TWO EXTENSIONS TO

FORWARD START OPTIONS VALUATION

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TALK OUTLINE

1. Forward start options.

2. Literature review.

3. Purpose.

4. Affine jump-diffusion (AJD) financial model.

5. FFT approach.

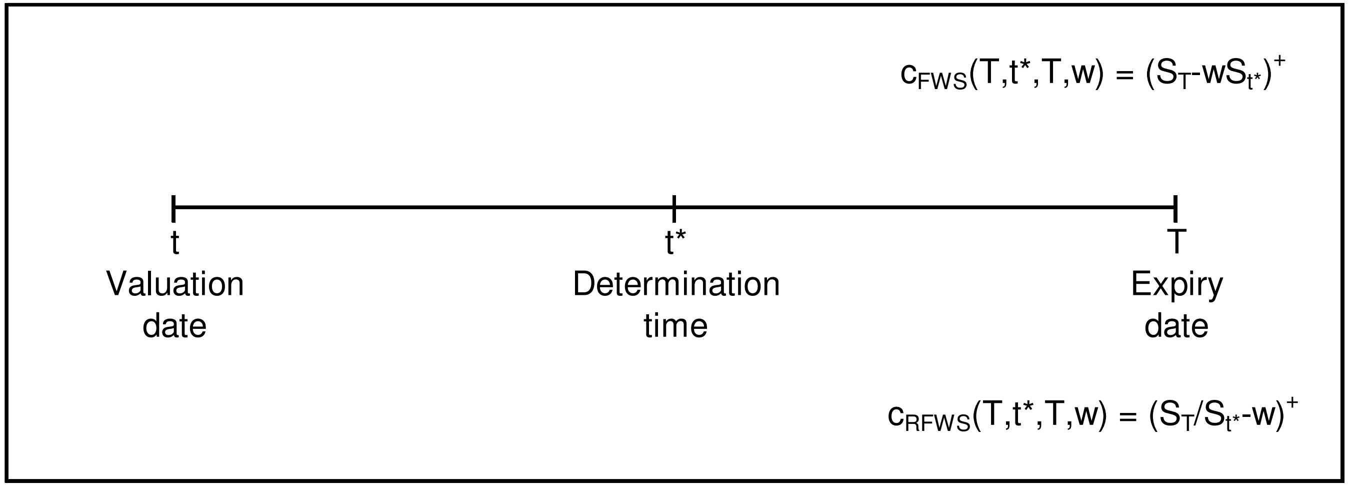
6. Direct integration approach (main result).

7. Numerical results.

8. Conclusions.

FORWARD START OPTIONS

- Two types of European-style FS (call) options:



LITERATURE REVIEW

- Rubinstein (1991): under Black-Scholes-Merton framework.

- Kruse and Nögel (2005):

  - under Heston (1993) SV model; but

  - two 2-dim integrations.

- Mercurio and Moreni (2005): solves integration wrt SV.

LITERATURE REVIEW (cont)

- Hong (2004) approach:
  - single 1-dim Fourier transform inversion;
  - requires characteristic function of the forward rate of return;
  - "applicable" to any exponential affine Lévy model;
  - BUT requires *model dependent* optimization of a dampening factor (α) to ensure square-integrability.

LITERATURE REVIEW (cont)

- Haastrecht and Pelsser (2009):
  - Hong (2004) approach under
    * SV model of Schöbel and Zhu (1999);
    * Gaussian TS model of Hull and White (1993); and
    * a full correlation structure.

PURPOSE

- Alternative pricing methodology:
  - Valid under the general AJD framework of Duffie, Pan and Singleton (2000);
  - Only requires plain-vanilla option to be homogeneous of degree 1 in spot and strike;
  - Does not require any parallel optimization routine;
  - Yields a single (and exact) Fourier inversion $\Longrightarrow$ no truncation error;
  - Straightforward to implement (e.g. Gaussian quadrature);
  - Better accuracy-efficiency trade-off than the usual Hong (2004) approach.

