On-line Mathematics: Visions and Opportunities, Issues and Challenges, and Recommendations

White Paper based on
The Fields Institute for Research in Mathematical Sciences'
Mathematics On-line Working Meeting
November 15-17, 2001

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Introduction

Forty years ago groups of mathematics educators were experimenting with and advocating for the computer delivery of mathematics lessons (Suppes, 1965). In these early projects, computer assisted instruction [CAI] packages were delivered from large mainframe computers to elementary grade pupils sitting at teletype terminals located in their schools. The vast technological gulf between the university based developers of such systems and the school teachers who would use them and the public in general meant that the promised immanent educational revolution failed to appear.

The technological landscape has changed drastically during the passing four decades. Today the network connected personal computer has become a common household appliance and schools have classroom computers linked to the World Wide Web [Web]. Sophisticated software makes computer communication and information access easy and appealing for users of all ages. Surfing the Web, visiting chat rooms, and communicating by e-mail are becoming major recreational activities for the "net generation" (Tapscott, 1998).

This new technical reality has encouraged some to again predict a coming educational revolution, but future directions are not clear. On the small-scale side of the picture, individual teachers are using the Web to provide their classes with access to current information and experiences that lie outside the classroom (Heide & Stilborne, 1999). On a larger-scale, educational institutions - school boards, colleges, and universities - are beginning to convert regular courses for delivery via the Web, and some authors are predicting the demise of these traditional education providers in the face of competition from agencies and corporations dedicated to the production of Web-based courses (Downes, 2000).

Noting the wide range of predictions related to the on-line delivery of education, in early 2001 the Fields Mathematics Education Forum established a working group to explore the issues and produce a statement concerning the teaching and learning of mathematics via the Web. As part of this exercise the working group organized a symposium, the Mathematics On-line Working Meeting, November 15-17, 2001. This report is the outcome of the deliberations of the 60 (?) mathematics educators who attended the meeting.

History

The Fields Institute, named after the Canadian mathematician John Charles Fields, was established in 1992 as a research centre for the mathematical sciences. The Institute, with primary funding from the Ontario Ministry of Training, Colleges and Universities and the federal Natural Sciences and Engineering Research Council [NSERC], provides visiting mathematicians with on-going highly focussed mathematics research seminars and opportunities to investigate the industrial and business applications of their work. Included in the Institute's mandate is the support of mathematics education at all ages, from the beginning of schooling through to graduate and post-doctoral study. To further this goal, the Institute, since early 1997, has coordinated and sponsored the Fields Mathematics Education Forum. Here in regular meetings, mathematics educators from the elementary and secondary school systems, colleges, and universities meet to exchange ideas and develop proposals for the improvement of teaching and learning in the discipline. The Forum has come to play a significant role in Ontario education, organizing symposia and workshops addressing key issues and contributing to the development of the province's new secondary school mathematics curriculum (Ontario Ministry of Education, 2000; Ontario Ministry of Education and Training, 1999).

Context: Change in Mathematics Education

Since the early 1980s there has been an on-going debate with repeated calls for change in mathematics teaching and learning at both the school and university levels (National Council of Teachers of Mathematics [NCTM], 1980; Schoenfeld, 1983). This change program, which is most fully developed in the Principles and Standards for School Mathematics (NCTM, 2000), has been led in North America by the National Council of Teachers of Mathematics [NCTM] and the Mathematical Association of America [MAA], and locally in Ontario by the Ontario Association for Mathematics Education [OAME], the Ontario Mathematics Coordinators' Association [OMCA], and the Ontario Colleges Mathematics Association [OCMA]. With participants drawn from the active members of these associations, the debates and positions of the Fields Mathematics Education Forum reflect the trends of the larger change movement. In 1997 a Fields Institute sponsored workshop issued a white paper, Mathematics Education for the 21st Century (Langford, Long & McDougall, 1997), which called for a curriculum that delivered "a context-rich, problems-based environment, showing the relevance of mathematics to other disciplines" (p. 14). Mathematical modelling would be a central theme of the workshop's proposed new program and students would have "opportunities to pose their own problems and solve them through investigations and project-based learning" (p. 14). The white paper set out "the integration of information technologies into the curriculum as a priority" (p. 15) and in calling for every classroom to have "access to information technology resources on the Internet" (p. 16) signalled an interest in mathematics on-line.

