

# CODING INTO RAMSEY SETS

JORDI LÓPEZ-ABAD

In [1] W. T. Gowers has formulated and proved a Ramsey-type result which lies at the heart of his famous dichotomy for Banach spaces. He defines a family  $\mathcal{G}$  of weakly Ramsey sets of block sequences and shows that every analytic set of block sequences belongs to  $\mathcal{G}$ , though his dichotomy can be deduced from the fact that every  $G_\delta$  set of block sequences, i.e. countable intersection of open sets, belongs to  $\mathcal{G}$ . We show that  $\mathcal{G}$  is not closed under taking complements and that the full generality really appears at the  $G_\delta$  level. More precisely, we supply a rather direct proof of Gowers' result that  $\mathcal{G}$  contains all analytic sets as a direct consequence of the fact that  $G_\delta$  sets of block sequences belong to  $\mathcal{G}$ . This fact can explain why the only known applications of this technique are based on very low-ranked Borel sets (open, closed,  $F_\sigma$  or  $G_\delta$ ). We also show, answering a question of Gowers ([1]), that under a suitable large-cardinal assumption every definable set of block sequences belongs to  $\mathcal{G}$ .

## REFERENCES

- [1] W. T. Gowers, An infinite Ramsey theorem and some Banach-space dichotomies. Preprint.

UNIVERSITÉ PARIS 7- DENIS DIDEROT abad@logique.jussieu.fr

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