Convenience Yield-Based Pricing of Commodity Futures

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1. The objectives and results

Background

- Convenience yield is often used to describe the value to hold commodities as is explained in e.g., Geman(2005).

- Following the notion, convenience yield is useful to represent the linkage between spot and futures prices.

- On the other hand, asset pricing theory offers a concept of stochastic discount factor to determine financial instrument prices like futures prices written on spot prices.

- The relationship between spot and futures prices is expressed by two ways: convenience yield and stochastic discount factor.

- Putting two concepts together, convenience yield may play an alternative role of a stochastic discount factor.

Literature Survey

• Risk neutral valuation is applied to price commodity derivatives e.g., Schwartz and Smith(2000), Yan(2002), Casassus and Collin-Dufresne(2005), Korn(2005), Eydeland and Wolyniec (2003).

• The application does not necessarily seem satisfactory because commodity market is incomplete by the illiquidity. A further task is needed to select a stochastic discount factor (SDF) for derivative pricing.

• A familiar SDF selection is a utility-based approach: SDF is assigned to unspanned risk and the derivative is uniquely priced. Davis(2001) and Cao and Wei(2000) use the method to price weather derivatives in commodities. It depends on utility function and optimal consumption.

• To avoid the problem and incorporate market incompleteness into commodity futures pricing, we employ good-deal bounds (GDB) of Cochrane and Saarequejo(2000). The point is in the restriction on the SDF variance: an upper bound characterized by the maximum Sharpe ratio is always required to be more than or equal to Sharpe ratios of all assets in the market.

• Kanamura and Ohashi(2009) applied GDB to weather derivatives.

Literature Survey

- GDB may be useful to price incomplete market assets in that it does not rely on utility function and optimal consumption. The method is still dissatisfactory because the maximum Sharpe ratio that binds SDF is unknown and must be given exogenously. A pricing scheme for commodities is desirable if the Sharpe ratio, i.e., the restriction on SDF, is given using commodity market data.

- We consider that two-way concept on intertemporal relationship between spot and futures prices, i.e., convenience yield can implicitly characterize SDF, will be beneficial to determine the Sharpe ratio.

The objectives

This paper proposes the convenience yield-based pricing for commodity futures.

By using the pricing method, we conduct empirical analyses of crude oil, heating oil, and natural gas futures traded on the NYMEX in order to assess the incompleteness of energy futures markets.

We apply the market price of risk embedded in energy futures markets to the Asian call option pricing on crude oil futures.

The results

(1) We propose a convenience yield-based pricing for commodity futures, which embeds the incompleteness of commodity futures markets in convenience yield.

(2) Empirical analyses of crude oil, heating oil, and natural gas futures on the NYMEX show that the fluctuation from incompleteness is partly owed to convenience yield.

(3) It is shown that the additional Sharpe ratio, which represents the degree of market incompleteness and is also used for derivative pricing written on energy prices, is obtained from the NYMEX data.

(4) We numerically price the Asian call option using market price of risk estimated from crude oil futures prices.

2. The convenience yield-based pricing of commodity futures

Model Setup

The relationship between spot and futures prices is expressed by two ways: convenience yield and stochastic discount factor.

Model Setup: Convenience yield

We employ Gibson Schwartz (1990) two-factor model based on convenience yield (δ) to represent the spot price (S).

$$\frac{dS_t}{S_t} = (\mu - \delta_t)dt + \sigma_1 dw_t, \tag{1}$$

$$d\delta_t = \kappa(\alpha - \delta_t)dt + \sigma_2 du_t, \tag{2}$$

where $E_t[dw_t du_t] = \rho dt$. Using Ito's lemma to equation (1), we obtain

$$S_T = S_t e^{(\mu-\alpha-\frac{1}{2}\sigma_1^2)(T-t)+\frac{1}{\kappa}(1-e^{-\kappa(T-t)})(\delta_t-\alpha)+\int_t^T(\sigma_1+\frac{\sigma_2\rho}{\kappa}(1-e^{-\kappa(T-s)}))dw_s+\int_t^T(\frac{\sigma_2\sqrt{1-\rho^2}}{\kappa}(1-e^{-\kappa(T-s)}))dz_s}. \tag{3}$$

We assume that the fluctuation due to convenience yield is spanned by both of complete and incomplete parts:

$$du_t = \rho dw_t + \sqrt{1-\rho^2} dz_t. \tag{4}$$

Model Setup: SDF

As is well known, commodity markets may demonstrate incompleteness because of the illiquidity.