Continuing discussion has refined and strengthened the Forum's image of mathematics learning, and this vision was taken as the starting point for discussions at the Mathematics On-line Working Meeting. As an initial task, participants were presented with the following vision statement:

> We want students to be immersed in a mathematics culture that gives them an opportunity to: a) learn, use, and refine inquiry, investigation, experimentation and problem solving processes, and b) develop the tools/skills/habits of a life-long learner, learn significant concepts and procedures (with understanding) that they can then use in an integrated, authentic fashion to conduct inquiries, experiments, investigations and problems.

and were asked to identify the characteristics of on-line environments that would support such mathematics learning.

Context: On-line Learning

The recent flurry of research reports, government and corporate statements, and popular press articles concerning the present and future state of on-line learning suggest that there is divergence in opinions concerning both the nature and the viability of the enterprise. Teachers who employ the Internet in their lessons do so by integrating Web-based activities into their on-going curricular program. Their students use information accessed via the Web in larger classroom-based projects and to a lesser degree use the Internet for inter-school collaborative activities and for publishing personal work (Becker, 1999). On-the-other-hand task forces involving leaders from government, business, and education (Ontario Ministry of Education/Ontario Ministry of Training, Colleges and Universities, 2000, June; Web-based Education Commission, 2000) see a much larger enterprise with full courses being delivered via the Web. University generated reports suggest use of the Web could have positive financial

returns and reduce the cost pressures being experienced by tertiary level educational institutions (Task Force on Learning Technologies, 2000). In many ways, differing opinions on the best directions for on-line learning reflect the larger debates concerning the balance between direct instruction and constructivist approaches. Research (Becker, 1999) shows that Internet use is greatest for teachers who regularly employ constructivist practices such as those advocated by the NCTM, OAME, and the Fields Mathematics Education Forum (Langford, Long & McDougall, 1997). Unfortunately it appears that the developers of Websites do not take as strong a constructivist position. A survey of 436 Websites designed to support learning in mathematics, science, and technology (Mioduser, Nachmias, Lahav & Oren, 2000) found that "a traditional, hierarchical, highly structured, and directed instruction mode still prevails. Only 28% of the sites support inquiry-based learning" (p. 62).

The advocates for large-scale course delivery via the Web claim that Web-based education "holds extraordinary promise" (Web-based Education Commission, 2000, p. iii), but recent research suggests there are issues to be addressed. Schollie (2001) found that although 95% of Alberta students, parents, and teachers participating in on-line schooling were satisfied or very satisfied with the quality of education delivered, in fact, in mathematics, these pupils scored below regular school students on standardized achievement tests.

On-line course development is not a simple task and within the Ontario university system, movement to Web-based courses has been slow. Of the 17 distance education mathematics courses listed in the Ontario Council for University Lifelong learning [OCULL] (2001) course directory, only 3 are offered on-line. Universities are finding that, with the high costs of course development and maintenance, Web-based programs are not the revenue generating ventures anticipated. During 2001 both the Massachusetts Institute of Technology (MIT, 2001, April 4) and New York University (N.Y.U., 2001, November 30) announced that they were abandoning their for-profit programs to market on-line courses.

Innovative teachers have developed very productive and educationally sound mathematics projects integrating on-line activities, but movement from this starting point to wider and more extensive programs such as full course delivery is not a simple step. Mathematics educators need to think about, discuss, and clearly describe the types of Web-based mathematics learning environments required. The Fields Institute Mathematics On-line Working Meeting is one step in this direction.

Working Meeting Schedule

Thursday November 15, 2001

7:30 PM Public Forum

Ann Heide, University of Ottawa
Active Learning in the Digital Age Classroom

Dr. Jonathan Borwein FRSC, Simon Fraser University
Collaborative On-line Mathematics: Wishing and Hoping

9:30 PM Reception

Friday November 16, 2001

8:30 - 9:00	Coffee
9:00 - 9:30	Welcome and Introduction
9:30 - 10:30	Working Groups - Mixed Instructional Levels
10:30 - 10:50	Break
10:50 - 12:00	Working Groups - Mixed Instructional Levels
12:00 - 1:00	Lunch Break
1:00 - 1:30	Debriefing of Morning Session & Review of Afternoon Activities
1:30 - 3:10	Working Groups - Instructional level
3:10 - 3:30	Break
3:30 - 4:20	Presentations and Posters - first session
4:20 - 5:00	Presentations and Posters - second session
5:00	Reception

Saturday November 17, 2001

8:30 - 9:00	Coffee
9:00 - 10:30	Working Groups - Instructional level
10:30 - 10:50	Break
10:50 - 11:40	Presentations from Working Groups
11:40	Next steps

Discussion Questions and Tasks

Visions and Opportunities (Friday morning)-Mixed Instructional Level Groups

> We want students to be immersed in a mathematics culture that gives them an opportunity to: a) learn, use, and refine inquiry, investigation, experimentation and problem solving processes, and b) develop the tools/skills/habits of a life-long learner, learn significant concepts and procedures (with understanding) that they can then use in an integrated, authentic fashion to conduct inquiries, experiments, investigations and problems.

