GUIDANCE FOR TEACHERS AND STUDENTS COMING TO UNIVERSITY

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Summary. The new curriculum demands that students will have to develop a more mature attitude towards their own learning, in that they will need to take the initiative in going to books or the web to ensure sufficient algebraic and geometric facility. In particular, ability to deal with inequalities and to sketch graphs will be very helpful at a higher level. Properly implemented, the new curriculum promises to improve the prospects of matriculating students provided the injunction to engage in investigations and problem solving is taken seriously and the students develop the ability to think divergently, make connections and communicate effectively. Students should learn to pay attention to detail and how to check and troubleshoot their work. While formal proofs are not mandated for most of the high school curriculum, students must be able to proceed in a logical way from the data of a problem or situation to the desired conclusions, and to indicate this clearly in their writing. However, a large part of the success of the new curriculum depends on the integrity of the grading system, and this is an issue that teachers within the various boards need to address with some urgency.

1. **General comments.** While it is recognized that high school teachers have many students who will not be going on to university and that their needs should not be compromised for the sake of those who are, it is nevertheless reasonable to assume that the Grade 12 courses designated as U courses will be taught at a level that will adequately prepare students for university and that students not able to attain the appropriate level will either be failed or encouraged to take a more suitable program.

To succeed at University, students will require not only arithmetic and algebraic facility, but also an attitude towards mathematics that sees it as more than a box of tools to be blindly and mechanically applied and an ability to manipulate and communicate ideas. In fact, the way in which students approach mathematics is sufficiently critical that even with a reasonable technical ability, students may not succeed because they do not have the judgment that allows techniques to be used effectively.

Under the old curriculum, in many classrooms there was a lot of emphasis on doing type examples. But often this was at a low level without a sense of context, so that it was difficult for students to retain whatever skill they managed to muster. The thrust of the new curriculum, if properly implemented, promises to improve the situation considerably. The emphasis on experimentation, hands-on experience, use of technology and communication should allow students to take better ownership of the material and contextualize it. Some critics will complain that the greater emphasis on process will be at the expense of students gaining the needed background of knowledge and technique for university. This may well be true, but there are certain things to keep in mind as to how this might be compensated.

Criticism of the skills of entering university students is not new. Even when they were emphasized, if it was not the goal of teachers and students to foster understanding, they could not be remembered in any reasonable kind of mental construct and deployed effectively. For some students, this was ameliorated by having to prepare for summative examinations that allowed students to review a whole body of material and perhaps experience a sense of a number of pieces falling into a coherent whole.

However, the design of a syllabus should be thought of similar to the creating of a caricature. A good political cartoon with a few appropriate strokes can capture the essence of the subject; a poor one is often overdrawn and even then the person depicted becomes unrecognizable. Cognizant that time will not permit covering a detailed syllabus, the challenge for the teacher will be to try to ensure that essential pieces are put in place that students can build upon. For example, students will still need to be thoroughly familiar with trigonometric relationships in order to handle the material of first year calculus and science courses. However, they cannot expect their teachers to spoonfeed them the one or two dozen formulae that they should have at their fingertips; rather the student needs to understand the sort of thing that is possible with the trigonometric relations and be able to devise strategies that will first allow her to reconstruct important properties and secondly to retain them for use. Secondly, students will still need to practise to attain proficiency, but they will need to become more responsible for accessing exercises on their own, doing what is necessary and checking their work. The teacher will not have the time in class to cover all the minutiae, and will need to become more of a strategic coach.

The new curriculum, well implemented, should be beneficial. It enjoins that "students have opportunities to learn in a variety of ways", that they "explore concepts individually and cooperatively; independently and with teacher direction; through hands-on activities." It recognizes differences in the types of knowledge and the diversity of approaches that may be had. It requires work to be done in context, with "rich environments" that "open the door for students to see the 'big ideas' of mathematics". There is to be an emphasis on the use of technology and on communication.

This puts a burden of maturity on the student. The broadest agreement on the effect of the new curriculum for students matriculating to tertiary education is that the students will be one year less mature. If this lack of maturity turns out to be more than transitory, then there is no way that the new curriculum can succeed in preparing students for higher education. It *requires* greater autonomy, initiative, independence and seriousness on the part of the student. Attaining this must be built into the secondary regime and any attempt to compromise or water down these expectations will lead to disaster.