Following Cochrane(2001) which can generally express the market incompleteness, we assume that Λ_t is given by

$$\frac{d\Lambda_t}{\Lambda_t} = -rdt - \phi dw_t - \nu dz_t, \tag{5}$$

where ν represents the incompleteness of the market. By using Ito's lemma to equation (5), we obtain

$$\frac{\Lambda_T}{\Lambda_t} = e^{-(r+\frac{1}{2}\phi^2+\frac{1}{2}\nu^2)(T-t)-\int_t^T \phi dw_s-\int_t^T \nu dz_s}. \tag{6}$$

Model Setup: Linkage between CY and SDF

While Schwartz(1997) introduced risk neutral measure to price commodity futures, the ambiguity may remain in the existence of such probability measure.

Hence, we chose more comprehensive representation of the futures prices using stochastic discount factor (SDF).

The futures prices F_t^T are in general represented as follows:

$$F_t^T = E_t\left[\frac{\frac{\Lambda_T}{\Lambda_t} S_T}{E_t\left[\frac{\Lambda_T}{\Lambda_t}\right]}\right], \tag{7}$$

where we denote the SDF by Λ_t at time t.

The convenience yield-based pricing of commodity futures (CY-based model)

We have the futures price as follows:

$$F_t^T = S_t e^{\Upsilon(t,T)-\Omega(t,T)\delta_t}, \tag{8}$$

$$\begin{aligned}\Upsilon(t,T) = (r-\alpha+\frac{\sigma_2^2}{2\kappa^2}-\frac{\sigma_1\sigma_2\rho}{\kappa}+\phi\frac{\sigma_2\rho}{\kappa}+\frac{\nu\sigma_2\sqrt{1-\rho^2}}{\kappa})(T-t)+\frac{\sigma_2^2}{4\kappa^3}(1-e^{-2\kappa(T-t)})\\ +(\alpha\kappa+\rho\sigma_1\sigma_2-\frac{\sigma_2^2}{\kappa}-\phi\sigma_2\rho-\nu\sigma_2\sqrt{1-\rho^2})\frac{1-e^{-\kappa(T-t)}}{\kappa^2},\end{aligned} \tag{9}$$

$$\Omega(t,T) = \frac{1-e^{-\kappa(T-t)}}{\kappa}. \tag{10}$$

Model Implication

The point of this model stands on the inclusion of incompleteness parameter ν into spot-futures price relationship.

We obtained a futures pricing method not using risk neutral measure, but using convenience yield-based incompleteness parameter ν.

3. Empirical studies for energy prices

Data

We use the daily closing prices of WTI crude oil (WTI), heating oil (HO), and natural gas (NG) futures traded on the NYMEX.

Each futures product includes six delivery months – from one month to six months.

The covered time period is from April 3, 2000 to March 31, 2008.

The data are obtained from Bloomberg.

Incompleteness check using Kalman Filter

By examining the relationship between ϕ (complete market price of risk) and ν (incomplete market price of risk), we can find the degree of incompleteness of energy markets.

The parameters of the CY-based pricing are estimated using the Kalman filter.

Both log transformed spot prices (x_t) and convenience yields (δ_t) are unobservable.

Log transformed futures prices (y_t) are observable.

Time and measurement update equations

Time update equations:

$$x_t = x_{t-1} - \Delta t \delta_t + (\mu - \frac{1}{2}\sigma_1^2)\Delta t + \sigma_1 \epsilon_t \equiv f_1(x_{t-1}, \delta_{t-1}, \epsilon_t). \quad (11)$$

$$\delta_t = (1 - \kappa \Delta t)\delta_{t-1} + \kappa \alpha \Delta t + \sigma_2 \eta_t \equiv f_2(x_{t-1}, \delta_{t-1}, \eta_t). \quad (12)$$

The measurement update equation in the KF system is obtained from the futures-spot price relationship.

$$y_t = x_t - \Omega(t, T)\delta_t + \Upsilon(t, T) + \xi_t \equiv h_1(x_t, \delta_t, \xi_t). \quad (13)$$

Note that $V[\xi_t]$ = diag$[m_1, m_2, m_3, m_4, m_5, m_6]$ (Diagonal matrix).

Maximum likelihood estimation (WTI)

The parameters of the CY-based pricing for crude oil are estimated by the maximum likelihood method:

	μ	σ_1	κ	α	σ_2	ρ	ν
Est	0.563	0.544	1.629	0.093	0.636	0.857	-1.404
(S.E.)	0.000	0.001	0.001	0.002	0.000	0.002	0.000
	m_1	m_2	m_3	m_4	m_5	m_6	
Est	2.197E-4	1.628E-5	1.000E-6	1.000E-6	1.000E-5	7.753E-6	
(S.E.)	1.854E-5	3.004E-6	1.893E-6	1.312E-6	2.747E-5	3.655E-6	
LL	5.461E+4						
AIC	-1.092E+5						
SIC	-1.092E+5						

All parameters except m_3, m_4, and m_5 for WTI are statistically significant.