1. What characteristics should an on-line course possess in order to support the above vision?
2. What current 'best' classroom practices and new practices do we wish to see manifested in on-line courses?
3. What is the classroom/on-line experience that we are trying to create?
4. What on-line teaching/learning tools/technologies do we need to implement our vision? How may these be used to extend mathematics teaching and learning beyond the physical classroom and possibly improve how teachers teach and how students learn mathematics?

Issues and Challenges (Friday afternoon) - Instructional Level Groups

1. What are the issues and challenges around pedagogy?
2. What are the issues and challenges around implementation?
3. What have we learned about on-line learning environments for the teaching of mathematics? What are some of the exemplary practices in existing on-line learning initiatives (i.e. use of technology, use of resources)?
4. How do we ensure students get an opportunity to interact effectively with the subject matter, their teacher, and co-learners, witness/ learn from the model behaviour of others reflect on/communicate their actions/experiences/learning, and assess their knowledge and skills in an effective, ongoing, and authentic fashion?
5. How do we overcome outmoded perceptions of teaching, learning, outmoded perceptions of subjects such as mathematics, outmoded views of the roles of teachers and students, outmoded views of what constitutes an effective learning task/environment?
6. How do we accommodate different learning styles of students, how can we obtain the tools and proper environment for this ideal learning and delivery"

Recommendations (Saturday morning) - Instructional Level Groups

1. Where should we be five years from now?
2. What do we need to do to get there? (i.e., communication, further forums, research, development projects)?

This paper, which reflects the consensus of the Working Meeting participants, is organized using the three themes employed to structure our discussions: Visions and Opportunities, Issues and Challenges, and Recommendations.

Brief Summary of the Report

In this section we summarize some of the key ideas that emerged from the Working Meeting. The summary is organized around key questions about online learning of mathematics. The section following the summary offers reflections provided by members of The Fields Institute for Research in Mathematical Sciences. Feedback was sought on the summary from representative members with elementary, secondary, college and university mathematics education experience and expertise.

What should be the focus of online mathematics learning?
The focus should be on mathematical problem solving and the study of mathematical relationships. Learning should be situated in activity that is authentic for the discipline of mathematics. On-line activities should allow for exploration of concepts and relationships.

What advantages does online learning offer?
On-line learning has the potential of providing interactive exploration based mathematics experiences where multiple representations are linked and explored dynamically. Well designed on-line multimedia investigations of mathematics concepts and relationships will be educational for both students and teachers, helping them develop a broader understanding of mathematics content and process.

What should be the nature of online learning in mathematics education?
There should be a balance of on-line experiences, including demonstration, performance and exploration. The design of learning should be modular, and it should be possible to access and use modules in many different ways (in a fully on-line course, in a partially on-line course, and as a supplement to a classroom-based course). Students should have choices based on ability and interest. Teachers should have the choice to use a variety of strategies in their goal to help students develop a solid understanding of mathematics. Although there will be some need to create fully on-line courses, we envision on-line learning modules being used primarily as supplements and enhancements of classroom based courses.

How would the role of the teacher change in online mathematics learning?
The role of the teacher in the on-line environment is crucial, with significant impact on the nature of the learning environment, what students attend to mathematically, how the on-line resources are used by students, and what they learn and what they value. The teacher will play a key role in creating an on-line culture that values and encourages students' sense making and construction of mathematical knowledge. Teacher education and professional development will play a role in the quality of online mathematics experiences for students.

How will we ensure the successful design and implementation of online mathematics learning?
There should be careful consideration and application of technology so that it benefits mathematics education. Mathematics pedagogy should be paramount in decisions to adopt technological tools. There should be sufficient time and resources allocated to the design, implementation and evaluation and research of on-line mathematics learning. There needs to be close collaborations between experts in mathematics, mathematics pedagogy, and on-line technology at all stages of development of on-line mathematics learning. There should ready and equitable student access to on-line learning.

Reflections on the Report

The following is a summary of reflections on the report provided by members of The Fields Institute for Research in Mathematical Sciences. Feedback was sought from representatives from elementary, secondary, college and university members.

