CHRISTOPHER CROKE
University of Pennsylvania

The Boundary Rigidity Problem

This series of talks is an introduction to the boundary rigidity problem. A long term goal would be to determine a Riemannian metric on a manifold with boundary from the distances between its boundary points. This would have applications in areas from medical imaging to seismology. Unfortunately, it is not always possible to do this. The boundary rigidity problem is to determining when it is possible. We consider Riemannian manifolds (M,B,g) with boundary B and metric g. We let d, the "boundary distance function", be the real valued function on BxB giving the distance in M (i.e. the "chordal distance") between boundary points. The question is whether there is a unique g for a given d (up to an isometry which leaves the boundary fixed). We will talk about the various conjectures, theorems and counter examples that have been developed over the years.

VICTOR ISAKOV
Wichita State University

Carleman Estimates

We will discuss weighted L^2-estimates of solutions of general partial differential equations of second order. We introduce the so-called pseudo-convexity condition for the weight function and give examples of such functions for elliptic and hyperbolic operators. Then we formulate Carleman estimates with boundary terms, and give an elementary proof for a particular case of the Helmholtz operator. This proof illustrates the general case and gives new estimates with constants not depending on the wave number.

Uniqueness and Stability in the Cauchy problem

Here, following the classical Carleman idea, we apply Carleman estimates to derive uniqueness results and stability estimates of the continuation of solutions to partial differential equations. We give the counterexample of Fritz John which shows importance of pseudo-convexity and outline recent progress in increased stability for the Helmholtz equation.

Applications to Inverse Problems and Optimal Control

By studying an "adjoint" problem we show that uniqueness of the continuation implies the so-called approximate controllability by solutions of PDE. For hyperbolic equations we will derive from Carleman estimates a stronger property called an exact controllability and its dual which is a Lipschitz stability estimate of the initial data by the lateral boundary data. Finally we outline the method of Bukhgeim-Klibanov which under certain conditions transform Carleman estimates into uniqueness results for unknown source terms and coeffieints of hyperbolic PDE. In conclusion we discuss open problems and further possibilities of Carleman estimates.

HIROSHI ISOZAKI
Tokyo Metropolitan University

Inverse Problems and Hyperbolic Manifolds

I am going to talk about recent results on inverse spectral problems related with hyperbolic manifolds.
[1] *Hyperbolic geometry and local DN map.* Consider the equation of conductivity $\nabla \cdot (\gamma \nabla u) = 0$ in a bounded domain $\Omega \subset \mathbf{R}^3$. Let Λ_γ be the associated Dirichlet-Neumann map, $\Lambda_\gamma : u|_{\partial\Omega} \to \gamma \partial u/\partial n|_{\partial\Omega}$, n being the unit outer normal to $\partial\Omega$. Take x_0 from the boundary of the convex hull of Ω, and let $B(x_0, R) = \{y; |y - x_0| < R\}$. If $\Lambda_{\gamma_1} = \Lambda_{\gamma_2}$ on $\partial\Omega \cap B(x_0, R)$ for some $R > 0$, then one can show that $\gamma_1 = \gamma_2$ on $\Omega \cap B(x_0, R)$. This means that the local knowledge of the DN map determines γ locally. This theorem has important applications in practical problems and is proved by using isometries of Möbius transformations in $\mathbf{H}^3$ (a joint work with G. Uhlmann).
[2] *Equivalence of inverse boundary value problems in euclidean and hyperbolic spaces.* The inverse boundary value problem for the Schrödinger operator in $\mathbf{R}^n$ is equivalent to that in $\mathbf{H}^n$. Moreover the inverse boundary value problem and the inverse scattering problem are equivalent in $\mathbf{H}^n$. Using this fact, one can solve the inverse spectral problems by utilizing spectral properties of Laplacians on hyperbolic spaces.
[3] *Local conformal deformations of hyperbolic metric.* Take a bounded contractible open set in any hyperbolic manifolds. Then one can solve the associated inverse boundary value problem. If the manifold is non-compact, one can introduce some spectral data at infinity of this manifold to reconstruct the local conformal deformation of the hyperbolic metric. As an example, we can deal with the inverse scattering at the cusp neighborhood at infinity.

SLAVA KURYLEV
Loughborough University

Gel'fand Inverse Boundary Problem in Multidimensions

Gel'fand inverse boundary problem consists of determination of an unknown elliptic operator on a bounded domain/manifold from the restriction to the boundary of its resolvent kernel. This kernel is assumed to be known, as a meromorphic operator-valued function, for all values of the spectral parameter. In our lectures we concentrate on the case of a Laplace operator on an unknown Riemannian manifold. Using the geometric version of the Boundary Control method we show that the Gel'fand inverse boundary problem is uniquely solvable and provide a procedure to recover the manifold and the metric. Using the theory of geometric convergence, we also study geometric conditions on an unknown manifold to guarantee stability of this inverse problem.

ADRIAN NACHMAN
University of Toronto

Introduction to Inverse Problems

This talk will give a graduate level introduction to the inverse boundary value problem of Calderon, its applications to medical and geophysical imaging, and its analysis using exponentially growing solutions of an elliptic equation. Several open problems in the field will also be presented. In the anisotropic case, the problem becomes one of recovering a metric in a Riemannian manifold with boundary from the corresponding Dirichlet-to-Neumann map for the Laplace-Beltrami operator. This leads to beautiful connections to differential geometry which will be further brought out in several of the lectures in the Symposium.

GUNTHER UHLMANN
University of Washington

The Dirichlet to Neumann Map and the Boundary Distance Function

We will consider in these introductory lectures the inverse boundary problem of Electrical Impedance Tomography (EIT). This inverse method consists in determining the electrical conductivity inside a body by making voltage and current measurements at the boundary. The boundary information is encoded in the Dirichlet-to-Neumann (DN) map and the inverse problem is to determine the coefficients of the conductivity equation (an elliptic partial differential equation) knowing the DN map. We will also consider the anisotropic case which can be formulated, in dimension three or larger, as the question of determining a Riemannian metric from the associated DN map. We will discuss a connection of this latter problem with the boundary rigidity problem which will be the topic of C. Croke's lectures. In this case the information is encoded in the boundary distance function which measures the lengths of geodesics joining points in the boundary of a compact Riemannian manifold with boundary.