AJD FRAMEWORK

- As in Duffie et al. (2000):
  - Markovian model factors $X \in \mathbf{D} \subseteq \mathbb{R}^n$:

$$dX_t = [K_0(t) + K_x(t) \cdot X_t]\, dt + \sigma(X_t, t) \cdot dW_t^{\mathbb{Q}} + dZ_t^{\mathbb{Q}}, \quad (1)$$

$$\sigma(X_t, t) \cdot \sigma(X_t, t)' = H_0(t) + \sum_{k=1}^{n} H_x^{(k)}(t)\,(X_t)_k\,, \quad (2)$$

with $K_0(t) \in \mathbb{R}^n$, $K_x(t)\,, H_0(t)\,, H_x^{(k)}(t) \in \mathbb{R}^{n \times n}$.

  - Jump-arrival intensity: $(l_0(t) \in \mathbb{R},\ l_x(t) \in \mathbb{R}^n)$

$$\lambda(X_t, t) = l_0(t) + l_x(t)' \cdot X_t. \quad (3)$$

  - Short-term interest rate: $(\rho_0(t) \in \mathbb{R},\ \rho_x(t) \in \mathbb{R}^n)$

$$r(X_t, t) = \rho_0(t) + \rho_x(t)' \cdot X_t. \quad (4)$$

AJD FRAMEWORK

- Underlying asset $S_t = \exp[(X_t)_1]$ pays continuous (but deterministic) dividend-yield $\delta \in \mathbb{R}$.

- Hence, $X_t = (\ln(S_t), Y_t)$, where $Y_t \in \mathbf{D}_y \subset \mathbb{R}^{n-1}$.

- **Assumption 1** (homogeneity requirement):

$$(K_x(t))_{i,1} = \left(H_x^{(1)}(t)\right)_{i,j} = (l_x(t))_1 = (\rho_x(t))_1 = 0, \qquad (5)$$

  for $i, j = 1, \ldots, n$.

- Very general AJD framework!

AJD FRAMEWORK

- Therefore, and based on Duffie et al. (2000, Proposition 1):

$$\begin{aligned}\psi(u,t,T;X_t) &= \mathbb{E}_{\mathbb{Q}}\left\{\exp\left[-\int_t^T r(X_s,s)\,ds\right]\exp\left(u'\cdot X_T\right)\middle|\,\mathcal{F}_t\right\} \\ &= \exp\left[\alpha(t,T;u)+u_1\ln(S_t)+\beta_y(t,T;u)'\cdot Y_t\right] \quad (6)\end{aligned}$$

where

  - u_1 is the first element of vector $u \in \mathbb{C}^n$; and
  - $\beta_y \in \mathbb{C}^{n-1}$ and $\alpha \in \mathbb{C}$ satisfy known complex-valued ODEs.

- **Proposition 1** (marginal characteristic functions):

$$\begin{aligned} & f_j\left(T, \phi; S_t, Y_t\right) \\ = & \mathbb{E}_{\mathbb{Q}_j}\left[e^{i\phi \ln(S_T)} | \mathcal{F}_t\right] \\ = & \exp\left[\lambda_{c,j}\left(t, T; \phi\right) + i\phi \ln\left(S_t\right) + \lambda_{y,j}\left(t, T; \phi\right)' \cdot Y_t\right], \end{aligned} \quad (7)$$

for $\phi \in \mathbb{C}$, $j = 1, 2$,

  - where $\lambda_{c,j}\left(t, T; \phi\right)$ and $\lambda_{y,j}\left(t, T; \phi\right)$ are simple functions of δ, $\beta_y \in \mathbb{C}^{n-1}$ and $\alpha \in \mathbb{C}$;

