92	3.88	25.36
Panel B: Merton model					
1	12.86	12.86	9.53	13.34	25.25
2	-2.87	-3.93	0.27	-2.27	-19.39
3	2.68	4.33	-1.82	1.22	6.49
4	-1.64	-3.22	2.69	-1.02	-2.79
5	-13.35	-13.39	-13.06	-13.24	-13.38
6	34.32	38.74	27.98	32.98	72.73

banks are thus not supported by our data.[25] We find, however, that under both models banks with a larger fraction of interbank assets should hold more capital. It is remarkable that correlations of similar magnitude and significance for the network as well as the Merton model, especially given that the latter does not incorporate interbank data. Interestingly, the correlation is negative and only marginally significant for interbank liabilities. Our results are therefore consistent with a bank's macroprudential requirement being an insurance against potential losses caused by its interbank counterparties, and not against losses it may cause to them. Default probability is not correlated with the macroprudential capital ratio in the network model. Intuitively in crisis situations in which many banks default because of counterparty risk in the interbank market and write-downs caused by fire sales, banks' resilience to contagion is more important than overall default probability. Default probability, however, is more important in the Merton model, where no direct contagion channels are modeled and multiple defaults are solely driven

[25] This finding, however, may be due to the high concentration in the Canadian banking sector. The banks in our sample are relatively large, holding between 5% and 26% of total assets in the banking sector.

Table 5. Correlation between macroprudential capital ratio, defined as capital over total assets, and bank characteristics. One, two, and three starts correspond to significance at the ten, five, and one percent level, respectively. Loss distributions are computed for the macro stress scenario. Panel A shows the results for the network model including asset fire sales and a minimum price of the illiquid asset $P_{min} = 0.98$, Panel B is for the Merton model.

Bank characteristic	Component VaR	Incremental VaR	Shapley value		Δ*CoVaR*
			Expected loss	VaR	
Panel A: Network model					
Total Assets (TA)	-0.24	-0.35	-0.35	-0.35	-0.60
Interbank assets/TA	0.73*	0.79**	0.78**	0.78**	0.91***
Interbank Liabilities/TAs	-0.61*	-0.78**	-0.78**	-0.78**	-0.51
Total PD	0.27	0.08	0.07	0.07	0.45
Tier 1/TA	0.90***	0.96***	0.96***	0.96***	0.72*
Tier 1/Risk weighted assets	-0.45	-0.26	-0.24	-0.25	-0.47
Panel B: Merton model					
Total Assets (TA)	-0.83**	-0.84**	-0.80**	-0.84**	-0.96***
Interbank assets/TA	0.85**	0.86**	0.86**	0.85**	0.86**
Interbank Liabilities/TAs	-0.52	-0.51	-0.49	-0.52	-0.39
Total PD	0.84**	0.85**	0.85**	0.85**	0.93***
Tier 1/TA	0.54	0.52	0.56	0.53	0.32
Tier 1/Risk weighted assets	-0.08	-0.09	-0.15	-0.07	-0.07

by the correlation of interbank asset values. Finally, macroprudential capital requirements are not significantly related to regulatory capital ratios, but, at least for the network model, strongly correlated with a leverage ratio measured as Tier 1 capital over total assets. This supports the current international initiatives to regulate leverage ratios.

Compared to both, the observed capital levels and the benchmark case where all banks have the same Basel regulatory capital ratio, all macroprudential capital allocations reduce the default probability of the average bank. Table 6 shows the default probabilities of the six banks under the observed capital ratio, the "Basel equal" benchmark, as well as under the five macroprudential mechanisms. It is interesting to note that all risk allocation mechanisms work well and bring a substantial improvement relative to the existing regulatory framework despite the heterogeneity in macroprudential capital requirements across risk allocation mechanisms. For the network model (Panel A), the Component VaR method reduces the PDs for each bank, the risk allocation mechanisms based on Shapley values decrease the PD the most. Δ*CoVaR* is

Table 6. Individual bank default probability under macroprudential capital allocation mechanisms (in %). Loss distributions are computed for the macro stress scenario. Panel A shows the results for the network model including asset fire sales and a minimum price of the illiquid asset $P_{min} = 0.98$, Panel B is for the Merton model.

