NXP problem for MITACS-Fields workshop: Construct stable, passive and realizable reduced order models for electronic circuits

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Abstract The super node algorithm performs model order reduction based on physical principles. Although the algorithm provides us with compact models, its stability has not thoroughly been studied yet. The loss of stability is a serious problem because simulations of the reduced network may encounter artificial behavior which render the simulations useless. The question is whether the method can be remedied in such a way that stability, and also passivity, are guaranteed.

1 Introduction

Model order reduction is of high importance within the electronics industry, as it allows much more realistic simulations. An example is the coupled simulation of an electronic circuit with its interconnects (see Figure 1) that affects the circuit behaviour considerably at high frequencies. In this case the Maxwell equations are used to describe the behaviour of the interconnect structure, leading to a large system of equations. The latter is reduced to obtain a low order model that describes the dominant behaviour, and this compact model is then coupled to the electronic circuit.

Although model order reduction for linear problems is quite mature, there are still many unsolved problems. Currently, research focuses on linear problems with many inputs or specific structures, parameterized model order reduction, techniques for coupled problems, and methods for nonlinear problems (see [1] for an extensive overview). Most of these methods are based on Krylov subspace techniques or trun-

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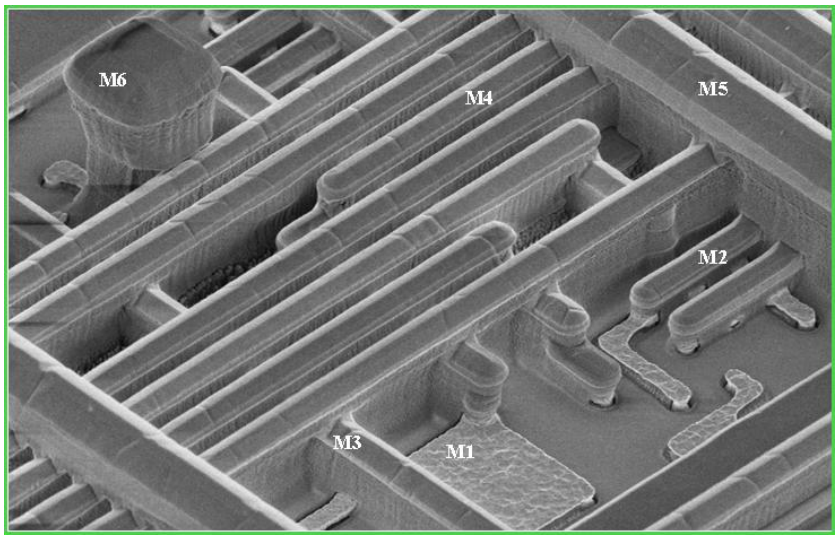

Fig. 1 Interconnect structure

cated balanced realization, originating from the fields of numerical linear algebra and systems and control theory, respectively.

There are also physically inspired methods, and one of these is the so-called super node algorithm. This method can be used to capture the behaviour of a printed circuit board into a compact model. After a boundary element discretisation on a relatively fine mesh, only a set of super nodes is kept whose distance is determined by the maximum frequency considered, making it very efficient [2]. The problem is that the resulting system is not always stable and passive, which means that time domain simulations with the reduced order model could explode at some point in time. Indeed, this phenomenon is observed, and is one of the reasons the super node algorithm is not used. We feel that it should be possible to remedy the super node algorithm, and come up with a stable and passive version.

An additional constraint is that the reduced order model should be in the form of an RLC circuit, so that it can easily be incorporated into a circuit simulator. The latter question also holds more generally for reduced order models obtained via more traditional approaches (Krylov, TBR), but remains unsolved till now. It would be very advantageous if reduced order models, both for the super node algorithm and for Krylov subspace algorithms, could be cast into the form of an electronic circuit consisting of resistors, capacitors and inductors. A combination of methods from graph theory, optimization and numerical analysis could lead to the solution of this problem, is our feeling.

2 Derivation of the model used in FASTERIX

In the design of electronic systems, the electromagnetic field concept is replaced by the electric circuit concept. This implies that the electromagnetic field in an electronic system is locally described by voltages and currents that satisfy Kirchhoff's laws. Circuit simulations that play an important role in circuit design make use of this electronic concept.

FASTERIX is a layout simulation tool for EMC (electromagnetic compatibility) modelling. It transforms PCB properties into an equivalent RLC circuit model which is then reduced into a compact one with approximately the same behavior. As a reduction technique it uses the so called 'super node algorithm'.