Visions and Opportunities

Our pedagogical vision is that students construct a solid understanding of mathematics and a confidence that mathematics makes sense as a whole rather than in isolated bits. This is possible when students are immersed in a mathematics learning culture that gives them the opportunity to:

- Learn, use and refine inquiry, investigation, experimentation and problem solving processes;
- Understand significant mathematics concepts, procedures and relationships that they can then use in an integrated, authentic fashion.

Such a vision of mathematics pedagogy has been persistent in mathematics education research and championed, in part or in whole, under such reform banners as discovery learning, problem solving and, most recently, constructivism. This vision is currently embedded to a significant degree and in various forms in mathematics curriculum documents across Canada and the United States. It seems rather obvious that this vision should guide us in the design on on-line mathematics teaching and learning. However, this vision has also been elusive in attempts to implement it on a large scale.

On-line learning offers the opportunity to expand the boundaries and methods of mathematics education. Presently, the landscape of on-line mathematics learning is changing quickly. Our vision is that the design, development and implementation of on-line mathematics education is well thought out and care is taken that on-line mathematics education does not set an unintended and possibly an undesired pedagogical direction. Well-designed on-line mathematics learning will require clear learning goals and cooperation between people who have technical, mathematical and pedagogical experience and expertise.

Below we outline our visions for on-line mathematics education in greater detail. The visions are organized around the four commonplaces of education, namely, the student, the teacher, the subject matter (mathematics) and the learning environment.

Student

Students learning mathematics on-line need to experience models of thinking that focus on mathematics relationships and problem solving. They also need to be able to assess their level of understanding at critical stages of the learning process:

- Students need opportunities to experience thinking and attitudes associated with mathematical activity. Some of this will be possible through appropriate interactive multimedia experiences.
- The on-line human interactions – between teacher and students, and among students – will play a key role in the mathematics thinking and attitudes that are modeled and encouraged.
- Students need opportunities to problem solve and express their math thinking and ideas. On-line design needs to take this into account.
- Students need opportunities to check their understanding. Feedback and assessment should be built into on-line activities.

We need to attend to existing mindsets of students toward on-line activity and generally computer-based activity.

- Many students associate computers with games and immediate feedback and gratification. This will not be the case with open-ended on-line explorations where there is not a game to be won and there is no immediate feedback of correct or incorrect answers. This is a re-training issue for students that we envision to be explicitly addressed in on-line learning. Part

of the solution is appropriate introductory activities that accustom students to think mathematically on-line.

Teacher

Historically, mathematics education reform has focused on teaching tools and resources without sufficient attention to the role that teachers play. Reform efforts that tried to create "teacher-proof" change in mathematics teaching have not been successful.

- Our goal is not to create teacher-proof on-line learning where it is assumed that the embedded instructional design will function as 'designed' regardless of the teacher.
- The role of the teacher in the on-line environment is crucial. The teacher's role will have significant impact on the nature of the learning environment, what students attend to mathematically, how the on-line resources are used by students, and what they learn and what they value.
- We envision the teacher playing a key role in creating an on-line learning culture that values and encourages students' sense making and construction of mathematical knowledge.
- The role of the teacher is crucial for creating a feeling of community and facilitating collaborative learning.

On-line mathematics activities may also be useful in teacher professional development and education. They can offer opportunities to improve teachers' knowledge and understanding of mathematics and increase their awareness of what it means to do mathematics.

- Well-designed on-line mathematics learning will be educational for both students and teachers and may act as pedagogical models that may affect and change a teacher's practice in the direction of the pedagogical goal outlined above. For example, appropriately designed multimedia investigations of mathematics concepts and relationships will help teachers gain new understandings of the mathematics they teach and what is important to attend to mathematically. Such on-line learning can form the basis for the mathematical development of teachers to broaden their understanding of mathematics concepts and relationships and mathematics activity.

Learning Environment

On-line mathematics learning should not necessarily be seen as meeting all of students' learning needs. We should be thinking of more options than fully on-line courses.

- Although there will be some need to create fully on-line courses, we envision on-line learning modules being used as supplements and enhancements of classroom-based courses. This may be especially the case for elementary and secondary school applications.
- On-line learning design should not be a reproduction of a textbook.
- The design of learning should be modular. It should be possible to access and use the modules in many different ways. For example, a module may be used in a fully on-line course, in a partially on-line course, and as a supplement to a classroom-based course.
- On-line support, feedback and assessment should be timely. Some of this may be automated. However, the role of the teacher and peers in this process is crucial for creating a feeling of community and facilitating collaborative learning.