Bank	Observed capital	Basel equal	Component VaR	Incremental VaR	Shapley value		Δ*CoVaR*
					Expected loss	VaR	
			Panel A: Network model				
1	6.19	8.84	6.12	4.07	4.00	3.99	8.06
2	10.22	10.00	7.69	8.49	8.34	8.38	10.71
3	8.95	8.49	8.14	8.52	8.57	8.56	8.16
4	10.16	8.61	6.47	6.18	6.28	6.22	8.01
5	7.27	7.23	7.19	7.36	7.34	7.35	8.43
6	11.73	10.46	8.40	8.83	8.80	8.80	6.32
Average	9.09	8.94	7.34	7.24	7.22	7.22	8.28
			Panel B: Merton model				
1	7.12	9.14	4.78	4.78	5.32	4.70	3.14
2	2.25	2.23	2.52	2.62	2.22	2.46	4.56
3	8.38	7.92	7.29	6.69	9.18	7.87	5.94
4	2.19	1.85	2.31	2.43	2.00	2.26	2.40
5	1.92	2.02	3.97	3.98	3.91	3.95	3.98
6	38.21	35.29	12.18	10.02	15.75	12.89	1.56
Average	10.01	9.74	5.51	5.09	6.40	5.69	3.59

the most effective risk allocation mechanism under the Merton model (Panel B) where it cuts the average PD more than in half. Macroprudential capital requirements using component and incremental VaR perform roughly equally well in both models.

Under the network model all five macroprudential capital requirements also reduce the probability of a financial crisis. Panel A of Table 7 presents the probability of multiple bank defaults for the five capital allocation mechanisms. Especially incremental VaR and the Shapley value allocations and the Δ*CoVaR* reduce the probability of multiple bank failures significantly. Under Shapley value based capital allocations the probability of five or six banks defaulting can be reduced from 7.13%, which is based on current banks' capital levels, to 5.64%. This corresponds to a 21% reduction in the probability of a financial crisis. For all risk allocation mechanisms this decrease in the likelihood of multiple defaults is not driven by a simple decrease in default correlation as the probability that one or two banks default does not increase. It is

Table 7. Probability of multiple bank defaults under macroprudential capital allocation mechanisms (in %): The table shows the probabilities that one to six banks will default simultaneously. Loss distributions are computed for the macro stress scenario. Panel A shows the results for the network model including asset fire sales and a minimum price of the illiquid asset $P_{min} = 0.98$, Panel B is for the Merton model.

Number defaults	Observed capital	Basel equal	Component VaR	Incremental VaR	Shapley value Expected loss	Shapley value VaR	$\Delta CoVaR$
Panel A: Network model							
1	3.63	3.66	3.15	3.44	3.41	3.43	3.46
2	1.34	1.26	1.02	1.19	1.17	1.18	1.10
3	0.99	0.77	0.63	0.76	0.75	0.76	0.70
4	1.14	0.77	0.71	0.93	0.92	0.93	0.87
5	2.48	1.56	1.66	2.42	2.46	2.46	1.89
6	4.66	5.66	4.27	3.21	3.18	3.16	4.80
≥ 5	7.13	7.22	5.93	5.63	5.64	5.62	6.69
≥ 4	8.27	7.99	6.64	6.56	6.56	6.55	7.56
Panel B: Merton model							
1	28.10	25.60	12.11	11.11	14.04	12.50	8.55
2	8.54	8.74	4.64	4.22	5.63	4.84	2.91
3	2.91	3.06	1.98	1.84	2.33	2.06	1.20
4	0.91	0.90	0.80	0.75	0.86	0.81	0.53
5	0.35	0.35	0.35	0.34	0.38	0.36	0.21
6	0.13	0.14	0.13	0.12	0.13	0.13	0.07
≥ 5	0.48	0.49	0.48	0.46	0.51	0.49	0.28
≥ 4	1.39	1.39	1.28	1.22	1.37	1.29	0.81

interesting to note that under the network model scenarios with three to four defaults are very unlikely under any capital allocation including observed capital levels. Because of the explicit modeling of contagion channels an adverse shock is either contained resulting in one or two isolated defaults or, once a critical number of banks is affected, wipes out the whole banking system. AFS and network contagion make the default correlation in the network model state dependent and increasing in the number of defaulted banks. In the Merton model (Panel B) we can observe a monotone decline in the probability of multiple bank failures. The absence of an explicit model of contagion as well as the assumption of a multivariate normal distribution for bank asset values with a state independent correlation structure makes multiple defaults less likely. Most macroprudential capital requirements therefore show no significant decrease in the probability of a crisis. The reduction in in individual bank PDs documented in Table 6 reduces

Table 8. Difference between capital requirements under a risk attribution analysis and macroprudential capital requirements (fixed point) as a percentage of observed tier 1 capital. Loss distributions are computed for the macro stress scenario. Panel A shows the results for the network model including asset fire sales and a minimum price of the illiquid asset $P_{min} = 0.98$, Panel B is for the Merton model.

