ARITHMETIC AND GEOMETRY OF ALGEBRAIC VARIETIES

WITH SPECIAL EMPHASIS ON

CALABI–YAU VARIETIES AND MIRROR SYMMETRY

MARCH 4–5, 2006

ABSTRACTS

MARCH 4, 2006

9:30am: Ling Long (Iowa State University)

The modularity of certain elliptic modular surfaces

In this talk, we will explain how to obtain the modularity of special elliptic modular surfaces whose monodromy groups are noncongruence subgroups of $SL(2,\mathbf{Z})$. Applications of these modularity results will be also discussed. Joint work with A.O.L. Atkin, W.C. Li, and Z. Yang.

10:30am: Remke Kloosterman (University of Hannover and Queen's University)

The L-series of a cubic fourfold

Let X be a cubic fourfold. Suppose X contains a surface T that is not a complete intersection. In this case, we call X "special". Over the field of complex numbers, and under certain conditions on the degree of T, Hassett proved that the variety $F(X)$ of lines on X is isomorphic to the desingularized second symmetric product $S[2]$ of a K3-surface S. In this talk we prove a stronger statement: Suppose X and S are as above and S is defined over K, a subfield of the complex numbers. Suppose that S has a K-rational point. Then X has a model over K and $F(X)$ is isomorphic to $S[2]$ over K. In the case S is defined over $\mathbf{Q}$, it has a rational point and the Picard number of S equals 20, we can use the above result to prove that the "interesting" part of the L-series of X equals the L-function of a Hecke eigenform.

11:30am: Matt Papanikolas (Texas A& M University)

Hypergeometric functions over finite fields and counting points on varieties

First studied by Greene and Stanton in the 1980's, finite field hypergeometric functions are constructed as certain sums of products of Jacobi sums. Work of Ahlgren, Koike, Ono, and others have shown in certain examples that values of these hypergeometric functions are closely related to counting points on some Calabi–Yau manifolds over finite fields as well as to Fourier coefficients of modular forms. Our overall goal is to explain these phenomena, and we consider additional examples of values of 4F3-hypergeometric functions and investigate how they count points on families of varieties whose Picard-Fuchs equations are essentially hypergeometric. Joint work with Frechette.

2:00pm: Roya Beheshti–Zavareh (Queen's University)

Rational surfaces in smooth hypersurfaces

It is conjectured that a general hypersurface of degree n in $\mathbf{P}^n$ is not unirational for $n > 3$. I'll talk about some questions, arising from this conjecture, on rational surfaces contained in a smooth hypersurface of degree n in $\mathbf{P}^n$. I'll also talk about the birational geometry of the space of rational curves of low degree contained in such a hypersurface. This is joint work with Jason Starr.

3:00pm: Marco Aldi (Northwestern University)

Seidel's mirror map for abelian varieties

We introduce Seidel's mirror map and show how to compute it in the case of abelian varieties. The map depends on a symplectomorphism ρ representing the large complex structure monodromy. For the example of the two-torus, different families of elliptic curves are obtained by using different ρ which are (affine) linear in the universal cover. The case of Kummer surfaces is also considered.

4:00pm: Steven Lu (UQAM)

The birational geometry of log Calabi–Yau and log abelian varieties

The birational geometry of orbifolds and more generally logarithmic varieties has posed much challenge and there are many unanswered questions even in terms of the various definitions that can be imposed. We impose the least restrictive definition that comes naturally from the study of holomorphic geometry, the fundamental group and classification theory.

An example of the theory is our generalization of Kawamata's theorem that characterizes projective varieties birational to abelian varieties to the logarithmic context. It says for example that a quasiprojective varieties has zero log-Kodaria dimension and maximal log-irregularity if and only if its albanese map is generically injective (or finite) and its image misses at most a codimension two subset of the semi-abelian albanese. We will also discuss sharpenings of the theorem posed by Kollar via plurigenera.

5:00pm: Helena Verrill (Louisiana State University)

The A_n family of Calabi–Yau threefolds

I will discuss some aspects of geometry, modularity, and Picard-Fuchs equation of of the Calabi-Yau $n-2$ fold given by the resolution of

$$(x_1 + x_2 + ... + x_n)(a_1/x_1 + ...a_n/x_n) = t$$

where a_i are parameters and x_i coordinates of $\mathbf{P}^{(n-1)}$.

MARCH 5, 2006

9:30am: Chuck Doran (University of Washington)

Modular invariants for lattice polarized K3 surfaces

We study the class of complex algebraic K3 surfaces admitting an embedding of $H + E8 + E8$ inside the Neron-Severi lattice. These special K3 surfaces are classified by a pair of modular invariants, in the same manner that elliptic curves over $\mathbf{C}$ are classified by the J-invariant. Via the canonical Shioda-Inose structure we construct a geometric correspondence relating K3 surfaces of the above type with abelian surfaces realized as cartesian products of two elliptic curves. We then use this correspondence to determine explicit formulas for the modular invariants. As a consequence we provide the proper geometrical description of the F-theory/heterotic string duality for $H + E8 + E8$-polarized K3 surfaces. This is a joint work with Adrian Chingher.

10:30am: Andreas Rosenschon (State University of New York at Buffalo)

Motives (after M. Nori)

We explain the construction of a category of motives due to Madhav Nori.

11:30pm: Matt Kerr (University of Chicago)

Periods of the Milnor regulator and irrationality proofs

We describe how to use toric data to construct relative higher Chow cycles in $CH^n(X, n)$ for families of CY $1, 2$, and 3-folds; and how to derive the inhomogeneous Picard-Fuchs equations satisfied by the regulator "periods" of such elements. In two special cases this leads to a motivic reworking of the Apery-Beukers irrationality proofs for $\zeta(2)$ and $\zeta(3)$, the latter containing some beautiful geometry. Time permitting, we will also describe other applications and relations to Mirror Symmetry. This is joint work with Charles Doran.

12:30pm: Mike Roth (Queen's University)

Accumulation of rational points and curvature

This talk will explore a possible connection between a local invariant measuring the accumulation of rational points, and a local invariant measuring curvature.