On-line mathematics learning should be flexible and multi-dimensional. Motivation and learning is enhanced when students have choices based on their level of performance, interest, and

learning style. An on-line learning environment lends itself to offering students options.

- On-line activities should be flexible. We should avoid lock-step learning. Students should have some choices based on ability and interest. Teachers should have some choice to use a variety of strategies in their goal to help students construct a solid understanding of mathematics. We need to allow for opportunities for personal expression in learning and teaching.
- We can and should provide a variety of approaches on-line, in some cases in ways that we cannot in the classroom.
- We should allow flexible entry and flexible paths to completion.
- An on-line course should change with participant input. This means that the amount of course material would be considerably more extensive than the amount visible or used at any given time. As participants engage with course materials and with each other, the nature and scope of the course would change to reflect:
  - The interests of the participants
  - The level of expertise of the current group
  - Progress made to-date by individuals
  - Learning styles prevailing in the group
- Opportunities for tailoring the course to reflect the needs of individuals or to respond to a shift across a group of participants.

On-line technologies offer the potential of combining student interactive exploration of mathematics relationships and opportunities to form discussion and collaborative learning communities. We should take advantage of the strengths of on-line technologies when we design on-line mathematics learning environments.

- On-line activities should allow for exploration of concepts and relationships. This seems quite appropriate for a technologically based course or module. Multiple representations can be linked and explored dynamically, for example.
- There should be careful consideration and application of technologies and how they may best benefit mathematics education. For example, web based technology may facilitate communication, interactivity, illustrating/simulating concepts that would be difficult to illustrate in other media, and using electronic memory for diagnosis and tracking, to give a few examples.
- There should be access to mathematical objects that can be manipulated by students and teachers. It should be easy to communicate mathematically.

We need to be careful that our on-line instructional design is based on sound pedagogy and not simply on what the technology can or cannot do. Care needs to be taken that the technology is transparent and does not take student attention away from mathematics.

- Instructional design should take priority over technological form.
- We should not try to do too much. The design focus should be on what is most important for mathematics and how to do it well. What is possible technologically may not be what is desirable pedagogically.
- Simplicity should be a focus of design – technology should not become an obstacle when engaging in on-line learning.
- The interface should be intuitive, transparent, and user-friendly.
- The language used in an on-line learning environment should be invitational.
- There should be an efficient archival resource – e.g., of resources and past discussions.
- On-line tools should be compatible, so that the user does not have to carry out complex

technical tasks to make communication possible.
- There should be links to other resources beyond the course site – to provide contexts for problems and give access to data.
- There should be a significant help section: e.g., glossary, technical help, program help.

Communication on-line offers both opportunities and obstacles. There are opportunities to communicate in a variety of ways for different purposes. There are obstacles associated with text-based communication interfaces, where it is difficult to expressed ideas with mathematical language and graphical representation.
- Where appropriate, various forms of on-line collaboration should be available so the on-line experience is that of a community of learners rather than individuals doing and learning mathematics in isolation. In some cases, where on-line experiences are enhancements of regular classroom settings, this community may be created mostly in the classroom settings. In other cases, on-line discussions and sharing should be used. In the latter cases, appropriate on-line tools (allowing for text, symbolic and visual representations) will be necessary.
- There is a need to consider a variety of communication tools – discussion, whiteboard, audio, video – and their possible advantages in an on-line learning environment.
- On-line learning should allow for both group learning and individual learning.
- There should be various forms of discussion possible – private, public, synchronous, asynchronous.

Mathematics

Traditionally mathematics teaching has focused on facts, skills and procedures. Though these constitute an important part of mathematics knowledge, mathematics is primarily with problem solving and with connections and relationships among mathematics concepts and representations.
- The primary focus of mathematics activity should be on problem solving. Learning should be situated in activity that is authentic for the discipline of mathematics.
- The content and context of mathematics should engage the learner. There is a meaningful context in which mathematics is explored and understood.
- Students should be motivated to think mathematically. Care needs to be taken to ensure that the focus of on-line activity is on mathematics.