Part of the maturity needed is a willingness for students not to expect complete understanding immediately and to be willing to work to gain it and to get a robust control over the mathematics they need. The advice given by Beatrice to the Pilgrim in the third book, *Paradise*, of Dante's *Divine Comedy*, puts the matter very well:

> you must sit at the table yet awhile
> because the food that you have taken in
> is tough and takes time to assimilate.
>
> Open your mind to what I shall reveal
> and seal it in, for to have understood
> and not retained, as knowledge does not count.
>
> *Canto V, 37-42*

There will be a small number of your students who will be going into a demanding program such as honours mathematics or engineering science. To gauge the sort of thing that such students may be having to deal with early in the year, you may be interested in calling up the notes distributed to first year engineering science students at the University of Toronto on the foundations of analysis; they can be found at

http://courses.ece.toronto.edu/mat194h1f.

2. **Grades.** An important part of the success of the new curriculum in preparing students for university is the integrity of the system of evaluation. If individual schools have the sole responsibility for submitting a final grade for Grade 12 students, it is inevitable that students will receive higher grades than are warranted by their understanding and skill, and will be misled by them. Individual teachers must be freed from the dilemma of assigning a grade that accords with their professional judgment and assigning (in an atmosphere of "grade inflation") one that will ensure a place in university for a student who would be reasonably expected to succeed. They need to be freed from the importuning of students, parents and principals.

Ideally, the comparability of grades should be decided at the level of the board. While the judgment of teachers as individuals might be subverted, the judgment of teachers as a body can be relied upon. Therefore, the mathematics heads of each school district should devise an effective policy that will ensure comparable standards from one school to another. There are many ways that this can be achieved. There may be common examinations. Schools may use external examiners from other schools. An adjudicating body may collect samples of work across the system and make a report. Projects might be presented through "fairs" which involve the participation of all schools within a certain area.

3. **Proofs and reasoning.** Some teachers point out that under the new curriculum, students will not have a good introduction to proof until the Geometry and Discrete Mathematics Course (MGA4U). This is far too late. However, this criticism should not be read to mean that there should be a solid dose of formal proof earlier on, or a return to the traditional topics and approach of Euclidean geometry. If we take this curriculum as it stands, we should see that there will be lots of occasions for students to make statements and justify them by logical and cogent argumentation. During their schooling, students should become aware that mathematics involves a flow of ideas, proceeding from what is known through a succession of statements depending on other known facts and reasoning, to what has to be established. The preparation for proof is the fostering of a state of mind that does not see mathematics as arbitrary, but as a coherent body of material which can be understood, and part of this understanding is attained through reasoning. Every act of communication of the students should embody a clear progression of ideas and consistent use of notation and concepts. If students are to be encouraged to investigate and communicate, then they are bound to be making conjectures and coming to an understanding about the truth of the situation.

While the curriculum may be light on *particular* proofs before the Grade 12 program, there is the need for teachers to be opportunistic in exploiting situations that arise in class in which it is natural for students to make justifications and their experience at formal reasoning can be advanced.

4. **Information.** A broad theme of the curriculum is *information.* Students should be continually asking the following questions and learning how to answer them explicitly:

- what information do I have?
- what information do I need?
- what sort of information is appropriate to the context?
- what is the most convenient way of getting the information?
- how can I use what I know as a lever to find out what I need to know?
- what information is tied up in the mathematical expression or diagram that I am dealing with?
- how can I manipulate the mathematical artifact to make manifest the information that is latent in it?
- is the information I am getting of any use?
- is the information in the right form? does it have the right precision?
- have I displayed the information so that it is intelligible to the reader?

5. **Communication.** Teachers can very greatly improve the prospects of their students by being very particular about how they present their solutions. Students must be aware that they are communicating with a reader who should not be assumed to be a mind reader. Complete mathematical and English sentences must be used, and there should be paragraphing, so that related material can be followed easily. All symbols not defined in the problem or given by convention must be defined in the solution; where appropriate, units should be specified. Problems in which a particular context has to be put into a mathematical form should have solutions which are prefaced by a set of statements that carefully relate the mathematical symbolism to the entities in the problem and concluded by a statement written in ordinary English that states whatever conclusions are pertinent to the situation. In a solution, students must indicate the logical relationships between the statements, making use of such formulations as "therefore", "since", "implies"; solutions should have logical flow that move from what is given or known by logical steps to what has to be established. No student should be given an A or a grading at the top level without being able to communicate clearly. This point is so crucial that the habit of clear and sufficient communication needs to be fostered right from the beginning of high school. A subtle part which students need to learn from continual experience is to know which details need to be included on the solution so that the essential points are covered and which can be glossed over or left to the reader.

