Pricing a European Gas Storage Facility Using a Continuous-Time Spot Price Model with GARCH Diffusion

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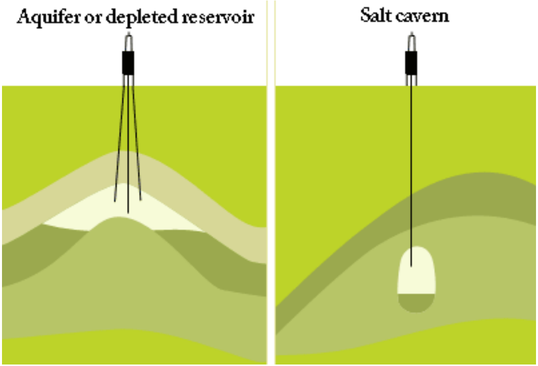
Gas is stored in salt caverns or depleted oil/gas reservoirs



Source: http://www.iz-klima.de.

- Gas storage: salt cavern, depleted oil/gas reservoir, aquifers, LNG storage.
- The main storage characteristics are injection, withdrawal and cycling rate.
- Salt caverns, for example, allow high injection/withdrawal rates.

Liquid spot markets offer more profit at a higher risk

- Historically, in Europe, gas is bought in the summer and sold in the winter.
- But: The traded volume rose on European gas hubs by 57 % from 2007 to 2008 (IEA, 2009). The Dutch Title Transfer Facility (TTF) established itself as the main continental hub in Europe.
- New chances for storage operators: Facilities can be used as physical hedges for option trading on short-term gas markets.
- These new strategies are complex and riskier.
- Two major questions arise:
  1. What is an adequate model for the short-term gas price?
  2. Given new operational possibilities: How to price a storage facility and how to determine the optimal strategy?

Agenda

1. A TTF day-ahead gas price model

2. Pricing a gas storage facility
   2.1 A review of methods for gas storage pricing
   2.2 The pricing algorithm

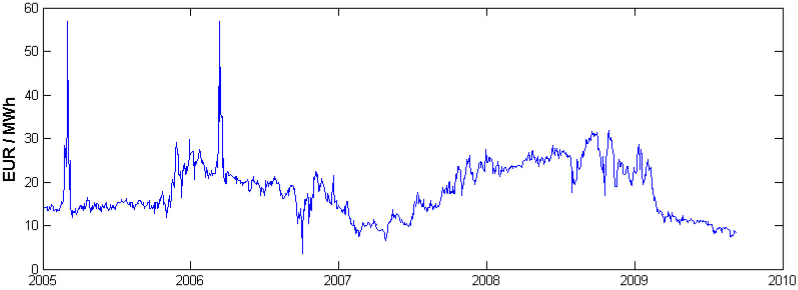
3. A German example
   3.1. The Trianel GmbH
   3.2. Running the algorithm

4. Conclusion

Agenda

Characteristics of the TTF day-ahead price

The TTF Day-Ahead Price Index



Source: APX Group.

We see:

- There is little evidence of seasonality,
- extreme price jumps disappear after 2006.

A new model for the TTF day-ahead price

- We fit an Ornstein-Uhlenbeck process and see that volatility is not constant over time.
- There are different ways to incorporate dynamic volatility, e.g. regime-switching models, Poisson-jumps, rolling volatility, ...
- In discrete world GARCH (see Engle, 1982) works very well. Drost & Werker (1996) derive a continuous-time limit.
- We use their approach and our model for the price P at time t reads

$$\begin{aligned} dP_t &= \lambda(\mu - P_t)dt + \sigma_t dW_t^{(1)}, \\ d\sigma_t^2 &= \theta(\omega - \sigma_t^2)dt + \sqrt{2\eta\theta}\sigma_t^2 dW_t^{(2)}, \quad dW_t^{(1)}, dW_t^{(2)} \sim \mathcal{N}(0, t), \end{aligned}$$

  with $\lambda, \theta, \eta > 0, \mu, \omega \in \mathbb{R}$.
- Risk-adjustment is done by modifying μ and ω (see e.g. Heston, 1993).