The approach of compact equivalent circuits makes FASTERIX an excellent and fast program for the frequency analysis of the electromagnetic behavior for PCB's. Although the super node algorithm provides us with compact models, the problem of carrying out simulations in the time domain still exists. One can think of using another reduction techniques instead of the super node algorithm, which can preserve stability and passivity. The advantage of the super node algorithm is that it is inspired by physical insight into the models, and produces reduced RLC circuits depending on the maximum predefined frequency. This fact makes the algorithm an attractive technique for EMC modelling.

FASTERIX translates electromagnetic properties of a PCB into a circuit model which is described by the system of Kirchhoff's equations

$$(\mathbf{R} + s\mathbf{L})I - \mathbf{P}V = 0 \tag{1}$$

$$\mathbf{P}^T I + s\mathbf{C}V = J \tag{2}$$

where $\mathbf{R} \in \Re^{\varepsilon \times \varepsilon}$ is the resistance matrix, $\mathbf{L} \in \Re^{\varepsilon \times \varepsilon}$ is the inductance matrix, $\mathbf{P} \in \Re^{\varepsilon \times \eta}$ is an incidence matrix, $\mathbf{C} \in \Re^{\eta \times \eta}$ is the capacitance matrix, $I \in C^{\varepsilon}$ is the vector of currents flowing in the branches, $V \in C^{\eta}$ is a vector of voltages at the nodes. Vector $J \in C^{\varepsilon}$ collects the terminal currents flowing into the interconnection system. Value s is a complex number with negative imaginary part: $s = -j\omega$. Matrices $\mathbf{R}$, $\mathbf{L}$, $\mathbf{C}$ are symmetric and positive definite.

In order to obtain an equivalent network model, the set of all nodes in the circuit described by (1)-(2) is subdivided into two subsets N and N' where N denotes the subset of nodes which are retained in the reduced circuit, and N' is for all other nodes. Due to this, vectors V, J and matrices $\mathbf{P}$, $\mathbf{C}$ are partitioned into blocks, see [2], chapter 8.

If we consider the voltages at the nodes (from the subset N) as an input $v^{(p)}$, and currents flowing into the system through the nodes as an output $i^{(p)}$, we arrive at the following system:

$$\left(\underbrace{\begin{pmatrix} \mathbf{R} & -\mathbf{P}_{N'} \\ \mathbf{P}_{N'}^T & \mathbf{0} \end{pmatrix}}_{\mathbf{G}} + s \underbrace{\begin{pmatrix} \mathbf{L} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_{N'N'} \end{pmatrix}}_{\mathbf{C}} \right) x = \begin{pmatrix} \mathbf{P}_N \\ -s\mathbf{C}_{N'N} \end{pmatrix} v^{(p)}, \tag{3}$$

$$i^{(p)}(s) = \begin{pmatrix} \mathbf{P}_N^T & s\mathbf{C}_{N'N}^T \end{pmatrix} x + s\mathbf{C}_{NN} v^{(p)}, \tag{4}$$

where $x = \begin{pmatrix} I & ,V_{N'} \end{pmatrix}^T$. It should be noted that in (3) the matrix $\mathbf{G}$ is positive real, and matrix $\mathbf{C}$ is positive semi-definite. It is enough to prove that the unreduced system is stable.

The currents $i^{(p)}$ obtained each time under a unity voltage $v^{(p)}$ at a particular input terminal, the others being grounded, constitute the columns of the desired admittance matrix $\mathbf{Y}$. The following holds

$$\mathbf{Yv} = \mathbf{i}, \tag{5}$$

where the the matrix $\mathbf{v} = \begin{pmatrix} v1 & \dots & v_N \end{pmatrix}$ corresponds to the identity matrix; columns of the matrix $\mathbf{i}$ are unknown vectors i^p. Thus, matrix $\mathbf{Y}$ describes the reduced circuit at a particular frequency ω.