6. **Practice.** One advantage of the web is the appearance at various sites of advice and practice problems for students coming to university. Students planning to take a science, engineering or mathematics program at university should make it their business to track these down and make use of them. A lot of the exercises are very basic, so regardless of what is on the syllabus, any student who has difficulty with them and does not seek assistance is asking for trouble. Some engineering science undergraduates at the University

of Toronto were hired to set up a site, and this can be found at *http:// www.ecf.utoronto.ca/ wangq/engsci*. There are other sites set up by Canadian universities:

- *http://www.math.unb.ca/#stud*
- *http://conway.math.unb.ca/Placement/test*
- *http://math.usask.ca/readin/menu.html*
- *http://www.mast.queensu.ca/diagnostic.html*.

As part of an Australian Mathematical Methods Computer Algebra Systems (CAS) pilot study, there is quite a nice collection of problems on graphing. Consult the site:
http://www.vcaa.vic.edu.ca/vce/studies/MATHS/caspilot.htm.

[If your search software is persnickety, try `http://www.edfac.unimelb.edu.au/DSME/CAS-CAT/proj.html` or enter via a Google search on `caspilot`. Look for the section entitled *Sample examination papers*.]

For those students who want to have more advanced practice, they can go to the website of the Canadian Mathematical Society at *http://www.cms.math.ca* and click on the International Mathematical Talent Search (IMTS) or the Mathematical Olympiads Correspondence Program (MOCP), also called *Olymon*. Not all of the problems in either collection are Olympiad headbreakers; some of them yield to cogent reasoning, some experimentation and a reasonable algebraic or geometric facility.

7. **Advanced Functions and Introductory Calculus (MCB4U).** Most universities and colleges will have first year calculus courses that "start from scratch", even if the early stages are covered rather quickly. In any case, the technical material covered in the MCB4U course is very slight, and the teacher who sees her role as merely covering the pittance of techniques given in the syllabus will have failed its intent miserably.

But there are some very important ideas that, properly grounded in this course, will provide students with a very strong basis for the study of university calculus.

(a) *Characteristics of functions.* The course deals with three different types of functions: polynomials, exponentials and logarithms. The students should come away from the course with a clear idea of what the characteristic properties of each of these types, and how they differ from one another. For example, one characteristic of polynomials is that their roots can be related to how they factor. Partly for this reason, it is worth spending some time and effort on the remainder and factor theorems. To prepare for the remainder theorem, students should review long division of one polynomial by another and write out the relationship between the divisor, divided, quotient and remainder, *viz.*, when $g(x)$ is divided into $f(x)$, one obtains a quotient $q(x)$ and a remainder $r(x)$ for which $f(x) = q(x)g(x) + r(x)$ where the degree of r is less than that of q. If $g(x) = (x - a)$ is linear, then there is a constant remainder. At this stage, the remainder theorem can be adduced with the factor theorem as a corollary. (Here is an example of proof that should not be missed.)

There are two main things that teachers will want to draw from the factor and remainder theorem. The first is that it is a theorem about *polynomials* and (at least at this level) does not apply to other functions, such as trigonometric, exponential and logarithmic functions. Secondly, the factor theorem will play an important role in the computation of limits of rational functions (see below).

One of the characteristics of the exponential function is the self-similarity of its graph. The identity $a^{c+x} = a^c a^x$ has the following geometric interpretation: The graph of $y = a^{c+x}$ can be obtained in two different ways from the graph of $y = a^x$. The first is to translate c units to the left; the second is to dilate it in the $y-$direction by a factor a^c. This has the following consequences for our understanding of exponential growth (or decay). Suppose we have two populations that exhibit the same percentage rate for exponential growth; then the ratio of the size of the two populations will remain constant, and when the smaller population attains to the size of the larger, it will mimic the behaviour of the larger with a lag in the independent variable. When you think about the behaviour of systems that undergo such growth (such as money invested as at a specified rate of interest), this property seems entirely reasonable, but it should be brought out. A characteristic of exponential growth and decay is that the system changes by the same factor over equal lengths of time; this is what makes the concept of *half-life* and *doubling time* intelligible. Polynomials do not have this property.