On-line technology can offer environments where students explore dynamic representations and simulations that model mathematical relationships. We need to consider which mathematics topics or areas of study we should focus on when first implementing on-line learning.
- Mathematics is the study of relationships. On-line technology appears naturally advantageous in this regard in that hypertext is good at delineating relationships.
- On-line learning can provide interactive, exploration-based mathematics experiences that help teachers and students see mathematics differently and broaden their understanding of mathematics concepts and of mathematics activity.
- There should be a balance of on-line experiences, including demonstration, performance and experimentation.
- Initial implementation should address key areas of mathematics and topics that may not be well understood (i.e., functions, rational numbers, and integers, to name a few). Consideration should be given to topics that best lend themselves to the representations possible on-line.

Issues and Challenges

The meeting began with a stimulating discussion of our vision and opportunities for mathematics online education. In the next session our focus was switched to the "reality check" of identifying issues and challenges needed in order to attain our vision. The make-up of the groups was also changed at this time to participants who were all at the same level of instruction. Each group's conclusions portrayed some unique and also some over-lapping perspectives. We found that there are many issues and challenges facing teachers, students and administrators in choosing some type of online learning. A broad issue is the perception that certain fundamental aspects of good mathematical pedagogy and curriculum will be changed when this is not necessary or even desired by educational experts. The greatest challenge is to ensure that online environments provide students with quality learning experiences. The supporting educational system has to be prepared to change and adapt to the demands that will be created with this type of learning.

The points the participants identified as issues and challenges are organized in areas of the student, the teacher, the learning environment and the subject matter of mathematics. Many of the points are applicable for all online learning environments and others are unique to only mathematics education.

<u>Student</u>

Our objective is to have online learning as one of the important ways for students to become successful in learning how to do mathematics as well as understand, appreciate and apply mathematics. This creates many challenges.

Students are involved in ensuring their own success in learning mathematics on-line.

- Issue: Online learning is dependent on student skills such as the ability to read well and be technologically proficient. In mathematics, algebraic graphing software and spreadsheet software are used extensively depending on the age of the student. Working in groups but through the medium of technology is a necessary capacity used extensively in on-line learning. Challenge: In order to be successful, students will require mastery of certain skills before taking on an online course. This presents a challenge for elementary and secondary schools to ensure that their graduates are prepared for online learning.

Educators have a responsibility to set up an environment that supports students learning mathematics.

- Issue: Students display many different learning styles when learning mathematics. A visual learning style is emphasized in online learning whereas tactile manipulation for kinesthetic learning is difficult for students to experience in this environment. Challenge: As online learning is not as holistic as face-to-face learning it is a challenge to accommodate the students' needs.
- Issue: Students want choice in the different versions of online learning. They may want to work at their own rate or they may require parts of courses - "what I need when I need it". Challenge: Many different types of online learning challenge the role of teachers, the instructional design of the courses or modules, and the administrative support for such initiatives.
- Issue: Students need ready and equal access to the Internet network in online learning.

Challenge: This presents demands on our educational technology system to ensure that students are all treated fairly in their availability to the Network.

- Issue: In this type of environment, there are barriers to student contact with peers and the teacher. Challenge: Educators must ensure that students' communication skills and access to others are not diminished.
- Issue: Students desire a safe and secure environment in taking a fully online course. Challenge: A student's personal vulnerability needs to be taken into consideration through the design and the student's involvement in any type of online course.

The mathematics content is fundamental to students learning mathematics.

- Issue: The mathematics content the students learn may be undermined in the midst of all this change to a different approach to learning. Challenge: The mathematics curriculum must be upheld and be paramount in ensuring quality mathematical learning

Teacher

The teacher's role changes to more of a facilitator and the degree of change varies depending on the type of online learning. Teachers need to maintain their professionalism of quality fundamental pedagogical practices in teaching mathematics online.

Teachers want professional development and access to research that they can apply to the teaching of on-line mathematics. Assistance from peers and experts in online learning becomes necessary for teachers when adapting to this new role.

- Issue: It is paramount that an online mathematics course be thoroughly organized and developed ahead of time. Challenge: More emphasis is put on certain aspects of the teacher's practice such as the advance preparation and complete lesson plans.
- Issue: Assessment strategies are altered to fit online learning in mathematics. Challenge: Consideration has to be given to the challenges presented by assessment through technology while maintaining quality evaluation. Also, it must be taken into consideration when developing assessment by technology that this can result in more opportunities for students to cheat.
- Issue: It is essential to have research into matters that arise for discussion such as the ideal class size, the teacher's role and the use of textbooks. Challenge: Many teachers need guidance in the actual act and type of research such as action research where they learn as they teach and observe.
- Issue: Instructional design in a technological environment takes on a new role in remote teaching. Challenge: The teacher needs new skills and knowledge for this task.
- Issue: A sharing of information with other teachers becomes even more important in this new environment. Challenge: Teachers need support in seeking advice and gathering examples of best practices from experienced role models.