  - and

EMM	Numeraire
$\mathbb{Q}^S \equiv \mathbb{Q}_1$	$S_t e^{\delta t}$
$\mathbb{Q}_T \equiv \mathbb{Q}_2$	$P\left(t, T\right)$

AJD FRAMEWORK

- Plain-vanilla options:
  - Duffie et al. (2000, Equation 3.5) would involve 2 Fourier transform inversions;
  - Instead, can use Lee (2004, Theorem 5.1), Attari (2004, Equation 14) or Kilin (2007, Equation 14):

$$c_t\left(K,T;S_t,Y_t\right)=S_t e^{-\delta(T-t)}-\frac{KP\left(t,T\right)}{2}-K\Omega\left(t,K,T;S_t,Y_t\right), \tag{8}$$

where

$$\begin{aligned} & \Omega\left(t,K,T;S_t,Y_t\right) \\ = \ & P\left(t,T\right)\frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{e^{-i\phi \ln(K)} f_2\left(T,\phi;S_t,Y_t\right)}{\phi^2+i\phi}\right]d\phi. \end{aligned} \tag{9}$$

FFT APPROACH

- **Proposition 2** (Hong (2004)):

$$
\begin{aligned}
& c_{FWS}(t, t^*, T, \omega) \qquad (10) \\
= \;& \omega e^{-\delta(T-t)} S_t \frac{e^{-\alpha \ln(\omega)}}{\pi} \\
& \text{Re}\left[\int_0^\infty e^{-iu\ln(\omega)} \frac{g_1(t^*, T, u - i(\alpha - 1); S_t, Y_t)}{\alpha(\alpha - 1) - u^2 + i(2\alpha - 1)u} du\right],
\end{aligned}
$$

where $\alpha \in \mathbb{R}_+$, and

$$
g_j(t^*, T, \phi_z; S_t, Y_t) := \mathbb{E}_{\mathbb{Q}_j}\left[e^{i\phi_z z(t^*, T)} \middle| \mathcal{F}_t\right] \qquad (11)
$$

is the characteristic function of the forward rate of return

$$
z(t^*, T) := \ln\left(\frac{S_T}{S_{t^*}}\right),
$$

for $j = 1, 2$ and $\phi_z \in \mathbb{C}$.

FFT APPROACH

- **Proposition 3**: $g_j(t^*, T, \phi_z; S_t, Y_t)$ can be obtained from the (marginal) characteristic function of the additional state variables Y (and independently of S_t!):

$$
\begin{aligned}
h_j\left(T, \phi_y; Y_t\right) &= \mathbb{E}_{\mathbb{Q}_j}\left(e^{i\phi_y' \cdot Y_T} | \mathcal{F}_t\right) \\
&= \exp\left[l_{c,j}\left(t, T; \phi_y\right) + l_{y,j}\left(t, T; \phi_y\right)' \cdot Y_t\right], \quad (12)
\end{aligned}
$$

  - for $j = 1, 2$, where $\phi_y \in \mathbb{C}^{n-1}$, and
  - $l_{c,j}\left(t, T; \phi_y\right)$ and $l_{y,j}\left(t, T; \phi_y\right)$ are simple functions of δ, $\beta_y \in \mathbb{C}^{n-1}$ and $\alpha \in \mathbb{C}$.

DIRECT INTEGRATION APROACH

- **Proposition 4**:

$$\begin{aligned}
& c_{FWS}\left(t,t^*,T,\omega\right) \\
= \;& S_t e^{\delta t}\mathbb{E}_{\mathbb{Q}^S}\left[\left.\frac{c_{FWS}\left(t^*,t^*,T,\omega\right)}{S_{t^*}e^{\delta t^*}}\right|\mathcal{F}_t\right] \\
= \;& S_t e^{-\delta(t^*-t)}\mathbb{E}_{\mathbb{Q}^S}\left[\left.\frac{c_{t^*}\left(\omega S_{t^*},T;S_{t^*},Y_{t^*}\right)}{S_{t^*}}\right|\mathcal{F}_t\right] \\
= \;& S_t e^{-\delta(T-t)} - \omega S_t e^{-\delta(t^*-t)}\left\{\frac{1}{2}\mathbb{E}_{\mathbb{Q}^S}\left[\left.P\left(t^*,T\right)\right|\mathcal{F}_t\right]\right. \\
& \left.+\mathbb{E}_{\mathbb{Q}^S}\left[\left.\Omega\left(t^*,\omega S_{t^*},T;S_{t^*},Y_{t^*}\right)\right|\mathcal{F}_t\right]\right\} \\
= \;& S_t e^{-\delta(T-t)} - \omega S_t e^{-\delta(t^*-t)}\left\{\frac{1}{2}\mathbb{E}_{\mathbb{Q}^S}\left[\left.P\left(t^*,T\right)\right|\mathcal{F}_t\right]\right. \\
& \left.+\mathbb{E}_{\mathbb{Q}^S}\left[\left.\Omega\left(t^*,\omega,T;1,Y_{t^*}\right)\right|\mathcal{F}_t\right]\right\}. \qquad (13)
\end{aligned}$$

DIRECT INTEGRATION APROACH

- **Proposition 5**:

$$
\begin{aligned}
& \mathbb{E}_{\mathbb{Q}^S}\left[\Omega\left(t^*, \omega, T; 1, Y_{t^*}\right)\middle| \mathcal{F}_t\right] \\
= \; & \mathbb{E}_{\mathbb{Q}^S}\left\{\frac{P\left(t^*, T\right)}{\pi} \int_0^{\infty} \operatorname{Re}\left[\frac{e^{-i\phi \ln(\omega)} f_2\left(T, \phi; 1, Y_{t^*}\right)}{\phi^2 + i\phi}\right] d\phi \middle| \mathcal{F}_t\right\} \\
= \; & \mathbb{E}_{\mathbb{Q}^S}\left\{\frac{P\left(t^*, T\right)}{\pi} \int_0^{\infty} \operatorname{Re}\left[\frac{e^{-i\phi \ln(\omega)}}{\left(\phi^2 + i\phi\right) P\left(t^*, T\right)}\right.\right. \\
& \left.\left.\exp\left(\alpha\left(t^*, T; (i\phi, \underline{0})\right) + \beta_y\left(t^*, T; (i\phi, \underline{0})\right)' \cdot Y_{t^*}\right)\right] d\phi \middle| \mathcal{F}_t\right\} \\
= \; & \frac{1}{\pi} \int_0^{\infty} \operatorname{Re}\left\{\frac{\exp\left[\alpha\left(t^*, T; (i\phi, \underline{0})\right) - i\phi \ln(\omega)\right]}{\phi^2 + i\phi}\right. \\
& \left.\mathbb{E}_{\mathbb{Q}^S}\left[\exp\left(\beta_y\left(t^*, T; (i\phi, \underline{0})\right)' \cdot Y_{t^*}\right)\middle| \mathcal{F}_t\right]\right\} d\phi. \qquad (14)
\end{aligned}
$$

DIRECT INTEGRATION APROACH

- Explicit and single 1-dim integral pricing solution (even for $n > \mathbf{1}$);

- Modulo to the specification of $\beta_y(t, T; u) \in \mathbb{C}^{n-\mathbf{1}}$ and $\alpha(t, T; u) \in \mathbb{C}$;

- Quadratic term on the denominator $\Longrightarrow$ fast rate of decay;

- Closed-form solutions for functions $\beta_y(t, T; u) \in \mathbb{C}^{n-\mathbf{1}}$ and $\alpha(t, T; u) \in \mathbb{C}$ under the Bakshi, Cao and Chen (1997) model:

  - Stochastic volatility; Stochastic interest rates; Jumps in the asset returns;

  - Nests Heston (1993) model.