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There are three different types of storage pricing methods

1. **Monte Carlo simulation:**
   - Price paths are simulated; testing of different price models is simple.
   - Deriving the optimal strategy requires complex extensions like Least Squares Monte Carlo.
2. **Binomial tree method:**
   - Forward differencing method is applied.
   - Requires large computational resources to store the tree structure.
   - It is difficult to incorporate flux limiters.
3. **Partial differential equations:**
   - The method is capable of handling hyperbolic equations.
   - It is relatively easy to incorporate flux limiters.
   - When starting from the Bellman equation, we have a necessary and sufficient condition for optimality (see e.g. Bellman, 1957).

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The variables

Variable	**Description**
P	Gas price
t, T	Current and terminal time
σ^2	Volatility
$I_{min} \leq I \leq I_{max}$	Inventory level with physical boundaries
$c_{min}(I), c_{max}(I)$	Control boundaries depending on the inventory level (pressure inside the storage)
$c_{min}(I) \leq c(I,P) \leq c_{max}(I)$	Control variable; ($c < 0 \rightarrow$ injecting, $c > 0 \rightarrow$ withdrawing)
$a(I,c)$	Dynamic costs of injection/withdrawal; If $c \geq 0 \rightarrow a(I,c) = 0$, if $c < 0$ then $a(I,c) = K, K \in \mathbb{R}^+$

The current storage value is the sum of discounted future cash flows

- Real options theory: Applying option valuation methods e.g. to investment decisions or to operate industrial facilities.
- Option characer of a gas storage: "Inject, withdraw or do nothing?"
- The storage value V in time t is computed as

$$V(P, \sigma^2, I, t) = \max_{c(P,I,t)} E\left[\int_t^T e^{-\rho(\tau-t)}(c - a(I,c))P_\tau d\tau\right].$$

  E is the expectation over price and volatility paths under the risk-neutral measure. As $t \to 0$ we obtain the current storage value and optimal strategy.
- For the price dynamics we use the model of Section 1.
- Inventory level dynamics: $dI = -cdt$.

Introducing a time step *dt* yields the Bellman equation

The trick is to split up the intregral...

$$V = \max_c E\left[\int_t^{t+dt} e^{-\rho(\tau-t)}(c-a(I,c))Pd\tau + \int_{t+dt}^{T} e^{-\rho(\tau-t)}(c-a(I,c))Pd\tau\right]$$

Following this procedure, we arrive at the Bellman equation which is a **necessary and sufficient optimality condition**:

$$V = \max_c E\left[(c-a(I,c))Pdt + e^{-\rho dt}V(P+dP,\sigma^2+d\sigma^2,I+dI,t+dt)\right].$$

- Then simplification via Ito's Lemma and $dt \to 0$.
- Solution via backward induction and finite differences.

A two-step mechanism to compute the storage value

Starting in terminal time T we proceed backwards in time. At time t:

Step 1: Identify the optimal strategy c_{opt} by solving

$$\begin{array}{rl} \max_c & [-cV_I + (c - a(I,c))P] \\ \text{s.t} & c_{min}(I) \leq c \leq c_{max}(I). \end{array}$$

Step 2: Solve the following equations

$$\begin{array}{rcl} (c_{opt} - a)P - c_{opt} V_I - \rho V + V_t + 0.5\sigma^2 V_{PP} & & \\ + \lambda\theta \left(\sigma^2\right)^2 V_{\sigma^2\sigma^2} + \eta(\mu - P)V_P + \theta(\omega - \sigma^2)V_{\sigma^2} & = & 0, \end{array}$$

where ρ is the long-run risk-free interest rate.