Factors outside of the teacher's immediate environment can have a great effect on their role as an on-line educator.

- Issue: The issues involved in the intellectual property become even more important in such a teaching environment. Challenge: Teachers need guidelines as to the actual ownership and copyright of their own developed material.
- Issue: The teacher wants to feel safe and secure in teaching fully online modules or courses. Challenge: Protocols for students and teachers need to be developed for online relationships.

- Issue: In this change to the educational environment, teachers are in need of great assistance. Challenge: The teachers' unions and professional bodies need to be involved and proactive in supporting and protecting teachers.
- Issue: There could be an issue with society's perception that the role of students and their teachers has completely changed. Challenge: All those involved in education must be united in the message to society that the interaction between teachers and students is still fundamental to quality learning of mathematics.

Learning Environment

A partnership of experts in discipline, technology, pedagogy and design is required for this new learning environment. There is a great need for commitment from the supporting structure of administration.

On-line learning of mathematics needs all the various educators cooperating in developing an exciting, appealing and pedagogically-sound environment.

- Issue: There is a monopoly of certain online technological tools that may influence the choice of pedagogy. Challenge: Educators must ensure that good mathematical pedagogy is paramount in any decision to adopt a technological tool.
- Issue: Decisions are made as to the degree to which online learning is included in a course. Challenge: It is imperative that a concerted effort is made to involve all the relevant educational professionals in the degree to how much online learning is involved in any educational pursuit.
- Issue: Given the time constraints in any educational environment, educators are pressured to make quick decisions on the timing and the extent of online learning. Challenge: There is a need for input from all educators in deciding when and how to include online learning in the course.
- Issue: Students require ease in navigating through courses. Challenge: There are demands on the technology support staff in combination with educational expertise to ensure that the technology interface is user-friendly.

Support from administrators are paramount in ensuring success in on-line learning of mathematics.

- Issue: Meaningful material and necessary tools such as learnware need the support of time and funding to be developed properly. Challenge: Educators must be ensured that adequate financial support and resources is available in order for them to develop these quality tools and material for teaching mathematics on-line
- Issue: There is a need for educators to share and learn from each other in this environment. Challenge: Administration is asked to promote and facilitate these initiatives that involve learning from your peers.

Mathematics

The educational system needs to stay true to the subject matter of mathematics. The role of pedagogy must be in control not technology or the pursuit of efficiency.

The subject matter of mathematics should continue to stay all-important and not be diminished by the necessity of technology in on-line learning.

- Issue: Rich learning tasks are important in developing mathematical concepts. Challenge: Online technology tools need to support exploration and discussion in mathematics.
- Issue: Students should be able to easily communicate mathematics electronically. Challenge: The emphasis is on collaborative communication systems and easily manipulated mathematical objects in the online learning environment.

The on-line learning and teaching of mathematics may have a different look to the usual classroom practice but the outcome will be the same with students understanding and applying mathematics successfully.

- Issue: Best practices in a classroom may not adapt easily to online learning of mathematics. Challenge: The online environment must absorb only those actions of the classroom which fit quality mathematical learning, teaching and content.

Recommendations

Earlier in this paper, we described our vision and opportunities for on-line mathematics education as developed at the on-line mathematics conference. We focused on opportunities for students, teachers, mathematics as a subject, and the learning environment. In many cases, these visions model good face-to-face teaching practices with facilitation and on-line collaborative experiences. In this section, we outline the recommendations proposed by the conference participants.

Each public education level (elementary, secondary, college and university) discussed the issues and then provided recommendations for on-line mathematics learning. The following section will identify the recommendations by the following categories: teacher, student, mathematics, learning environment, implementation, and research. The next section will summarize the common recommendations among the groups for on-line mathematics learning.

Student

It is recommended that:
- appropriate software for elementary students is available, and
- a continuum of student on-line experiences, from elementary to post-secondary, be developed.

It is recommended that colleges:
- have policies that ensure they maintain intellectual ownership for students' research projects, and
- have appropriate use policies for the student and faculty use of technology.

Teacher

It is recommended that:
- on-line professional development for teachers be organized. A graduated series of developmental work for teachers needs to be created and teachers can pick the level that they are comfortable,
- on-line teacher learning experiences about good mathematics be developed. If teachers have a positive experience using the on-line programs themselves; there will be an easier transition for using that in their classrooms,
- boards should provide time for implementing these professional development experiences during the school week,
- experiences for teachers to become an effective moderator are provided, and
- administrative support for teachers' professional development be provided.