NUMERICAL RESULTS

- Heston (1993) model.

- 4 $\neq$ parameter settings:

  - Bakshi et al. (1997, Table III)—S&P 500 call option prices;

  - Broadie and Kaya (2006, Table 1)—S&P 500 futures option prices;

  - Broadie and Kaya (2006, Table 2)—equity option market;

  - Andersen (2007, Table 1)—long-dated currency options.

NUMERICAL RESULTS

- Proxy for the *exact* FS option price:
  - Quadratic exponential (and martingale-corrected) Monte Carlo scheme of Andersen (2007);
  - 32 steps per year and 10^7 paths.

- Proposed direct integration approach:
  - Gauss-Laguerre with 100 weights and abscissas;
  - Gauss-Lobatto adaptive quadrature of Gander and Gautschi (2000):
    * $[0, \infty) \rightarrow [0, 1]$ following Kahl and Jackel (2006, Equation 41);
    * Relative tolerance of 10^{-12}.

NUMERICAL RESULTS

- Hong (2004) approach:
  - FFT method:
    - Log-strike grid with $16,384$ prices and constant spacing of size 0.01.
  - Optimal dampening parameter α—Lord and Kahl (2007) algorithm.
  - Gauss-Lobatto quadrature is also tested.
  - Extension of the COS approximation of Fang and Oosterlee (2008):
    - Pdf of $z(t^*, T)$ is replaced by its Fourier-cosine series expansion with 10^4 terms;
    - Same integration range as in Fang and Oosterlee (2008).

Table 1: ATM FS options with $T - t = 2$ years, $t^* - t = 1$ year, $\delta = 0\%$ and $S_t = \$100$