Maximum likelihood estimation (WTI)

Since ρ is 0.857, it is shown that the fluctuation from incompleteness is partly owed to convenience yield.

While ν is obtained as negative value, taking into account

$$\frac{d\Lambda_t}{\Lambda_t} = -rdt - \phi dw_t - (-\nu)dz_t, \tag{14}$$

due to the symmetry of dz_t, i.e., $dz_t = -dz_t$ by definition, the absolute value of ν represents the incompleteness of the market, i.e., incomplete market price of risk.

The incompleteness of crude oil market is calculated as $|\nu| = 1.404$ using the market data.

Maximum likelihood estimation (HO)

	μ	σ_1	κ	α	σ_2	ρ	ν
Est	0.568	0.575	1.358	0.069	0.883	0.745	-1.041
(S.E.)	0.196	0.017	0.062	0.249	0.034	0.081	0.234
	m_1	m_2	m_3	m_4	m_5	m_6	
Est	3.619E-4	1.000E-5	2.936E-5	1.000E-5	1.181E-4	6.920E-4	
(S.E.)	6.725E-5	1.889E-5	1.810E-5	2.531E-5	3.485E-5	1.719E-4	
LL	4.325E+4						
AIC	-8.648E+4						
SIC	-8.651E+4						

The parameters except α, m_2, m_3, and m_4 are statistically significant. In addition to the existence of incompleteness from convenience yield, we also obtained the incomplete heating oil market price of risk using the market data.

Maximum likelihood estimation (NG)

	μ	σ_1	κ	α	σ_2	ρ	ν
Est	0.361	0.995	0.617	-0.416	2.061	0.829	-0.749
(S.E.)	0.299	0.022	0.089	0.660	0.081	0.012	0.252
	m_1	m_2	m_3	m_4	m_5	m_6	
Est	3.314E-3	5.368E-5	1.198E-3	1.033E-3	3.278E-5	2.884E-3	
(S.E.)	1.199E-4	2.124E-5	3.595E-5	3.133E-5	1.393E-5	9.812E-5	
LL	3.083E+4						
AIC	-6.163E+4						
SIC	-6.166E+4						

The parameters except μ and α are statistically significant. In addition to the existence of incompleteness from convenience yield, we also obtained the incomplete natural gas market price of risk using the market data.

Incompleteness Assessment of Energy Futures Markets

	$\lvert\phi\rvert$	$\lvert\nu\rvert$	$A = \sqrt{\nu^2 + \phi^2}$
Crude oil	0.924	1.404	1.681
Heating oil	0.883	1.041	1.365
Natural gas	0.302	0.749	0.807

Note that ϕ is calculated as $\phi = \frac{\mu - r}{\sigma_1}$ assuming $r = 0.06$.

For crude oil, the incomplete market price of risk (1.404) is a little greater than the complete market price of risk (0.924), i.e., the crude oil market should be spanned by both complete and incomplete markets. The pricing of derivative instruments on crude oil prices requests the Sharpe ratio of 1.681, which is about twice as large as the complete market price of risk (0.924) based on the GDB.

For heating oil and natural gas, the Sharpe ratios take a value of 1.365 and 0.807, respectively.

Empirical Study Implications

It was shown that the fluctuation from market incompleteness is partly owed to the fluctuation from convenience yield.

We could obtain the incompleteness market price of risk from the NYMEX market data.

4. Application of CY-based model to energy derivative pricing

Asian call option on energy futures prices

We price Asian call option on energy futures prices using the incompleteness market price of risk ν estimated from the NYMEX data.

In general, GDB pricing is expressed by

$$\underline{C}_t = E_t \int_{s=t}^{T} \frac{\underline{\Lambda}_s}{\underline{\Lambda}_t} x_s ds + E_t(\frac{\underline{\Lambda}_T}{\underline{\Lambda}_t} x_T), \quad \frac{d\underline{\Lambda}_t}{\underline{\Lambda}_t} = -rdt - \phi dw_t \mp \nu dz_t, \tag{15}$$

where $\mp$ represents lower and upper price boundaries, respectively.

We assume the average i-month futures price from times 0 to $\bar{T}$ as

$$I = \frac{1}{\bar{T}} \int_0^{\bar{T}} F^i(S, \delta, t) dt. \tag{16}$$

We set $\frac{d\underline{C}}{\underline{C}} = \mu_{\underline{C}} dt + \sigma_{\underline{C}w} dw + \sigma_{\underline{C}z} dz$. GDB pricing is transformed into

$$\mu_{\underline{C}} - r + \sigma_{\underline{C}w} \mp \sigma_{\underline{C}z} \nu = 0, \tag{17}$$

where $\mp$ represents lower and upper price boundaries, respectively. Note that $x_s = 0$.