It is recommended that Faculties of Education:
- teach teacher candidates about moderating learning in an e-learning environment and in a classroom,
- help students, as part of their course work, develop and update a public database of screened on-line resources,
- train teachers using on-line courses so they are familiar and comfortable with the technology, and
- provide programs that include experiences online.

It is recommended that colleges:
• create opportunities for dialogue among current practitioners to discuss what does and doesn't work in relation to this new format,
• create opportunities to share the information with other faculty such as technology information, curriculum, best practices, maybe informal faculty meetings that deal with issues,
• require appropriate teaching facility, availability, hardware, need physical plant to support this new modality, and
• provide courses for faculty on on-line learning and bring in experts to assist with development, design and move course from f2f to on-line.

Learning Environment

It is recommended that each school board:
• have a committee that deals with technical support for on-line learning. This committee should develop protocols and provide information to superintendents, technology technicians, coordinators, principals/vice principals, and school board members/trustees, and
• have a committee on assessment on-line professional development to explore what tools are being used to communicate understanding of mathematics.

It is recommended that:
• quality learning environments be developed, as poor instructional models entrenched in on-line modules may be difficult to undo.
• online experiences should use a modular approach where good modules should be of value to students, teachers and parents,
• multiple approaches to online learning be taken to develop a solid understanding and allow for teacher and student choice,
• a set of criteria to evaluate on-line learning packages and to guide users in the selection or development of on-line learning experiences (criteria to focus on the quality of the educational experience such on-line learning packages support) be developed, and
• development of online modules and programs should always include the technology expert.

It is recommended that colleges and universities:
• inform the public of new format of delivery both on-line, partial on-line through providing information, research, and marketing campaigns for parents, students, and other interested parties,
• provide technology assistance such as technologists to move to on-line courses, and
• provide on-line enhancements to courses such as on-line labs & tutorials, collaborative peer-to-peer help, collaborative mathematical exploration, on-line modules for students lacking particular skills & concepts, and inter-university sharing of specialized senior courses.

Mathematics

It is recommended that:
• appropriate mathematics software be used for the delivery of online experiences,
• support should be provided for a variety of initiatives and experiments in on-line course enhancement with a focus on the doing of mathematics,
• the focus needs to be on and must be seen to be on mathematics and not technology,
• linked software such that on-line mathematical objects can be collaboratively manipulated is

needed, and
• software is needed to translate captured graphics output from one program into the mathematical objects they represent to permit further manipulation by other participants using the same or different software.

Implementation

It is recommended that:
• a strand at OAME conferences and leadership conference be created in online learning of mathematics,
• provide some recognition such as getting certificates and recognition of their participation in this pioneering stage of on-line teaching in mathematics,
• a committee be created to evaluate online software. This committee would coordinate with other organizations to evaluate software and design introductory lessons using the software,
• school boards work together to create on-line learning activities,
• coordination of implementation suggestions for teachers be facilitated,
• the white paper be disseminated to Ministry of Education, Industry, School districts and OAME, OMCA, CMS,
• on-line mathematics subcommittees be formed to investigate database of web-based mathematics resources, On-line modules, and
• examples of on-line teaching resources, Federal government sponsorship and samples of what is happening be shared with interested people.

It is recommended that colleges:
• create opportunities for access to technicians/technologists to generate a common level of understanding,
• create opportunities for union involvement with regard to on-line teaching, recognition, workload, equity,
• advocate the need to be proactive, sit on committees, integrate selves into full college life and be knowledgeable,
• support on-line learning with time, money and additional resources for on-line learning,
• acknowledge that on-line learning is teaching and requires the same level of expertise, knowledge, dedication, professionalism,
• investigate cost benefit for conducting on-line university courses of the form the group envisions could cost more than traditional delivery because faculty involvement and monitoring would be greater, and
• use common software standards, protocols, and Open source code so that experiments can modify and adapt applications to particular needs and aims.

Research

It is recommended that:
• opportunities be provided for school boards to get research on on-line learning,
• a Fields forum, perhaps in the format of the CMESG conference, with some presentations, working groups and infusion of research about on-line mathematics teaching/learning, with follow-up groups be held,
• a subcommittee of Fields gather and disseminate information from teachers using on-line mathematics teaching/learning environments,

• a public database of web-based resources de created using existing lists of resources, and
• research that allows us to characterize current typical and exemplary on-line learning practices be conducted.

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