The inverse relationship between the logarithm and exponential functions means that the exponential properties $a^{x+y} = a^x a^y$ and $(a^x)^y = a^{(xy)}$ should have an echo for the inverse $\log_a x$ of a^x. A lot of attention should be paid to ensuring that the students internalize that $\log a^x = x$ and $a^{\log_a x} = x$; this might be verbalized as "$\log_a x$ is the power to which I must raise a to get x". One of the downsides to students no longer having to use tables of logarithms is that they have not really absorbed this property, nor have at their fingertips the most important characteristic property of the logarithm, that it is a device to convert products to sums: $\log_a xy = \log_a x + \log_a y$. The curriculum calls for examples involving logarithmic scales, and I strongly recommend that these be discussed.

The curriculum provides for some comparison of the growth of the three types of functions, and students should understand that, for $a > 1$, a^x grows faster with positive x than any power of x while $\log_a x$ grows slower. This can be investigated using technology, while at the same time making a first informal pass at the idea of $\lim_{x\to\infty} f(x)$.

While the curriculum does not specify it explicitly, you should make reference to the fact that the graph of $y = f(x)$ is the reflection of the graph of $y = f^{-1}(x)$ in the line $y = x$, and draw students' attention to this when $f(x) = a^x$ and $f(x) = x^n$ in particular.

(b) *The nature of the system of real numbers.* The progress of many students in first year calculus is confounded by an inability to understand the set of real numbers as a *continuum*, *i.e.,* capable of being modelled by the real line. Many students have a "birds-on- a-telegraph-wire" view of the number system, where each number sits next to a neighbour and there is a smallest positive number. The structure of the number system is deep and not easy for students to grasp, so teachers should be prepared to take whatever time is required to challenge the perceptions of their students. If appropriate, it may be useful to use physical examples that involve a time variable, because the smooth Newtonian flow of time may be more intuitive than the geometric line for some students.

(c) *Limit.* Students need to realize that the calculation of $\lim_{x\to c} f(x)$ involves the behaviour of the function f near the point c and that its value (if any) at c is utterly immaterial. The computer and calculator can be very useful in helping students explore examples and see how the values of a function might trend in the neighbourhood of a particular point. This perception of limit needs to be solidly in place before students are allowed to calculate limits in situations where the limit is simply $f(c)$. If this time lapse is not allowed, then students may come away not seeing much use for the idea of limit and be too much tied to the calculating limit by evaluating; it may also result in their assigning a limit an indeterminate form 0/0 as if this had a meaning (or to which they may assign a meaning without much real understanding of what is at stake).

In computing a "0/0" limit of rational functions, care must be taken to explicate each step. Suppose we need to evaluate

$$\lim_{x\to 2} \frac{x^3 - 8}{x^2 - 4} .$$

The solution relies on the continuity and other properties of polynomials. It should be noted that the numerator and denominator are both polynomials that vanish when $x = 2$; this means that the factor theorem can be applied. Let us first deal with the continuity aspects. A formal definition of continuity (especially using $\epsilon - \delta$) would be lethal at this stage, but students can acquire an informal intuition of continuity (geometry and physics would be helpful here) and realize that the continuity of a function f at a point c is *characterized* by $\lim_{x\to c} f(x) = f(c)$. It should not be difficult to persuade them that $\lim_{x\to c} k = k$ (for the constant function k) and $\lim_{x\to c} x = c$ and that certain combinations of continuous functions are continuous. This leads to the continuity of polynomials and of rational functions (subject to the usual caveat about dividing by zero).

An important point to be made is that a proper definition of a function requires specifying its domain of definition. In this case, one notes that $(x^3 - 8)/(x^2 - 4)$ is defined except when $|x| = 2$ and that on its domain, it is equal to $(x^2 + 2x + 4)/(x + 2)$. However, this latter function is both defined and continuous when $x = 2$, so that

$$\lim_{x\to 2} \frac{x^2 + 2x + 4}{x + 2} = \frac{2^2 + 2 \cdot 2 + 4}{2 + 2} = 3 .$$

In the calculation of $\lim_{x\to 2}((x^3-8)/(x^2-4))$, we are concerned with values of the function for x close to, but not equal to, 2. It does not matter what the value of the function is for $x = 2$, or whether it is even defined. Since for values distinct from 2,

$$\frac{x^3-8}{x^2-4} = \frac{x^2+2x+4}{x+2} \ ,$$

we can deduce that the required limit is 3. Students should be strongly encouraged to engage the underlying structure and not just take a mechanical, rote approach to this situation. And they should not use the expression 0/0 as a mathematical entity.

