Own-Company Stockholding and Work Effort Preferences of an Unconstrained Executive

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[1] Joint Work with Alexander Szimayer and John Gould.

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Motivation

- Share-based payments frequently used and controversial;
  (public interest: Are executives overpaid?)
- Finance and economics theory: principal-agent-problem;
  (principal = share holder, agent = executive)
- How do share-based payments (e.g.: stock options) increase the executive's incentive/effort?
  ("constrained executive": risk taking in own-company manipulated)
- "Base case" as first step: analyze "unconstrained executive" without any constraints on his compensation.
  ⇒ Insight how the agent can be controlled by the principal.

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Framework

Utility-maximizing Executive

- Endowed with an initial wealth v_0, which is invested in the money market account, a diversified market portfolio, and own company shares
- Value of his own company is influenced via work effort:
  - Gain in utility from the increased value of his direct shareholding
  - Loss in utility for his work effort $\rightarrow$ disutility term

Characterization of the Executive

- Risk aversion parameter γ
- Work effectiveness parameters:
  - Inverse work productivity κ
  - Disutility stress α

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- Money Market Account:

$$\mathrm{d}B_t = r\,B_t\,\mathrm{d}t\,, \quad B_0 = 1\,, \tag{1}$$

- Market Portfolio:

$$\mathrm{d}P_t = P_t\,(\mu^P\,\mathrm{d}t + \sigma^P\,\mathrm{d}W_t^P)\,, \quad P_0 \in \mathbb{R}^+\,, \tag{2}$$

- Company's share price process is a controlled diffusion with SDE

$$\mathrm{d}S_t^{\mu,\sigma} = S_t^{\mu,\sigma}\left(\mu_t\,\mathrm{d}t + \sigma_t\,\mathrm{d}W_t + \beta\left[\frac{\mathrm{d}P_t}{P_t} - r\mathrm{d}t\right]\right)\,, \quad S_0 \in \mathbb{R}^+\,, \tag{3}$$

where the drift μ_t and the volatility σ_t are controlled by the executive.

Individual influences the own company's share price.

$\hat{=}$ Gain in utility from the increased value of his direct shareholding.

Remark

W^P and W are two independent standard Brownian motions, but the instantaneous correlation between $S_t^{\mu,\sigma}$ and P_t is $\rho_t = \beta\,\sigma^P/\sqrt{\sigma^2 + (\beta\sigma^P)}$.

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Wealth Equation

For investment strategy $\pi = (\pi^P, \pi^S)$ and initial wealth $V_0 > 0$:

$$\mathrm{d}V_t^\pi = V_t^\pi \left((1 - \pi_t^P - \pi_t^S)\,\mathrm{d}B_t/B_t + \pi_t^P \mathrm{d}P_t/P_t + \pi_t^S \mathrm{d}S_t^{\mu,\sigma}/S_t^{\mu,\sigma} \right) . \tag{4}$$

Work Effort Choice and Disutility

Instanteneous disutility of work effort is reprensented by a Markovian disutility rate $c(t, v, \mu_t, \sigma_t)$ for control strategy (μ_t, σ_t).

$\Rightarrow$ The *optimal investment and control decision* is the solution of

$$\Phi(t, v) = \sup_{(\pi,\mu,\sigma)\in A(t,v)} \mathbb{E}^{t,v} \left[U(V_T^\pi) - \int_t^T c_u(\mu_u, \sigma_u)\,\mathrm{d}u \right] , \; (t, v) \in [0, T] \times \mathbb{R}^+. \tag{5}$$

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Dimension Reduction of the Maximization Problem

- Define Sharpe ratio as $\lambda = \dfrac{\mu - r}{\sigma}$.
- Minimize disutility rate for this fixed Sharpe ratio λ and obtain $c^\star(t, v, \lambda)$.
- Replace $c(t, v, \mu, \sigma)$ by $c^\star(t, v, \lambda)$.
- Restate the maximization problem (5) over the controls π and λ.

Lemma

Under sufficient assumptions on $c(t, v, \mu, \sigma)$, the minimization problem

$$\min_{\{\sigma>0:\mu=r+\lambda\,\sigma\}} c(t, v, \mu, \sigma)\,, \quad \textit{for } (t, v, \lambda) \in [0, T] \times \mathbb{R}^+ \times \mathbb{R}_0^+\,, \tag{6}$$

admits a unique solution $\sigma^\star(t, v, \lambda)$.

