Information asymmetry in Pricing of Credit Derivatives.

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OUTLINE OF THE TALK

OUTLINE

Introduction

- There exist two main approaches in the credit risk modelling: the structural approach and the reduced-form approach.
- The two approaches are related by the accessibility of information
  - on the underlying asset value process (Duffie-Lando 2001, Jeanblanc-Valchev 2005, Coculescu-Geman-Jeanblanc 2006, Guo-Jarrow-Zeng 2008), delayed or noisy observation of the underlying process,
  - on the default threshold (El Karoui 1999, Giesecke-Goldberg 2008), constant or random default barrier.

AIM study the impact of the information concerning the default threshold in the credit analysis, in addition to the partial observation of the underlying asset process.

INTRODUCTION

$(\Omega, \mathcal{A}, \mathbb{P})$ probability space.

- $(X_t)_{t\geq 0}$ positive continous-time process : asset value of the firm.
  $\mathbb{F} = (\mathcal{F}_t = \sigma(X_s, s \leq t))_{t\geq 0}$ satisfying the usual conditions
- Default threshold L, random variable in $\mathcal{A}$.
- Default time τ

$$\tau = \inf\{t : X_t \leq L\} \quad \text{where } X_0 > L$$

  with the convention that $\inf \emptyset = +\infty$
- We introduce the decreasing process X^* defined as

$$X_t^* = \inf\{X_s,\ s \leq t\}.$$

- We assume that the filtration $\mathbb{F}$ is generated by a Brownian motion B.

OUR FRAMEWORK

The manager of the firm has full information concerning the underlying asset of the firm and he chooses the default barrier.
$\Rightarrow L$ is a random variable set by the manager of the firm.

The investors on the market have different levels of information on the fundamental process $(X_t)_{t\geq 0}$ and on the default threshold L.

- Full information knowledge of $(X_t)_{t\geq 0}$ and L (manager of the firm)
- Noisy full information knowledge of $(X_t)_{t\geq 0}$ + noisy signal on L + default time observable
- Progressive information knowledge of $(X_t)_{t\geq 0}$ + default time observable
- Delayed information delayed information on $(X_t)_{t\geq 0}$ + default time observable

PRICING FRAMEWORK

$$\text{different level of information} \Rightarrow \begin{cases} \text{different filtration } (\mathcal{H}_t)_t \\ \text{different pricing measure } \mathbb{Q} \end{cases}$$

Evaluate a credit-sensitive derivative claim of maturity T :
the value process at time $t < \tau \wedge T$ is given by

$$V_t = R_t \mathbb{E}_{\mathbb{Q}} \left[C R_T^{-1} 1_{\{\tau > T\}} + \int_t^T 1_{\{\tau > u\}} R_u^{-1} dG_u + Z_\tau 1_{\{\tau \le T\}} R_\tau^{-1} \middle| \mathcal{H}_t \right] \quad (1)$$

where

- C ($\mathcal{F}_T$-measurable) represents the payment at the maturity T (if $\tau \geq T$)
- G ($\mathbb{F}$-adapted) represents the dividend payment
- Z ($\mathbb{F}$-predictable) represents the recovery payment at the default time τ
- R is the discount factor process.

OUTLINE

Full information

The manager knows the threshold L ω-wise from the beginning. Thus his information is given as the initial enlargement of the filtration $\mathbb{F}$ with respect to L :

Assumption (H^S)

$$\mathbb{G}^M = (\mathcal{G}_t^M)_{t\geq 0} \;\; with \;\; \mathcal{G}_t^M := \mathcal{F}_t \vee \sigma(L).$$

We assume that $\mathbb{P}(L \in \cdot|\mathcal{F}_t)(\omega) \sim \mathbb{P}(L \in \cdot)$ *for all* t *for* $\mathbb{P}$ *almost all* $\omega \in \Omega$.

Examples

- Assumption (H^S) is satisfied if L is independent of $\mathcal{F}_\infty$.
- Another example in a finite time horizon $T < T'$:

$$L = l_s 1_{]0,a[}(X_{T'}) + l_i 1_{]a,+\infty[}(X_{T'}), \;\; l_i < l_s$$

RISK NEUTRAL PRICING MEASURE FOR THE FULL INFORMATION

PROPOSITION

There exists a $\mathbb{G}^M$ adapted process $(\rho^M_\cdot(L))$ (called information drift) such that

- $Y^M(L) = \mathcal{E}(-\int_0^\cdot \rho^M_s(L)(dB_s - \rho^M_s(L)ds))$ is a probability density
- Any $(\mathbb{F}, \mathbb{Q})$-local martingale is an $(\mathbb{G}^M, Y^M(L)\mathbb{Q})$-local martingale.

THE PROGRESSIVE INFORMATION

This is the standard information structure in the credit risk analysis. Investors know at each time t whether or not default has occured. Thus their information is given as the progressive enlargement of filtration of $\mathbb{F}$ with respect to τ:

ASSUMPTION (H^N)

$$\mathbb{G} = (\mathcal{G}_t)_{t\geq 0} \ \ with \ \ \mathcal{G}_t = \mathcal{F}_t \vee \sigma(\{\tau \leq s\}, s \leq t).$$

REMARK

If L is independent of $\mathcal{F}_\infty$, the standard (H)-hypothesis is satisfied: every $(\mathbb{F},\mathbb{P})$ local martingale is also a $(\mathbb{G},\mathbb{P})$ local martingale.

