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Discrete fractional integral operators

We consider higher dimensional versions of discrete fractional integral operators first investigated by Stein and Wainger. Specifically, we define operators I_λ and J_λ for $0 < \lambda < 1$ by

$$I_\lambda f(n) = \sum_{m \in \mathbb{Z}_+^k} \frac{f(n - |m|^2)}{|m|^{k\lambda}}, \qquad J_\lambda f(n,t) = \sum_{m \in \mathbb{Z}_+^k} \frac{f(n - m, t - |m|^2)}{|m|^{k\lambda}};$$

here I_λ acts on functions of $\mathbb{Z}$, J_λ on functions of $\mathbb{Z}^{k+1}$. Our interest is in proving the boundedness of these operators from ℓ^p to ℓ^q for appropriate p, q, λ. We show how this can be done using complex interpolation and ideas originating from the circle method in number theory; furthermore we consider the case where $|m|^2$ is replaced by a more general quadratic form.