Asian call option on energy futures prices

Applying Ito's lemma to a price boundary $\underline{C}(S, \delta, I, t)$, we have

$$\begin{aligned}
\mu_{\underline{C}} &= \frac{1}{\underline{C}}\Big\{\frac{\partial \underline{C}}{\partial t} + (\mu - \delta)S\frac{\partial \underline{C}}{\partial S} + \kappa(\alpha - \delta)\frac{\partial \underline{C}}{\partial \delta} + \frac{1}{2}\sigma_1^2 S^2 \frac{\partial^2 \underline{C}}{\partial S^2} \\
&\quad + \frac{1}{2}\sigma_2^2 \frac{\partial^2 \underline{C}}{\partial \delta^2} + \rho\sigma_1\sigma_2 S \frac{\partial^2 \underline{C}}{\partial \delta \partial S} + \frac{1}{\bar{T}} F \frac{\partial \underline{C}}{\partial I}\Big\}, \\
\sigma_{\underline{C}w} &= \frac{1}{\underline{C}}\Big\{\mu S \frac{\partial \underline{C}}{\partial S} + \rho\sigma_2 \frac{\partial \underline{C}}{\partial \delta}\Big\}, \\
\sigma_{\underline{C}z} &= \frac{1}{\underline{C}}\Big\{\sigma_2\sqrt{1 - \rho^2}\frac{\partial \underline{C}}{\partial \delta}\Big\}.
\end{aligned}$$

Partial differential equation

$$-r\underline{C} + \frac{\partial \underline{C}}{\partial t} + \frac{1}{2}\sigma_1^2 S^2 \frac{\partial^2 \underline{C}}{\partial S^2} + \frac{1}{2}\sigma_2^2 \frac{\partial^2 \underline{C}}{\partial \delta^2} + \rho\sigma_1\sigma_2 S \frac{\partial^2 \underline{C}}{\partial \delta \partial S} + \frac{dI}{dt}\frac{\partial \underline{C}}{\partial I}$$
$$= (\delta - r)S\frac{\partial \underline{C}}{\partial S} + \left(\phi\rho\sigma_2 - \kappa(\alpha - \delta) + k\nu\sigma_2\sqrt{1-\rho^2}\,\mathrm{sgn}\left(\frac{\partial \underline{C}}{\partial \delta}\right)\right)\frac{\partial \underline{C}}{\partial \delta},$$

with the terminal payoff: $\underline{C}(S, \delta, I, \bar{T}) = f(I_{\bar{T}})$, where $k = -1$ and $+1$ generate the upper and lower price boundaries, respectively.

To obtain the GDB prices of the Asian call option, we set the payoff at maturity to be $f(I_{\bar{T}}) = \max(I_{\bar{T}} - K, 0)$ and, following Ingersoll(1987), $\frac{dI}{dt}$ to be: $dI = \frac{1}{\bar{T}}F(S, \delta, t)dt$.

Asian call option price

We computed Asian call option prices written on 1-month crude oil futures prices assuming that the strike price is 70 USD, the delta is zero, and interest rate is set to 6 %.

Futures Prices	70	80	90	100	110	120	130
Upper Price	1.47	7.31	17.40	26.36	35.23	44.15	52.71
No Risk Prem.	1.45	7.25	17.34	26.30	35.15	44.07	52.62
Lower Price	1.43	7.20	17.27	26.24	35.08	44.00	52.54
Upper Premium	0.02	0.05	0.07	0.06	0.07	0.08	0.08
Lower Premium	0.02	0.05	0.07	0.06	0.08	0.08	0.08
UP/NRPP (%)	1.08	0.72	0.39	0.24	0.21	0.17	0.16
LP/NRPP (%)	1.07	0.72	0.39	0.23	0.21	0.17	0.16

Outcomes from Asian call option prices

Both upper and lower risk premiums are small enough comparing with the level of the option prices. It may be easy to use for practitioners in the sense that the option price is priced using small price range.

We were able to obtain the risk premium based on the incompleteness of energy futures market implied from the CY-Based pricing method we proposed.

4. Conclusions and directions for future research

Conclusions

(1) We have proposed a convenience yield-based pricing for commodity futures, which embeds the incompleteness of commodity futures markets in convenience yield.

(2) Empirical analyses of crude oil, heating oil, and natural gas futures on the NYMEX showed that the fluctuation from incompleteness is partly owed to convenience yield.

(3) It was shown that the additional Sharpe ratio, which represents the degree of market incompleteness and is also used for derivative pricing written on energy prices, is obtained from the NYMEX data.

(4) We numerically priced the Asian call option using market price of risk estimated from crude oil futures prices.

Directions for future research

This paper only dealt with energy futures due to the availability of data. The concept in this paper can be extended to other commodity futures like agricultural futures. These empirical studies may be the next direction for our future researches.

Thank you.

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