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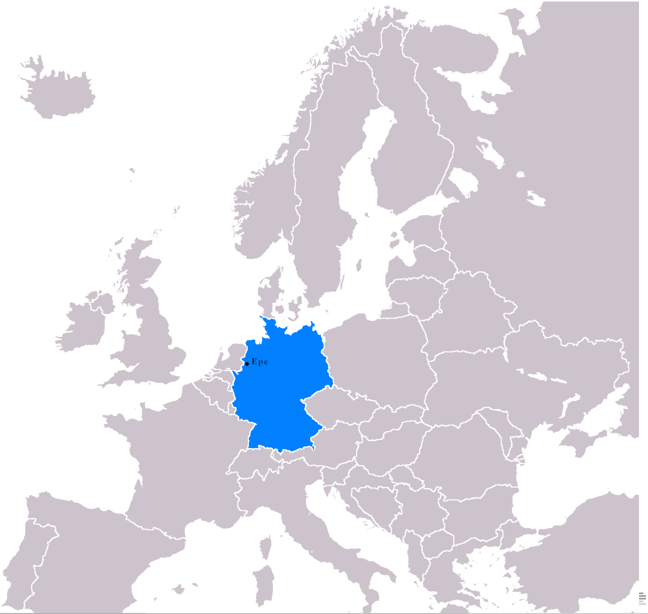
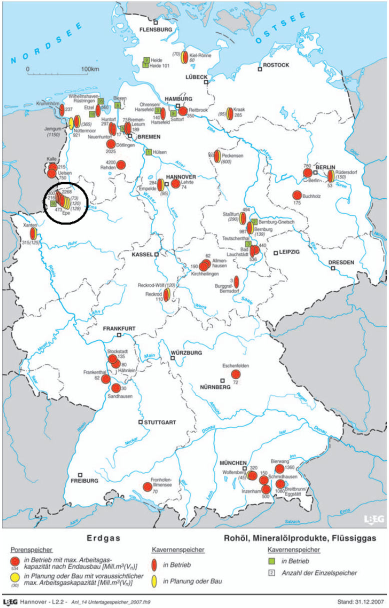
The location of Epe in Europe



The storage volume in Epe is the largest in Europe



Technical data of the caverns run by the Trianel GmbH

- Scenario: We are renting storage capacity for a period T. The gas market is liquid, i.e. we can sell and buy gas at any time.
- Type of gas: H-Gas with 87% - 99% methane and a caloric value of $11,5\ kWh/m^3$.
- The risk-free interest rate is about 1%.
- Storage volume: 314 million (M) m^3 (237M m^3 working gas).
- $c_{min} = -36.3612\sqrt{\frac{1}{I+77} - \frac{1}{314}}, \quad c_{max} = 0.4677\sqrt{I}.$
- We estimate the following parameters for the price process:

$$\begin{aligned} dP_t &= 0.01(17.76 - P_t)dt + \sigma_t dW_t^{(1)}, \\ d\sigma_t^2 &= 0.01(5.26 - \sigma_t^2)dt + 0.31\sigma_t^2 dW_t^{(2)}, \quad dW_t^{(1)}, dW_t^{(2)} \sim \mathcal{N}(0, t). \end{aligned}$$

- Terminal condition: $V(P, \sigma^2, I, T) = \mu I$.

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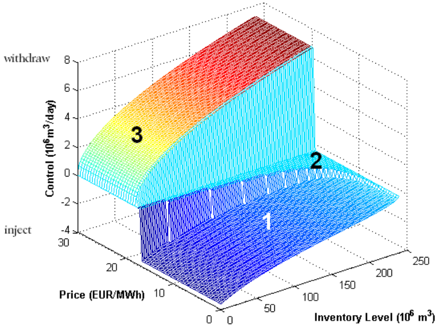
The control surface is split into three areas

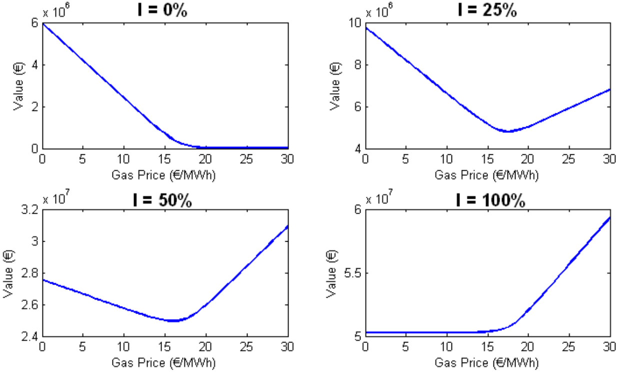


Area 1:
Future payoff exceeds the instant costs.