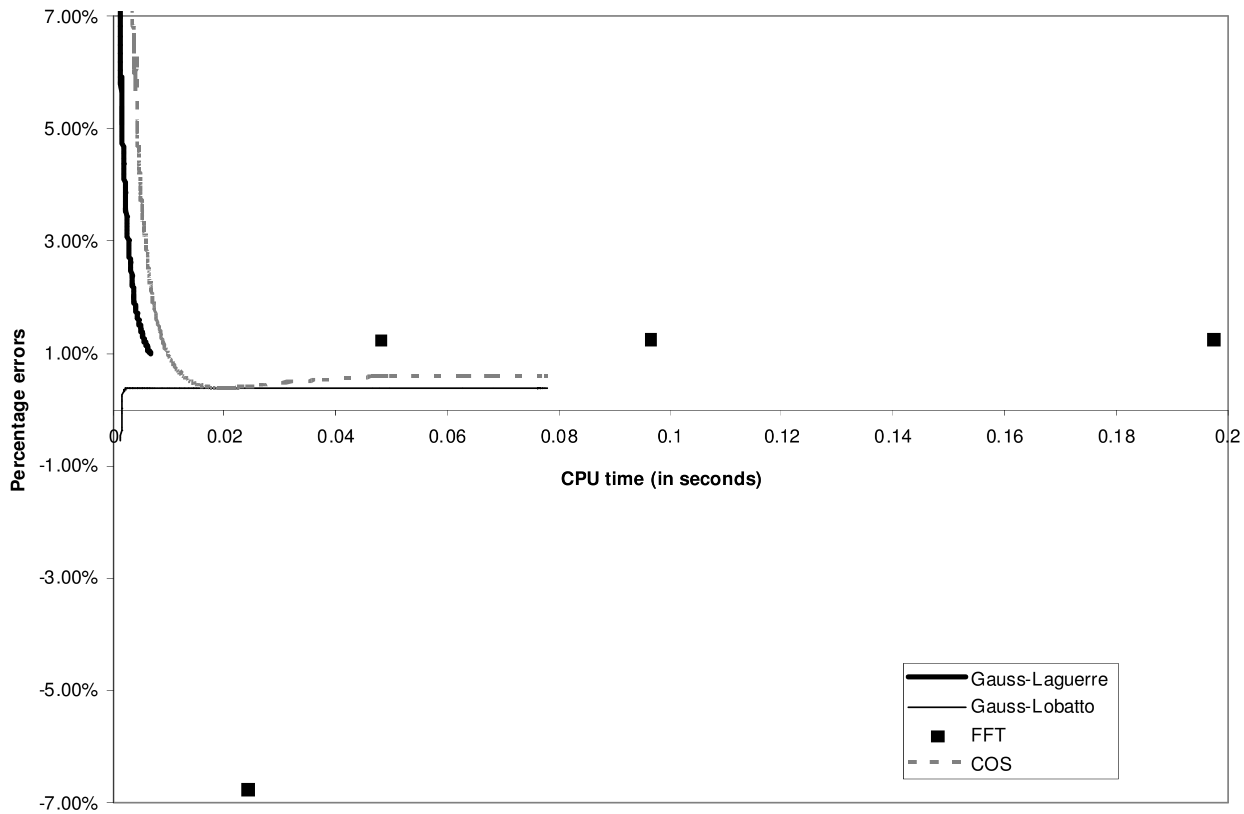
Model setup	ρ	r	Monte Carlo QEM scheme price	Monte Carlo QEM scheme %SE	Propositions 4 and 5 G-Laguerre %errors	Propositions 4 and 5 G-Lobatto %errors	Hong (2004) FFT %errors	Hong (2004) G-Lobatto %errors	Hong (2004) COS %errors
$\kappa_v = 1.15$	-0.64	0.04	8.517	0.006	-0.019	-0.019	-0.019	-0.019	-0.019
$\theta_v = 0.04$	-0.9	0.04	8.452	0.004	-0.001	-0.001	-0.001	-0.001	-0.001
$\sigma_v = 0.39$	0	0.04	8.617	0.011	-0.029	-0.029	-0.029	-0.029	-0.029
$v_t = 0.04/1.15$	-0.64	0.10	12.572	0.005	-0.024	-0.024	-0.024	-0.024	-0.024
	-0.64	0.00	6.095	0.009	-0.012	-0.012	-0.012	-0.012	-0.012
$\kappa_v = 6.21$	-0.7	0.03	6.954	0.004	0.001	0.001	0.001	0.001	0.001
$\theta_v = 0.11799$	-0.9	0.03	6.940	0.003	0.005	0.005	0.005	0.005	0.005
$\sigma_v = 0.61$	0	0.03	6.901	0.005	-0.003	-0.003	-0.003	-0.003	-0.003
$v_t = 0.010201$	-0.7	0.10	11.562	0.004	0.001	0.001	0.001	0.001	0.001
	-0.7	0.00	5.076	0.004	-0.004	-0.004	-0.004	-0.004	-0.004
$\kappa_v = 2$	-0.3	0.05	12.558	0.011	-0.033	-0.033	-0.033	-0.033	-0.034
$\theta_v = 0.18$	-0.9	0.05	11.996	0.006	-0.009	-0.009	-0.009	-0.009	-0.009
$\sigma_v = 1$	0	0.05	12.774	0.015	-0.022	-0.022	-0.022	-0.022	-0.029
$v_t = 0.09$	-0.3	0.10	15.504	0.010	-0.027	-0.027	-0.027	-0.027	-0.027
	-0.3	0.00	9.891	0.012	-0.043	-0.043	-0.043	-0.043	-0.043
$\kappa_v = 0.5$	-0.9	0.00	2.645	0.028	0.014	0.015	0.015	0.015	0.015
$\theta_v = 0.02$	-0.5	0.00	3.268	0.036	-0.008	-0.008	-0.008	-0.008	-0.008
$\sigma_v = 1$	0	0.00	3.927	0.056	-0.054	-0.054	-0.054	-0.054	-0.054
$v_t = 0.04$	-0.9	0.10	11.087	0.004	-0.013	-0.013	-0.013	-0.013	-0.013
	-0.9	0.03	5.120	0.012	0.069	0.068	0.068	0.068	0.068
Mean Abs. Percentage Error					0.020	0.019	0.019	0.019	0.020
CPU (seconds)			150879.70		0.05	**6.22**	1.96	**12.66**	1.75