3 Super Node Algorithm

The idea of the super node algorithm is to build a reduced circuit which consists of frequency independent elements. Such circuit is described by approximation of the admittance matrix $\mathbf{Y}$. In the first step, the system (3)-(4) is subdivided into two systems by introducing the following expansions of I and $V_{N'}$ in powers of (ik_0h)

$$V_{N'} = V_0 + V_1(ik_0h), \tag{6}$$

$$I = I_0 + I_1(ik_0h), \tag{7}$$

where k_0 is the free space wave number, and h is the mesh size. Equations (6)-(7) hold true when $k_0h \ll 1$, see [2] for details. Pairs (I_0, V_0) and (I_1, V_1) are obtained from two sets of equations which are found from (3)-(4) by gathering appropriate terms with orders $(ik_0h)^0$ and $(ik_0h)^1$:

$$(R + sL)I_0 - P_{N'}V_0 = P_N V_N, \tag{8}$$

$$-P_{N'}^T I_0 = 0, \tag{9}$$

$$(R + sL)I_1 - P_{N'}V_1 = 0, \tag{10}$$

$$-P_{N'}^T I_1 = s(C_{N'N'}V_0 + C_{N'N}V_N). \tag{11}$$

Thus, the expression for the admittance matrix is

$$Y = P_N^T(I_0 + I_1) + sC_{NN'}V_0 + sC_{NN}V_N. \tag{12}$$

Depending on the frequency range of interest, three types of approximations of Y can be distinguished. They are for low, high and full frequency range. We will consider the last two of them in details. The reduced circuits described by some approximation can be used in a circuit analysis program.

3.1 Admittance matrix for the high frequency range

To get an approximation of the admittance matrix for the high frequency range, the following expressions for $\mathbf{I}_0$, $\mathbf{V}_0$, $\mathbf{I}_1$ and $\mathbf{V}_1$ through frequency independent quantities $\mathbf{I}_{00}$, $\mathbf{V}_{00}$, $\mathbf{I}_{01}$, $\mathbf{V}_{01}$, $\mathbf{I}_{10}$ and $\mathbf{V}_{10}$ are introduced:

$$\mathbf{I}_0 = s^{-1}\mathbf{I}_{00} + s^{-2}\mathbf{I}_{01}, \tag{13}$$

$$\mathbf{V}_0 = \mathbf{V}_{00} + s^{-1}\mathbf{V}_{01}, \tag{14}$$

$$\mathbf{I}_1 = s\mathbf{I}_{10} + \mathbf{I}_{11}, \tag{15}$$

$$\mathbf{V}_1 = s^2\mathbf{V}_{10} + s\mathbf{V}_{11}, \tag{16}$$

where the pairs $(\mathbf{I}_{00}, \mathbf{V}_{00})$, $(\mathbf{I}_{01}, \mathbf{V}_{01})$, $(\mathbf{I}_{10}, \mathbf{V}_{10})$, $(\mathbf{I}_{11}, \mathbf{V}_{11})$ are found from four linear systems of equations. These equations can be easily found from (8)-(11) by gathering appropriate terms with coefficients (s^0, s^{-1}), (s^{-1}, s^{-2}), (s^2, s^{-1}), (s^1, s^0), see [?] for details. Thus the approximation of $\mathbf{Y}$ in the frequency range $\omega > \Omega$ is

$$\mathbf{Y}_{hf} \sim (s)^{-2}\mathbf{Y}_R + (s)^{-1}\mathbf{Y}_L + \mathbf{Y}_G + (s)\mathbf{Y}_C. \tag{17}$$

Each pair of nodes in the reduced circuit contains a branch, Fig. 5. Resistance R, inductance L, capacitance C, and resistance of conductance G can be found from the admittance matrices $\mathbf{Y}_R$, $\mathbf{Y}_L$, $\mathbf{Y}_G$ and $\mathbf{Y}_C$ in (17).

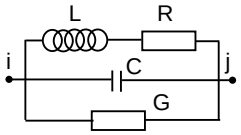

Fig. 2 High frequency branch between two nodes

3.2 Admittance matrix for the full frequency range

In [2], a method to obtain an approximation of the admittance matrix for the full frequency range was suggested. It is based on the construction of the stable approximation for the admittance matrix $Y_r l$. By introducing the null space of $P_{N'}^T$, from (3) we can find:

$$P_{N'}^T \mathscr{C} = 0, C^T P_{N'} = 0.$$

Then from (3) we can get:

$$I = \mathscr{C} \left(\mathscr{C}^T (R + sL) \mathscr{C} \right)^{-1} \mathscr{C}^T P_N V_N. \tag{18}$$

Let $A = \mathscr{C}^T R \mathscr{C}$, $B = \mathscr{C}^T L \mathscr{C}$ and A, $B \in \Re^{n \times n}$, where $n = |\varepsilon| - |N'|$. $\mathscr{C}$ has full column rank. Matrices A and B are symmetric positive definite.

