THE FIELDS INSTITUTE
FOR RESEARCH IN MATHEMATICAL SCIENCES

ABSTRACTS 1.2

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Optimization of light elastic structures

We consider a *mass optimization problem* which consists in finding the best distribution of a given amount of elastic material, in order to achieve the minimal compliance. The unkown mass distribution is then a nonnegative measure which may vary in the class of admissible choices, with total mass prescribed, and support possibly constrained in a given *design region.* Having in mind that lower dimensional structures could be optimal, we needed to introduce a formalism which covers, in a unified way, membranes, string and junctions. The phenomenon of appearance of low dimensional network structures was already remarked in the case of optimal mixtures of two materials, when the percentage of the strong one tends to zero (*light structures*). Moreover, because of capacitary arguments, concentrated loads are forbidden in the classical framework, but they become admissible as soon as we allow the distribution of material to be singular. Then in the framework of our mass optimization problems, we are allowed to consider the general case when for a load we take a given measure.

A first result is that we obtain the existence of an optimal mass distribution for which the elastic compliance is minimal. This optimal measure may present the interesting feature to be composed by terms of different dimensions. Moreover, we characterize these optimal solutions by means of a generalized version of the *Monge-Kantorovich* partial differential equation which in its scalar version describes a mass transfer problem.

Then we make the connection with the asymptotic limit of the problem of *shape optimization problem* when the total prescribed volume of a given material tends to zero, leading in the 2D-case to Michell trusses. We conclude by some numerical results based on a p- Laplacian like approximation.