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Dimension Reduction of the Maximization Problem

Theorem

Suppose

$$\Phi(t,v) = \sup_{(\pi,\mu,\sigma)\in A(t,v)} \mathbb{E}^{t,v}\left[U(V_T^\pi) - \int_t^T c_u(\mu_u,\sigma_u)\,\mathrm{d}u\right],\ (t,v)\in[0,T]\times\mathbb{R}^+$$

admits a $C^{1,2}$-solution Φ, then it is also the solution of the optimal control problem

$$\Phi(t,v) = \sup_{(\pi,\lambda)\in A'(t,v)} \mathbb{E}^{t,v}\left[U(V_T^\pi) - \int_t^T c_u^\star(\lambda_u)\,\mathrm{d}u\right],\quad (t,v)\in[0,T]\times\mathbb{R}^+,\quad (7)$$

where $c^\star$ is defined via

$$c^\star(t,v,\lambda) := c(t,v,r+\lambda\,\sigma^\star(t,v,\lambda),\sigma^\star(t,v,\lambda)) = \min_{\{\sigma>0:\mu=r+\lambda\,\sigma\}} c(t,v,\mu,\sigma).\quad (8)$$

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$$0 = \sup_{(\pi,\lambda)\in\mathbb{R}\times[0,\infty)} \Phi_t(t,v) + \Phi_v(t,v)\, v\,(r + \pi^S\,\lambda\,\sigma + [\pi^P + \beta\pi^S](\mu^P - r))$$
$$+ \frac{1}{2}\,\Phi_{vv}(t,v)\, v^2\,([\pi^S\,\sigma]^2 + [\pi^P\,\sigma^P + \beta\pi^S\sigma_P]^2) - c^\star(t,v,\lambda)\,, \qquad (9)$$

where $(t,v) \in [0,T) \times \mathbb{R}^+$, and $U(v) = \Phi(T,v)$, for $v \in \mathbb{R}^+$.

$\Rightarrow$ Maximizers $\pi^{P^\star}$, $\pi^{S^\star}$ and $\lambda^\star$ of (9) by establishing the FOCs:

$$\pi^{P^\star}(t,v) = -\frac{(\mu^P - r)}{v(\sigma^P)^2}\,\frac{\Phi_v(t,v)}{\Phi_{vv}(t,v)} - \beta\pi^{S^\star}(t,v)\,,$$
$$\pi^{S^\star}(t,v) = -\frac{\lambda^\star(t,v)}{v\sigma}\,\frac{\Phi_v(t,v)}{\Phi_{vv}(t,v)}\,, \qquad (10)$$

where $\lambda^\star$ is the solution of the implicit equation

$$\lambda\,\frac{\Phi_v^2(t,v)}{\Phi_{vv}(t,v)} + c_\lambda^\star(t,v,\lambda) = 0 \quad \text{for all } (t,v) \in [0,T] \times \mathbb{R}^+\,. \qquad (11)$$

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$$0 = \sup_{(\pi,\lambda)\in\mathbb{R}\times[0,\infty)} \Phi_t(t,v) + \Phi_v(t,v)\, v\,(r + \pi^S \lambda \sigma + [\pi^P + \beta\pi^S](\mu^P - r)) + \frac{1}{2}\Phi_{vv}(t,v)\, v^2\,([\pi^S \sigma]^2 + [\pi^P \sigma^P + \beta\pi^S \sigma_P]^2) - c^\star(t,v,\lambda)\,, \tag{9}$$

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Substituting the maximizers (10) in the HJB (9) then yields:

$$\Phi_t(t,v) + \Phi_v(t,v)\, v\, r - \frac{1}{2}(\lambda^\star)^2 \frac{\Phi_v^2(t,v)}{\Phi_{vv}(t,v)} - \frac{1}{2}(\lambda_P)^2 \frac{\Phi_v^2(t,v)}{\Phi_{vv}(t,v)} - c^\star(t,v,\lambda^\star) = 0\,, \tag{12}$$

where $\lambda_P := \dfrac{\mu_P - r}{\sigma^P}$.

$\rightarrow$

Goal:

Solve equation (12) for a special choice of the utility and disutility functions.