Do not teach l'Hôpital's Rule. The syllabus does not require it, and teaching it will do far more harm than good.

(d) *Graphs.* Pencil-and-paper graphing should be distinguished from computer or calculator graphing. The former is a *sketch* whose purpose is to succinctly convey the salient features of a function, while the latter is designed to give more precise information. In a sketched graph, information is appended selectively to highlight the significant aspects (extrema, asymptotes, intercepts). Such graphing may be useful before going to the computer or calculator, as it may give guidance as to the variable bounds and scale. This will also aid the student's intuition in comparing what is expected to what actually appears on the screen. Sketching a graph may also help a student become more sensitive to the role of symmetry and the ways in which functions may be formed by combining simpler functions. Note that, because of the limitations of precision, a calculator graph could actually be misleading if the function is changing rapidly. Encourage students to look at functions holistically, and where they arise in an applied problem, have the student conjecture the likely behaviour of the function before beginning to analyze it. My experience is that students who can graph well (as well as handle inequalities) are in pretty good shape in first year calculus.

(e) *Rates problems.* Students should be able to clarify the relationships among the variables, which are independent and which are dependent, and which may play an intermediary role. They should understand what a parameter is. They should be able to distinguish identities from conditional equations. Consider the following questions: *A rectangle of sides x and y has area 1; x is increasing at a rate of 2; at what rate is y changing when $x = 1/2$?* The implicit independent variable is the time, which we can denote by t; the dependent variables are x and y, and these are linked by the constraint $xy = 1$, so that each can be considered as dependent on the other. The given conditions are $x' = 2$ and $xy = 1$, and these are identities, true for all values of t on suitable domains. Since $xy = 1$ can be considered as an equation of *functions*, the functions on both sides must have the same derivative: $x'y + xy' = 0$, so that $y' = -(y/x)x'$, an identity involving the four functions x, y, x' and y'. Now we move to a particular situation, in which x is to assume the value 1/2. This occurs at that time when $y = 2$ and so we find that, at this particular time, $y' = -8$.

(f) *Optimization.* Optimization problems should be set up carefully. All variables not defined in the statement of the problem must be defined in the solution, with appropriate units indicated. All functions defined must be provided with a domain of definition. Students should distinguish identities from conditional equations. The character of extrema (whether maximum or minimum, whether global or local) must be justified explicitly by an *appropriate* method. Make sure that students check endpoints (this is one reason to be fussy about the domain of definition). Solutions must end with a sentence that relates the problem posed to the mathematical conclusions. Sketching a graph of the function to be optimized is highly recommended.

Encourage flexibility in justifying an extremum. Suppose that it is believed that a function $f(x)$ has a minimum when $x = a$. This can be checked in a number of ways, any of which in a given situation can be more tractable than others:
(1) $f(x) - f(a)$ can be analyzed and found (say through factorization or expressing in terms of squares) to be nonnegative for all x;
(2) $f'(x)$ can be analyzed to determine where it is positive and negative, so that intervals of increase and decrease of $f(x)$ can be determined;
(3) $f''(a)$ can be found; note, however, that this only gives *local* information about the behaviour of f and f' and so identifies relative rather than absolute extrema; other reasoning should be provided to ensure that the minimum is global;

(4) a theoretical argument can be used: $f(x)$ being continuous and differentiable assumes its extreme values on a closed interval, either at one of the endpoints or at a place where the first derivative vanishes; since this is generally a finite set of values, we can check them individually and select the place $x = a$ where the smallest value of f is assumed.

A popular method for checking extrema is to find out where the derivative vanishes and then to check specific values of the function on either side. I would discourage this unless the teacher is willing to spend the time making sure that the students know how to rigorously defend such a method. Usually this comes across to the student as a rote, mechanical method and inhibits their later willingness to seek understanding.