THE DELAYED INFORMATION

$$\mathcal{F}_t^D := \begin{cases} \mathcal{F}_0 & \text{if } t \leq \delta(t), \\ \mathcal{F}_{t-\delta(t)} & \text{if } t > \delta(t), \end{cases}$$

- constant delay time model : $\delta(t) = \delta$
- discrete observation model : $\delta(t) = t - t_i^{(m)}$, $t_i^{(m)} \leq t < t_{i+1}^{(m)}$ where $0 = t_0^{(m)} < t_1^{(m)} < \cdots < t_m^{(m)} = T$ are the only discrete dates on which the information is renewed.

The delayed information is the progressive enlargement of filtration of $\mathbb{F}^D$ with respect to τ:

ASSUMPTION (H^D)

$$\mathbb{G}^D = (\mathcal{G}_t^D)_{t \geq 0} \ \ with \ \ \mathcal{G}_t^D = \mathcal{F}_t^D \vee \sigma(\{\tau \leq s\}, s \leq t).$$

NOISY FULL INFORMATION

The investor observes the value of the firm $(X_t)_t$ and receives a noisy signal $(L_s = f(L, \epsilon_s))_{s\geq 0}$ on the threshold L.

ASSUMPTION (H^N)

$$\mathbb{G}^I = (\mathcal{F}_t^I \vee \sigma(\{\tau \leq s\}, s \leq t))_{t\geq 0} \;\; with \;\; \mathcal{F}_t^I = \cap_{u>t}(\mathcal{F}_u \vee \sigma(f(L, \epsilon_s), s \leq u))$$

- $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$ *is a given measurable function.*
- $\epsilon = \{\epsilon_t, t \geq 0\}$ *is independent of* $\mathcal{F}_\infty \vee \sigma(L)$.
- $\mathbb{P}(L \in \cdot|\mathcal{F}_t)(\omega) \sim \mathbb{P}(L \in \cdot)$ *for all t for* $\mathbb{P}$ *almost all* $\omega \in \Omega$.

EXAMPLE

Example in a finite time horizon $T < T'$:
$L_s = L + W_{g(T'-s)}$ with W an independent Brownian motion and
$g : [0, T'] \longrightarrow [0, \infty[$ a strictly increasing bounded function with $g(0) = 0$

Risk neutral pricing measure for the noisy information

Proposition

There exists a $\mathbb{F}^I$ adapted process $(\rho^I_\cdot)$ such that

- $Y^I = \mathcal{E}(-\int_0^\cdot \rho^I_s (dB_s - \rho^I_s ds))$ is a probability density
- Any $(\mathbb{F}, \mathbb{Q})$-local martingale is an $(\mathbb{F}^I, Y^I\mathbb{Q})$-local martingale.

Outline

AIM Compute the price of the contingent claim (C, G, Z) with maturity T given different sources of information :

$$V_t = R_t \mathbb{E}_{\mathbb{P}} \left[C R_T^{-1} 1_{\{\tau > T\}} + \int_t^T 1_{\{\tau > u\}} R_u^{-1} dG_u + Z_\tau 1_{\{\tau \leq T\}} R_\tau^{-1} \,\middle|\, \mathcal{H}_t \right]$$

$\longrightarrow$ $\mathbb{P}$ is the historical probability measure

$\longrightarrow$ $(\mathcal{H}_t)_{t\geq 0}$ describes the accessible information for the investors.

- $(\mathcal{H}_t)_{t\geq 0} = (\mathcal{G}_t^M)_{t\geq 0}$ for the full information,
- $(\mathcal{H}_t)_{t\geq 0} = (\mathcal{G}_t^I)_{t\geq 0}$ for a noisy full information.
- $(\mathcal{H}_t)_{t\geq 0} = (\mathcal{G}_t)_{t\geq 0}$ for the progressive information.
- $(\mathcal{H}_t)_{t\geq 0} = (\mathcal{G}_t^D)_{t\geq 0}$ for the delayed information.

Pricing for the full information

Proposition

We define $F_t^M(x) := p_t(x)1_{\{X_t^* > x\}}$ where $p_t(x)(\omega) = \frac{dP_t^L}{dP^L}(\omega, x)$,
$P_t^L(\omega, dx)$ = a regular version of the conditional law of L given $\mathcal{F}_t$,
P^L = the law of L.

The value process of the contingent claim (C, G, Z) given the full information $(\mathcal{G}_t^M)_{t\geq 0}$ is

$$V_t^M = 1_{\{\tau > t\}} \frac{\widetilde{V}_t^M(L)}{p_t(L)}$$

where

$$\widetilde{V}_t^M(L) = R_t E_{\mathbb{P}}\left[C R_T^{-1} F_T^M(x) + \int_t^T F_s^M(x) R_s^{-1} dG_s - \int_t^T Z_s R_s^{-1} dF_s^M(x) \,\middle|\, \mathcal{F}_t \right]_{x=L}.$$

Pricing for the progressive and delayed information

Proposition

We define $S_t := \mathbb{P}(\tau > t \,|\, \mathcal{F}_t)$.