Area 2:
Costs of injection are higher than the resulting benefit.

Area 3:
Immediate payoff exceeds future payoff.

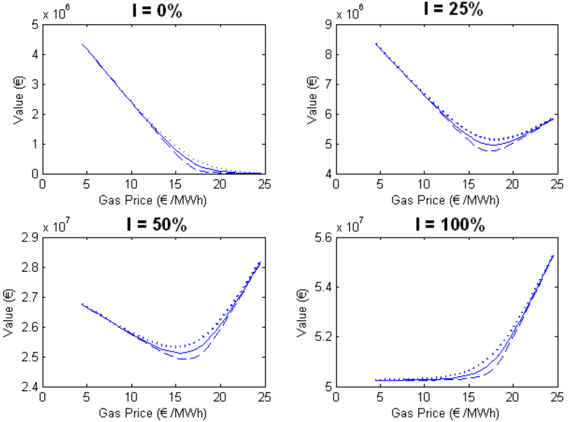
The value surface shows an option-like structure



For I = 0% the structure respects a put-option, for I = 100% we see the shape of a call option. Between these extremes the storage value has a straddle-like shape.

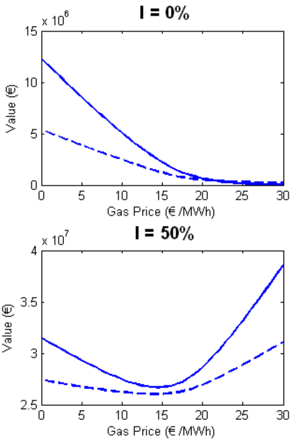
A sensitivity analysis

Higher volatility leads to higher storage value



The dashed line represents $\sigma^2 = 0.3$ at $t = 0$, the solid (dotted) line $\sigma^2 = 1.25(2.5)$.

Lower mean reversion leads to higher storage value

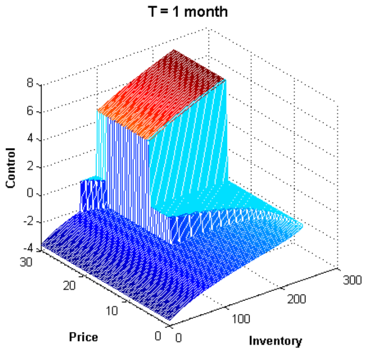


The solid (dashed) line shows the value surface for $\lambda = 0.014$ (0.1).

The sensitivity of further parameters

- The discounted long-run mean marks the border between withdrawing and remaining idle. It shifts as μ is changed.
- Although the volatility parameters influence the value surface, they do not influence the control surface. Every parameter shift that enhances volatility increases the storage value – however just around the discounted long-term mean. In the extreme regions, the effect diminishes.
- The terminal condition has – together with the lease period – a significant influence on both the control and the value surface.

An example for the terminal condition



$$V(P,\cdot,T,I) = \begin{cases} P \cdot (I - 100) & \text{if } I > 100, \\ 1.25 \cdot P \cdot (I - 100) & \text{if } I < 100, \\ 0 & \text{if } I = 100. \end{cases}$$

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The major results are:

1. The larger the volatility at $t = 0$, the higher the storage value. The effect, however, is decreasing for extremely high and low prices.
2. Mean reversion and all volatility parameters do not influence the control surface, but the value surface.
3. The long-term mean determines the structure of the control surface.
4. The terminal condition has – in combination with the lease period – a significiant influence on the control surface.

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Thank You for Your Attention!