The syllabus of MCB4U allows plenty of scope for teachers to bring out the underlying meaning and intelligibility of the processes of calculus and it is important that students get in the habit of making connections, being careful with detail and communicating explicitly.

8. Geometry and Discrete Mathematics (MGA4U). This will be a challenging course to teach, as it falls into three discrete parts with insufficient time to develop any of them beyond a superficial level. If there is an ongoing theme, it might be mathematical structure and the exploitation of that structure to explore mathematical ideas and to solve problems. Each section carries the danger of the students learning by rule rather than through understanding and the acquiring of an overview.

(a) *Geometry.* Physical models are probably the best way of introducing the notion of vector addition and relating this to the parallelogram of vectors. Underlying the facility of vectors is the apprehension of a vector as an "equivalence class of arrows" with magnitude and direction, which gets identified with a single vector rooted at the origin. A lot of students will find this uncomfortable. You cannot cover this formally ("the cure is worse than the disease"), so it will have do be done through lots of examples and solution of problems, even if this takes a lot of time.

The dot and cross products present a challenge. Each can be defined formally coordinate-wise or geometrically (the product of the magnitudes times a trigonometric function of the angle between them). However, the two formulations need to somehow be linked, as the student will need facility with both and may require the flexibility to move between them. The coordinatewise definition makes the algebraic properties (distributivity, commutativity or anti-commutativity) more manifest, and the geometric formulation provides a useful way for thinking about their meaning and significance.

The geometric and algebraic aspects of systems of linear equations should be linked. In particular, students should realize the significance of such systems having a unique solution, a singly- or doubly-infinite family of solutions, or no solutions at all. The part of the curriculum that treats row-reductions of augmented matrices touches on the theme of information. An augmented matrix should be seen as a way of coding a linear system, and row operations (which are reversible) as signifying manipulations transforming a given system to an equivalent system in which certain variables get isolated.

(b) *Proof.* While this appears as a separate part of the syllabus, a lot of work on proof can be woven elsewhere into the course, so that students can see conjecturing, analysis and proving as useful habits of mind for all of mathematical study. There seems to be quite a bit of discretion as to which mathematical results are the vehicle for teaching proof; individual topics are not as important as students getting exposure to come coherent body of mathematics and the deductive power that leads from basic facts to more complex and possibly unexpected conclusions. A lot of emphasis should be put on clarity of expression, organization of ideas, thoroughness, carefulness and flexibility in looking at situations from different standpoints.

(c) *Combinatorics.* In this part of the syllabus, writing up a result may be more difficult that obtaining it. Accordingly, some pains should be taken concerning how students can express their ideas. In induction arguments, a useful way to look at the induction step is as a kind of subroutine or protocol that allows one to certify arguments that can apply to any one of infinitely many numerical cases. To drive this home and underline the provisional status of the induction step, some students may find it helpful to write out a complete argument for all the steps up to, say, $n = 4$ or 5 from the $n = 1$ case.

For the university-bound student, important goals of this course are:

• being able to move between geometric and algebraic formulations in vectors;

• being able to relate conventions and processes (for example, with row reductions and combinatorial symbols);

• developing a strong sense of logical flow in solutions and arguments, with some knowledge contingent on other more basic knowledge;

• having a good command of logical implication and structure of arguments (particular indirect and induction arguments);

• writing well.

9. Mathematics of Data Management (MDM4U). The opening sentence of the description is worth quoting: "This course broadens students' understanding of mathematics as it relates to managing information." Important themes include how information can be effectively represented and how essential or representative information can be crystallized from a surfeit of data. Probability is seen as a intellectual artifact for dealing with uncertainty, its justification being that it seems to work in practice. Note that all applications of probability provide some assumptions about the situation at hand (equally likely events and "fairness" in the case of *a priori* probability and about stability of performance in *a posteriori* probability). The overriding goal is to help student become critical thinkers and recognize that models can be more or less effective and in any case represent a contingent view of reality.

As for the statistics part of the syllabus, adherence to the Hippocratic injunction to, above all, do no harm is a good policy. This course is not the place to either provide a cookbook compendium of statistical techniques or to explore the sophisticated theory of probability and statistics; these will be done at the appropriate time later in university. It is rather to have the students engage the ideas, make them aware of issues and get them discussing situations critically. It is also giving them a chance to have some fun.