Bank	Component VaR	Incremental VaR	Shapley value Expected loss	Shapley value VaR	Δ*CoVaR*
Panel A: Network model					
1	-18.68	3.56	3.08	3.23	-48.20
2	6.52	0.95	1.41	1.27	-24.73
3	-2.33	-4.04	-4.01	-4.01	9.21
4	21.71	7.05	6.86	6.91	22.87
5	-13.83	-5.79	-5.87	-5.85	-40.93
6	9.65	2.52	2.38	2.42	293.57
Avg. abs. diff.	12.12	3.98	3.94	3.95	73.25
Panel B: Merton model					
1	20.16	14.09	8.14	11.65	19.66
2	-17.29	-18.32	-4.18	-8.88	-25.04
3	18.78	20.18	0.83	7.47	-5.14
4	1.91	1.37	14.46	9.78	-11.97
5	-22.84	-21.08	-13.54	-16.99	-20.08
6	56.68	65.46	7.24	27.22	209.39
Avg. abs. diff.	22.94	23.41	8.06	13.66	48.55

the probability that three or fewer banks default simultaneously but does not change the probability of five or six banks default. Only the Δ*CoVaR* risk allocation mechanism reduces the probability of multiple bank defaults.

3.2 Attribution vs. fixed point

To show that the macroprudential capital requirements as they are computed throughout the paper using a fixed point can differ substantially from the capital that one would attribute to a bank using a simple risk attribution analysis, we compare fixed point macroprudential capital allocations $C^* = f(\Sigma(C^*))$ as defined in Equation (1) with the results of a simple risk attribution analysis $C^1 = f(\Sigma(C^0))$, where C^0 is the observed tier 1 capital for each bank.[26]

[26] C^1 can also be interpreted as the first iteration of an iterative procedure $C^i = f(\Sigma(C^{i-1}))$ which can be used to find the fixed point $C^* = \lim_{i\to\infty} C^i$.

Table 8 shows the difference between capital requirements under a risk attribution analysis and macroprudential capital requirements as a percentage of observed tier 1 capital, $(C^1 - C^*)/C^0$, for all risk allocation mechanisms. Deviations between risk attribution analysis and fixed point are about the same order of magnitude as deviations between macroprudential capital requirements and observed capital (Table 4). Capital requirements from a simple risk attribution analysis will thus roughly be as far away from the fixed point as currently observed capital levels. Looking at the average absolute differences, which we define as $mean(|C^1 - C^*|/C^0)$, we can see that under both models a risk attribution based on Shapley values comes closest to the fixed point while for the $\Delta CoVaR$ measure the risk attributions is furthest away from the fixed point. Comparing the results with the adjustments in Table 4 we see that the risk attribution analysis often seems to overadjust capital requirements relative to the fixed point. For banks 5 and 6, which have are required to hold less (more) capital under the fixed point, the simple risk attribution analysis requires them to hold even less (more) capital than under the fixed point. A risk attribution analysis can sometimes also go in the wrong direction. Consider, for example, bank 2 under the Shapley value EL mechanism: At the fixed point bank 2 has to increase its currently observed tier 1 capital by 0.27% , while the under the risk attribution analysis it is allowed to decrease capital by 4.18%-0.27%=3.91%. We thus can see the importance of considering the fact that overall risk as well as each bank's risk contribution changes when bank capital requirements change. Our analysis highlights that no matter which model or risk attribution mechanism is being used, macroprudential capital requirements should be computed based on a fixed point.[27]

3.3 Model Risk

Empirical verification of any changes in capital requirements, whether through a risk attribution analysis or based on a fixed point as it is done in this paper, requires a model of defaults in the banking system and is thus subject to model risk. The concern is that macroprudential capital requirements that might decrease systemic risk under one model increase systemic risk under another set of modeling assumptions. To control for model risk we take the macroprudential capital requirements estimated using one model and use the other model to compute bank PDs.