Table 2: FS options for different strikes, with $T-t=2$ years, $t^{*}-t=1$ year, $\delta=0\%$ and $S_t=\$100$

Model		Monte Carlo		Propositions 4 and 5		Hong (2004)		
		QEM scheme		G-Laguerre	G-Lobatto	FFT	G-Lobatto	COS
setup	ω	price	%SE	%errors	%errors	%errors	%errors	%errors
$\kappa_v=1.15$	0.50	52.036	0.008	-0.026	-0.026	-0.026	-0.026	-0.026
$\theta_v=0.04$	0.75	28.662	0.007	-0.025	-0.025	-0.025	-0.025	-0.025
$\sigma_v=0.39$	1.00	8.516	0.009	-0.014	-0.014	-0.014	-0.014	-0.014
$v_t=0.04/1.15$	1.25	0.750	0.059	0.016	0.016	0.016	0.016	0.016
$(\rho;r)=(-0.64;4\%)$	1.50	0.098	0.120	-0.012	-0.012	-0.013	-0.012	-0.012
$\kappa_v=6.21$	0.50	51.571	0.006	0.012	0.012	0.012	0.012	0.012
$\theta_v=0.11799$	0.75	27.625	0.006	0.001	0.001	0.001	0.001	0.001
$\sigma_v=0.61$	1.00	6.954	0.005	0.001	0.001	0.001	0.001	0.001
$v_t=0.010201$	1.25	0.127	0.038	-0.056	-0.056	-0.056	-0.056	-0.056
$(\rho;r)=(-0.7;3.19\%)$	1.50	0.0005	0.230	-0.053	-0.049	-0.173	-0.067	-0.053
$\kappa_v=2$	0.50	52.808	0.013	-0.023	-0.023	-0.023	-0.023	-0.023
$\theta_v=0.18$	0.75	30.636	0.013	-0.018	-0.018	-0.018	-0.018	-0.018
$\sigma_v=1$	1.00	12.556	0.016	-0.018	-0.018	-0.018	-0.018	-0.018
$v_t=0.09$	1.25	3.804	0.032	-0.073	-0.073	-0.073	-0.073	-0.074
$(\rho;r)=(-0.3;5\%)$	1.50	1.415	0.052	-0.077	-0.077	-0.077	-0.077	-0.079
$\kappa_v=0.5$	0.50	50.204	0.006	0.085	0.086	0.086	0.086	0.086
$\theta_v=0.02$	0.75	25.839	0.005	-0.103	-0.103	-0.103	-0.103	-0.103
$\sigma_v=1$	1.00	2.644	0.039	0.059	0.059	0.059	0.059	0.059
$v_t=0.04$	1.25	0.232	0.187	**1.897**	**1.868**	**1.868**	**1.867**	**1.868**
$(\rho;r)=(-0.9;0\%)$	1.50	0.081	0.198	**1.012**	**1.816**	**1.816**	**1.815**	**1.816**
Mean Abs. Percentage Error				0.179	0.218	0.224	0.218	0.218
MAPE (full truncation Euler MC)				0.109	0.068	0.074	0.069	0.068
CPU (seconds)		77859.93		0.08	7.83	1.95	41.49	1.78

Table 3: FS options for different times to determination and to maturity, with $\delta = 0\%$ and $S_t = \$100$