Then we can define

$$Y_{RL} = P_N^T I_0 = P_N^T \mathscr{C} (A + sB)^{-1} \mathscr{C}^T P_N V_N = \sum_{i=1}^{n} \frac{r_i}{(s - \lambda_i)}, \tag{19}$$

where $r_i = (P_N^T \mathscr{C} x_i)(y_i^T \mathscr{C}^T P_N)$ are residuals and λ_i are generalized eigenvalues of the generalized eigenvalue problem $Ax = \lambda Bx$.

It is supposed that the contributions Y_G and Y_C are taken from the high frequency range. Then

$$Y = Y_{RL} + Y_G + sY_C. \tag{20}$$

It means that now a branch between any pair of super nodes consists of $n+2$ connections: n of them have a resistor in series with an inductance; two others contain conductance and capacitor.

Thus, if n is large, evidently it can be time-consuming process to model circuit in a circuit analysis program. Also the numerical computations of all eigenvalues and eigenvectors of the generalized eigenvalue problem becomes expensive as soon as n becomes lager than a few hundreds. Thus, in practice an approximation of the admittance matrix can be obtained. In view of the expression (20), it is natural to look for an approximation of Y_{rl} with a number of terms $m << n$.

In a computer program, it can be done as follows: calculating m low and high eigenvalues λ_i of the generalized eigenvalue problem and some admittance matrices Y_k, for an appropriate chosen set of $m+2$ negative real values s. The set of these matched frequencies, s_k, consists of some large negative values and some small negative values.

An element of the branch admittance matrix is approximated by

$$y_{ij} = y_{G,ij} + s y_{C,ij} + \sum_{l=1}^{m} \frac{\widetilde{r}_{l,ij}}{(s + \lambda_l)} \tag{21}$$

where the coefficients $y_{G,ij}$, $y_{C,ij}$ and $\widetilde{r}_{l,ij}$ are obtained by solving the following set of $m+2$ equations:

$$y_{G,ij} + sy_{C,ij} + \sum_{l=1}^{m} \frac{\widetilde{r}_{l,ij}}{(s+\lambda_l)} = y_{ij}, \text{k=1,...m+2.} \tag{22}$$

An equivalent circuit which represents the admittance matrix consists of branches between every pair of circuit nodes. Each branch consists of m parallel connections of a series resistor R and inductor L, in parallel with a capacitor C, and a resistor of conductance G. Thus for the branch between the circuit nodes i and j

$$R_l = \lambda_l \widetilde{r}_{l,ij}^{-1},$$

$$L_l = \widetilde{r}_{l,ij}^{-1},$$

$$C = y_{C,ij},$$

$$G = y_{G,ij}.$$

3.2.1 Stability

For carrying out reliable simulations in the time domain, first it should be checked that the system is stable. The high frequency range model described by (17) is not stable, and it is usually used for simulations in the frequency domain. To show instability of it, we can show it through BIBO stability (bounded input - bounded output). The condition for BIBO stability is that the impulse response be absolutely integrable. The impulse response is the inverse Laplace transform of a given transfer function:

$$\mathscr{L}^{-1}\{Y(s)\} = Y_R \left\{\frac{1}{s^2}\right\} + Y_L \left\{\frac{1}{s}\right\} + \{Y_G\} + Y_C L^{-1}\{s\}, \tag{23}$$

Since this condition must apply to each element of $Y(s)$, it is easy to show that the inverse Laplace transform of the first element is not bounded, and corresponds to a ramp function which is in fact not bounded:

$$\mathscr{L}^{-1}\left\{\frac{1}{s^2}\right\} = tu(t). \tag{24}$$

From the other side, the full frequency model (20) is stable (but generally not passive). Thus, it can be already used for the simulations in the time domain.

3.2.2 Experiment

An illustration of possible stability violation is given below. The example consists of two printed striplines, which are parallel to each other. The striplines are 1 mm wide and 15 mm long. The maximum frequency for modeling the strips is 1MHz. The mesh is shown in Figure 1. The black dots are the super nodes, defined with the

mesh. A voltage source is at the two ports of the lower strip. The voltage is measured over the upper strip. The input pulse has a rise time of 1 picoseconds.