[27] In our algorithm for finding the fixed point we compensate for the problem of over-adjustment by changing capital from one iteration to the next only by a fraction $\gamma < 1$ of the adjustment that has been proposed by the risk allocation mechanism, i.e. we use $C^i = (1-\gamma)C^{i-1} + \gamma f(\Sigma(C^{i-1}))$, where we found $\gamma = 0.4$ to yield numerically stable results.

Table 9. Individual bank default probability under selected macroprudential capital allocation mechanisms (in %). Loss distributions are computed for the macro stress scenario. Panel A shows the results for the network model including asset fire sales and a minimum price of the illiquid asset $P_{min} = 0.98$, Panel B is for the Merton model.

Bank	Observed capital	Basel equal	Component VaR	Incremental VaR	Shapley value Expected loss	Shapley value VaR	$\Delta CoVaR$
Panel A: Merton Macroprudential Capital Requirements in the Network model							
1	6.19	8.84	2.55	2.61	3.08	2.41	0.66
2	10.22	10.00	11.71	12.60	9.55	11.26	29.41
3	8.95	8.49	8.81	8.78	8.77	8.98	9.93
4	10.16	8.61	11.24	12.30	8.88	10.82	16.64
5	7.27	7.23	10.72	11.29	9.35	10.44	15.62
6	11.73	10.46	4.81	4.07	5.48	4.98	0.31
Average	9.09	8.94	8.31	8.61	7.52	8.15	12.09
Panel B: Network Macroprudential Capital Requirements in the Merton model							
1	7.12	9.14	7.91	6.33	6.28	6.27	8.68
2	2.25	2.23	1.94	2.11	2.08	2.09	2.40
3	8.38	7.92	10.21	11.12	11.23	11.23	7.43
4	2.19	1.85	1.56	1.49	1.52	1.50	1.70
5	1.92	2.02	2.98	3.12	3.12	3.13	3.01
6	38.21	35.29	32.96	34.61	34.53	34.56	17.41
Average	10.01	9.74	9.59	9.80	9.79	9.80	6.77

Panel A of Table 9 shows that the Merton macroprudential capital requirements also reduce bank PDs in the network model. Except for $\Delta CoVaR$ we see a slight reduction in bank PDs across all risk allocation mechanisms, which is remarkable given difference in the model assumptions and that the models are calibrated using almost disjoint datasets. Panel B shows that the macroprudential capital requirements from the network model do at least not worse than observed capital levels under the Merton model. Only $\Delta CoVaR$ can reduce the average PD significantly.

3.4 Macroprudential Capital and observed losses

To check for robustness of our macroprudential capital specification we compare changes to bank capital to actual capital raised by banks as well as realized losses in market capitalization.

Table 10. Correlation between changes in macroprudential capital ratio, defined as macroprudential capital over observed tier-1 capital, and measures bank losses. Raised capital includes all issues that qualify as tier-1 capital between June 2008 and September 2009 as a percentage of observed tier 1 capital. Loss in equity value are returns on common shares between June 2008 and August 2008. Maximum loss is the return from June 2008 to the minimum shareprice between June 2008 and November 2010. One, two, and three stars indicate significance at the 10%, 5%, and 1% level, respectively.

Bank characteristic	Component VaR	Incremental VaR	Shapley value Expected loss	Shapley value VaR	$\Delta CoVaR$
Panel A: Network model					
Raised capital	0.65	0.78*	0.77*	0.77*	-0.03**
Loss in equity value	0.77*	0.92***	0.91**	0.91**	0.44
Maximum Loss	0.62	0.36	0.34	0.34	0.64
Panel B: Merton model					
Raised capital	-0.22	-0.28	-0.06*	-0.19	-0.32
Loss in equity value	0.32	0.27	0.45	0.34	0.27
Maximum Loss	0.10	0.11	0.20	0.10	0.18

For the former analysis we collect all securities issues by our sample banks between June 2008 and September 2009 that qualify as tier-1 capital. These include common shares, preferred shares, and issues of hybrid capital securities by a tier 1 trust.[28] Panel A of Table 10 shows a positive correlation between changes in bank capital as required under the macroprudential capital mechanism of the network model and actual capital raised as a percentage of observed tier 1 capital. All correlations, except for the ΔCoVaR are positive and three out of five are significant at the 10% level. Our measure thus has some predictive power to identify under-capitalized banks.

