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The Small Ball Inequality in all Dimensions

The Small Ball Inequality concerns a lower bound on the L norm of sums of Haar functions adapted to rectangles of a fixed volume. The relevant conjecture is improvement of the average case lower bound by an amount that is the square-root log of the volume of the rectangles. We obtain the first non-trivial improvement over the average case bound in dimensions four and higher. The conjecture is known in dimension 2, a result due to Wolfgang Schmidt and Michel Talagrand, with important contributions from Halasz and Temlyakov. Jozef Beck established a prior result in three dimensions, which argument we extend and simplify.

This question is related to (1) Irregularities of Distribution, (2) Probability and (3) Approximation Theory.