Model			Monte Carlo		Propositions 4 and 5		Hong (2004)		
			QEM scheme		G-Laguerre	G-Lobatto	FFT	G-Lobatto	COS
setup	$t^* - t$	τ	price	%SE	%errors	%errors	%errors	%errors	%errors
$\kappa_v = 1.15$	0.0625	0.5	5.495	0.005	0.002	0.002	0.002	0.002	0.002
$\theta_v = 0.04$	0.2500	0.5	3.785	0.010	-0.001	-0.001	-0.001	-0.001	-0.001
$\sigma_v = 0.39$	0.4375	0.5	1.663	0.015	-0.062	-0.078	-0.045	-0.078	-0.045
$v_t = 0.04/1.15$	0.6250	5.0	23.276	0.004	-0.001	-0.001	-0.001	-0.001	-0.001
$\rho = -0.64$	2.5000	5.0	15.797	0.008	-0.067	-0.067	-0.067	-0.067	-0.067
$r = 4\%$	4.3750	5.0	6.172	0.012	-0.122	-0.122	-0.122	-0.122	-0.122
$\kappa_v = 6.21$	0.0625	0.5	3.950	0.003	0.000	0.000	0.000	0.000	0.000
$\theta_v = 0.11799$	0.2500	0.5	2.837	0.006	0.008	0.008	0.008	0.008	0.008
$\sigma_v = 0.61$	0.4375	0.5	1.257	0.013	-0.010	-0.008	-0.008	-0.008	-0.008
$v_t = 0.010201$	0.6250	5.0	18.615	0.003	-0.007	-0.007	-0.007	-0.007	-0.007
$\rho = -0.7$	2.5000	5.0	12.757	0.006	-0.011	-0.011	-0.011	-0.011	-0.011
$r = 3.19\%$	4.3750	5.0	5.129	0.009	-0.011	-0.011	-0.011	-0.011	-0.011
$\kappa_v = 2$	0.0625	0.5	8.004	0.007	-0.009	-0.009	-0.009	-0.009	-0.009
$\theta_v = 0.18$	0.2500	0.5	5.490	0.014	0.011	0.011	0.011	0.011	0.011
$\sigma_v = 1$	0.4375	0.5	2.412	0.022	0.023	0.017	0.024	0.016	0.024
$v_t = 0.09$	0.6250	5.0	32.145	0.007	-0.007	-0.007	-0.007	-0.007	-0.007
$\rho = -0.3$	2.5000	5.0	22.627	0.015	-0.076	-0.076	-0.076	-0.076	-0.076
$r = 5\%$	4.3750	5.0	9.270	0.022	-0.096	-0.096	-0.096	-0.096	-0.096
$\kappa_v = 0.5$	0.0625	0.5	3.028	0.023	-0.324	-0.297	-0.291	-0.297	-0.297
$\theta_v = 0.02$	0.2500	0.5	1.796	0.040	-0.457	-0.144	-0.251	-0.144	-0.154
$\sigma_v = 1$	*0.4375*	*0.5*	*0.734*	*0.096*	*3.321*	*0.390*	*1.263*	*0.389*	*0.590*
$v_t = 0.04$	0.6250	5.0	7.080	0.009	-0.031	-0.031	-0.031	-0.031	-0.031
$\rho = -0.9$	2.5000	5.0	4.495	0.013	0.000	0.000	0.000	0.000	0.000
$r = 0\%$	4.3750	5.0	1.765	0.034	-0.103	-0.138	-0.139	-0.138	-0.138
Mean Abs. Percent. Error					0.198	**0.064**	0.103	**0.064**	0.072
CPU (seconds)			190814.54		0.07	**2.70**	2.41	**126.02**	2.05



Speed-accuracy trade-off

CONCLUSIONS

- The COS approximation can be biased in a low mean reversion setting.

- The QEM Monte Carlo scheme can be biased for deep out-of-the-money contracts.

- The adaptive Gauss-Lobatto quadrature scheme is the most robust integration method.

- The direct integration method proposed provides a better accuracy-efficiency trade-off than the usual Hong (2004) approach.

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