As a second test we compute losses in equity value defined as returns on common share prices from June 2008 until the end of August 2008, when uncertainty in the markets was probably the highest before the collapse of Fannie Mae, Freddie Mac, and Lehman. We set this cutoff also because 4 out of the 6 banks in our sample successfully raised tier-1 capital in the first week of September, proving their ability to recapitalize under adverse market conditions. We can see that losses are highly correlated with changes in capital implied by the five macro-prudential capital mechanisms. For incremental VaR and the two approaches based on Shapley value the correlation is significant at the 5% level. Finally we look at the maximum loss in common share prices that each bank has suffered between June 2008 and November 2010. We

[28] We cannot show the amount raised by each bank because we are not allowed to identify the banks.

find again positive, albeit not significant, correlations for both models.

4 Conclusions

One objective of macroprudential regulation is to internalize the externalities within the financial system. In this paper, we find that financial stability can be enhanced substantially by implementing a systemic perspective on bank regulation. All of the risk allocation mechanisms that we investigated yield to a substantial decrease in both the default probabilities of individual institutions and the probability of multiple bank defaults.

We explicitly recognize that overall risk of the system, default correlations, and banks' risk contributions will change once capital gets reallocated and therefore set macroprudential capital requirements as a fixed point for which capital allocations are consistent with the contributions of each bank to the total risk of the banking system, under the proposed capital allocations. To measure how overall risk changes with capital allocations we use two models of joint default in banking systems: the network mdoel explicitly considers contagion effects through network and asset fire sale externalities and is callibrated to regulatory data. The Merton model is estimated from bank stock prices and models joint defaults through the correlation structure of the banks' asset portfolios.

Our findings have important policy implications for future bank regulation. For our sample, all of the analyzed macroprudential capital mechanisms brought a substantial reduction for bank risk. It is therefore probably more important for policymakers to implement a systemic perspective on bank regulation rather than to find the best risk allocation mechanism.

Implementing macroprudential capital requirements in practice will not be easy. Bank regulators will need to collect a large amount of data and especially information on the interbank exposures between banks. While we were lucky to have that information for the Canadian banks, in many countries around the world, this information is unavailable. Another hurdle will be to base each bank's capital requirements not solely on that bank's characteristics. Banks will complain to be treated unfairly as banks with a similar asset mix will be charged with different capital allocations based on systemic importance. One possible way to implement macroprudential capital requirements in to augment existing capital ratios with a charge that is based on regulatory assessment, similar to FDIC premiums, which are in part determined by the the primary regulator's discretionary composite rating.[29]

[29]See also Acharya, Santos, and Yorulmazer (2010) for a mechanism to determine deposit insurance premiums in the presence of systemic risk.

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Appendix

A Fixed point convergence

In this appendix we provide evidence that the fixed point is well defined. Because of the nonlinear nature of the model that we use for clearing the interbank claims and computing bank PDs, we cannot explicitly prove the uniqueness of the fixed point that determines the macroprudential capital requirements. To check for robustness we performed a Monte Carlo analysis by using alternative starting values and found that all of them converged to the same fixed point.

To check for uniqueness of the fixed point we run a Monte Carlo simulation using alternative starting capital values C^0 and check that all of them converge to the same fixed point. Finding the fixed point is numerically intensive because we have to run a full Monte Carlo simulation in every iteration to get the joint loss and default distribution Σ. In this robustness check we draw 100 random capital endowments for the banks with the restriction that each bank has to be above the minimum capital requirement of 7% of risk weighted assets and that the total capital in the banking system stays the same. We find that our procedure converges to the same fixed point for all starting values. Figure 1 shows the norm of the distance to the fixed point $|C^i - C^*|$ over the first 15 iterations for the first 50 starting values and the Shapley value-expected loss risk attribution model. While we cannot provide a formal proof, evidence from our simulations makes us confident that for our data macroprudential capital requirements are well defined.

Figure 1. Convergence to the fixed point or alternative starting values: The graph shows the norm of the distance to the fixed point for the different iterations for the Shapley values with expected losses.