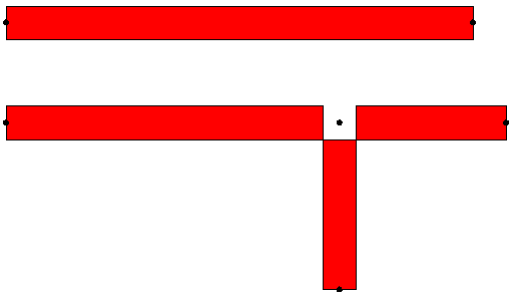

Fig. 3 High frequency branch between two nodes

In Figure 4, the simulation of the full non-reduced circuit in time domain is presented. All poles of the corresponding model are stable and stay in the right half plane. Hence the reduced circuit behaves unstable, see Figure 5 and contains a few unstable poles. This problem is currently under investigation. Observation shows that a few artificial large resistances and inductances appear at the step of finding residuals y_{ij} in (22). At this point the fact that passivity is not preserved does not play crucial role because of the reduced circuit already contains unstable poles.

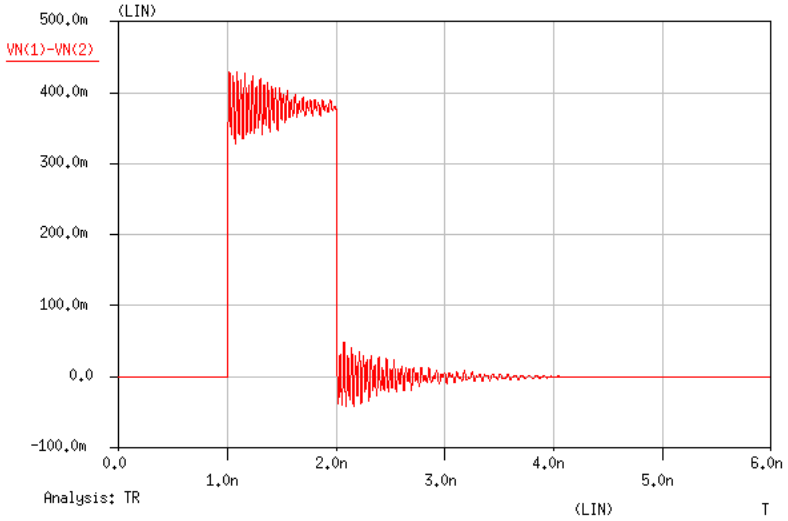

Fig. 4 Time domain simulation of the nonreduced citcuit

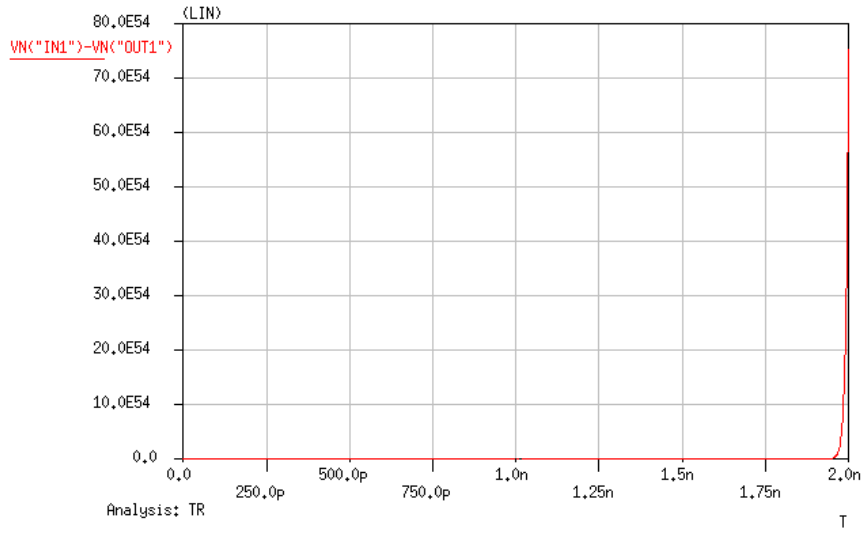

Fig. 5 Time domain simulation of the reduced circuit

4 References

[1] ”Model order reduction: theory, research aspects and applications”, W.H.A. Schilders, H.A. van der Vorst and J. Rommes, Springer Verlag (2008)

[2] W.H.A. Schilders and A.J.H. Wachters in the Handbook of Numerical Analysis, volume XIII, special volume on ”Numerical Methods in Electromagnetics”, Elsevier (2005)