Utility and Disutility Functions

The utility function U is assumed to be CRRA, in particular

$$U(v) = \begin{cases} \dfrac{v^{1-\gamma}}{1-\gamma}\,, & \text{for } \gamma > 0 \text{ and } \gamma \neq 1 \quad \text{„Power Utility“} \\[2ex] \log(v)\,, & \text{for } \gamma = 1\,, \qquad\qquad \text{„Log Utility“} \end{cases} \tag{13}$$

and the minimized disutility $c^\star$ satisfies:

$$c^\star(t, v, \lambda) = \kappa\, v^{1-\gamma}\, \frac{\lambda^\alpha}{\alpha}\,, \quad \text{for } \gamma > 0\,, \tag{14}$$

where $\kappa =$ inverse work productivity and $\alpha =$ disutility stress.

$\Rightarrow$ **Characterization of the executive** via κ, α and γ.

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The Power Utility Case: $\gamma > 0$ and $\gamma \neq 1$

- For $\alpha > 2$ and $\gamma \neq 1$ the separation approach

$$\Phi(t,v) = f(t)\,\frac{v^{1-\gamma}}{(1-\gamma)} \quad \text{with} \quad f(T) = 1$$

substituted in PDE (12) produces a Bernoulli ODE (for $n \neq 1$) of the form

$$\dot{f} = a_1\, f + a_n\, f^n\,.$$

- The solution is

$$f(t)^{1-n} = C\, e^{G(t)} + (1-n)\, e^{G(t)} \int_0^t e^{-G(s)}\, a_n\, \mathrm{d}s\,,$$

where $G(t) = (1-n)\int_0^t a_1(s)\,\mathrm{d}s$, and C is an arbitrary constant.

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→ **Solutions:**

$$\lambda^{\star}(t,v) = \left(\frac{1}{\kappa\,\gamma}\,f(t)\right)^{\frac{1}{\alpha-2}} \tag{15}$$

$$\pi^{P\star}(t,v) = \frac{\mu^P - r}{\gamma\,(\sigma^P)^2}\,, \qquad \pi^{S\star}(t,v) = \frac{\lambda^{\star}(t,v)}{\gamma\,\sigma^{\star}(t,v,\lambda^{\star}(t,v))}\,, \tag{16}$$

$$\Phi(t,v) = \frac{v^{1-\gamma}}{1-\gamma}\,f(t)\,, \tag{17}$$

where

$$f(t) = e^{(1-\gamma)\left(r+\frac{1}{2}\,\frac{\lambda_P^2}{\gamma}\right)(T-t)}\left(1 - \frac{(\alpha-2)\,\left(\frac{1}{\kappa\,\gamma}\right)^{\frac{2}{\alpha-2}}}{\alpha\,\left(2\,\gamma\,r+\lambda_P^2\right)}\,\left(e^{\frac{1-\gamma}{\alpha-2}\left(2\,r+\frac{\lambda_P^2}{\gamma}\right)(T-t)} - 1\right)\right)^{-\frac{\alpha-2}{2}}\,. \tag{18}$$

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The Log Utility Case: $\gamma = 1$

For $\gamma = 1$ (log-utility) the solution Φ can be derived by assuming an additive structure of the form

$$\Phi(t, v) = \log(v) + \varphi(T - t).$$

$\rightarrow$ **Solutions:**

$$\lambda^\star(t, v) = \kappa^{-\frac{1}{\alpha - 2}}, \quad \pi^{P\star}(t, v) = \frac{\mu^P - r}{(\sigma^P)^2}, \quad \text{and} \quad \pi^{S\star}(t, v) = \frac{\lambda^\star(t, v)}{\sigma^\star(t, v, \lambda^\star(t, v))}, \tag{19}$$

and value function

$$\Phi(t, v) = \log(v) + \left[r + \frac{1}{2} \left(\frac{\mu^P - r}{\sigma^P} \right)^2 + \frac{\alpha - 2}{2\,\alpha} \kappa^{-\frac{2}{\alpha - 2}} \right] (T - t)\,. \tag{20}$$

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Theoretical results are analyzed for practical insights:

- Investigate executive performance $\lambda^\star$ for sensitivities!
  (w.r.t.: work productivity κ^{-1}, disutility stress α)
- How much compensation is appropriate?
  (log-utility setting, indifference utility equivalence principle)

Parameters:

- investments:
  - risk-free rate: $r = 5\%$;
  - market portfolio: $\mu^P = 7\%$ and $\sigma^P = 20\%$;
  - own company: $\sigma^\star(t, v, \lambda^\star) = 40\%$;
- executive:
  - time horizon: $T = 10$ years;
  - initial wealth $v = \$5$ Mio.;
  - work productivity: $100 \leq \kappa^{-1} \leq 2000$;
  - disutility stress: $4 \leq \alpha \leq 6$;

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  - time horizon: $T = 10$ years;
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  - work productivity: $100 \leq \kappa^{-1} \leq 2000$;
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Optimal Effort $\lambda^\star$ under Log-Utility

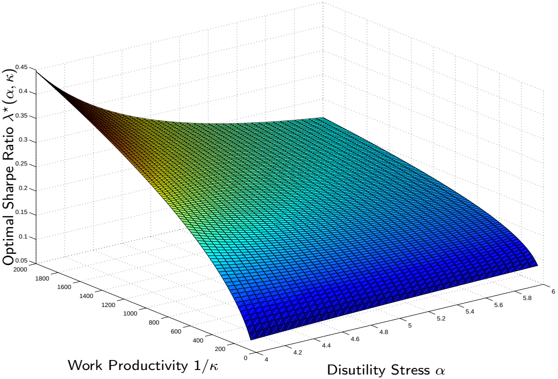

Figure: The optimal choice of the executive's effort parameter $\lambda^\star$ graphed against $1/\kappa$ and α.

Indifference Utility Approach for the Log-Utility Case

The executive's utility from his optimal personal investment and work effort decision is:

$$\Phi(0,v) = \log v + \left[r + \frac{1}{2}(\lambda^P)^2 + \frac{1}{2}(\lambda^\star)^2 \frac{\alpha-2}{\alpha} \right] T\,.$$

An outside investor's utility who invests optimally in the executive's portfolio strategy $\pi^\star$ (without spending work effort) is:

$$\hat{\Phi}(0,v) = \log v + \left[r + \frac{1}{2}(\lambda^P)^2 + \frac{1}{2}(\lambda^\star)^2 \right] T\,.$$

$\Rightarrow$ Loss of utility: $\Phi(0,v) - \hat{\Phi}(0,v) = -\frac{1}{\alpha}(\lambda^\star)^2 T$

$\Rightarrow$ Using the indifference utility argument $\Phi(0, v+\Delta v) = \hat{\Phi}(0,v)$ yields

$$\Delta v = v\left(e^{\frac{(\lambda^\star)^2\, T}{\alpha}} - 1\right) = v\left(e^{\frac{\lambda_0^2\, T}{\alpha}\left(\frac{\lambda_0^2}{\kappa}\right)^{\frac{2}{\alpha-2}}} - 1\right)\,.$$

$\Rightarrow$ Loss of utility is compensated.

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Executive's "Fair" Pay Δv under Log-Utility

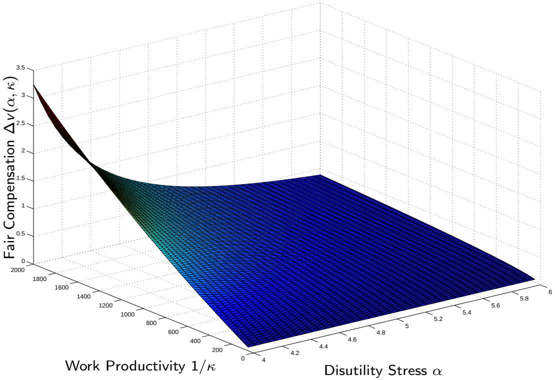

Figure: The executive's fair up-front cash compensation Δv (based on indifference utility) graphed against $1/\kappa$ and α; with initial wealth $v = \$5$ Mio. and $T = 10$.

Extensions of the “base case”:

- Closed-form solutions exist also for an exponential utility of wealth;
- Include consumption and time preferences (consumption and work effort) in the present model:
  - Log utility case $\gamma = 1$: Closed-form solution preserved.
  - Power utility case $\gamma \neq 1$: Solve an inhomogeneous Bernoulli ODE; works for $\alpha = 2\gamma + 2$.

Towards the “constrained executive”:

- Develop dynamic “game” with company determining executive’s own-company shareholding and executive controlling effort and other investment decision $\rightarrow$ Modeled as a Stackelberg differential game;
- Determine optimal mixed compensation (cash, shares, and options);

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