- The value process given the progressive information flow $\mathbb{G}$ is

$$V_t = 1_{\{\tau>t\}} \frac{R_t}{S_t} E_{\mathbb{P}} \left[R_T^{-1} S_T C + \int_t^T R_u^{-1} S_u dG_u - \int_t^T R_u^{-1} Z_u dS_u \,\middle|\, \mathcal{F}_t \right].$$

- The value process for a delay-informed investor is

$$V_t^D = \frac{1_{\{\tau>t\}}}{\mathbb{E}[S_t | \mathcal{F}_t^D]} \mathbb{E}_{\mathbb{P}} \left[\frac{R_t}{R_T} S_T C + \int_t^T \frac{R_t}{R_u} S_u dG_u - \int_t^T \frac{R_t}{R_u} Z_u dS_u \,\middle|\, \mathcal{F}_t^D \right].$$

Pricing for the noisy information

Proposition

We assume (H^N) with $L_t = L + \epsilon_t$, ϵ_t being a continuous process with backwardly independent increments and whose marginal has density q_t. The value process for the noisy full information flow $\mathbb{G}^I$ is given by

$$V_t^I = \frac{1_{\{\tau > t\}}}{\int_{\mathbb{R}} F_t^M(l) q_t(L_t - l) P^L(dl)} \int \widetilde{V_t^M}(l) q_t(L_t - l) P^L(dl)$$

where $\widetilde{V^M}$ and F^M are defined for the full information.

$$F_t^M(x) := p_t(x) 1_{\{X_t^* > x\}}.$$

$$\widetilde{V}_t^M(L) = R_t E_{\mathbb{P}} \left[C R_T^{-1} F_T^M(x) + \int_t^T F_s^M(x) R_s^{-1} dG_s - \int_t^T Z_s R_s^{-1} dF_s^M(x) \,\middle|\, \mathcal{F}_t \right]_{x=L}.$$

OUTLINE

1. Introduction
2. The informational structure
3. Pricing under the historical probability
4. RISK NEUTRAL PRICING
5. Numerical example

Risk neutral probabilities

- We assume that a pricing probability $\mathbb{Q}$ is given with respect to the filtration $\mathbb{F}$ of the fundamental process X (for example, $\mathbb{Q}$ such that X is an $(\mathbb{F}, \mathbb{Q})$ local martingale).
- We want to focus on the change of probability measures due to the different sources of informations and on its impact on the pricing of credit derivatives, $\Rightarrow$ without loss of generality, we take the historical probability $\mathbb{P}$ to be the benchmark pricing probability $\mathbb{Q}$ on $\mathbb{F}$ and $\mathbb{G}$.
- The pricing probability for the manager is $\mathbb{Q}^M$ where $\frac{d\mathbb{Q}^M}{d\mathbb{Q}} = Y^M(L)$ with $Y^M(L) = \mathcal{E}(-\int_0^{\cdot} \rho_s^M(L)(dB_s - \rho_s^M(L)ds))$
- The pricing probability for the noisy full information is $\mathbb{Q}^I$ where $\frac{d\mathbb{Q}^I}{d\mathbb{Q}} = Y^I$ with $Y^I = \mathcal{E}(-\int_0^{\cdot} \rho_s^I(dB_s - \rho_s^I ds))$
- We also take $\mathbb{Q}$ as the pricing probability for the delayed information

Risk neutral pricing for the full information

Proposition

1) Define $F_t^{\mathbb{Q}^M}(l) = 1_{\{X_t^* > l\}}$. Then the value process of a credit sensitive claim (C, G, Z) for the manager's full information under the risk neutral probability measure $\mathbb{Q}^M$ is given by

$$V_t^{\mathbb{Q}^M} = 1_{\{\tau > t\}} R_t \qquad \mathbb{E}_{\mathbb{P}}\Big[C R_T^{-1} F_T^{\mathbb{Q}^M}(x) + \int_t^T F_s^{\mathbb{Q}^M}(x) R_s^{-1} dG_s - \int_t^T Z_s R_s^{-1} dF_s^{\mathbb{Q}^M}(x) \; | \mathcal{F}_t \Big]_{x=L}.$$

2) Let ϵ be a continuous process with backwardly independent increments such that the probability law of ϵ_t has a density $q_t(\cdot)$ w.r.t. the Lebesgue measure. Let $\mu_{t,\theta}$ be the probability law of $\epsilon_\theta - \epsilon_t$. Then the value process for the insider's noisy full information under $\mathbb{Q}^I$ is given by

$$V_t^{\mathbb{Q}^I} = \frac{1_{\{\tau > t\}}}{\int_{\mathbb{R}} F_t^M(l) q_t(L_t - l) P^L(dl)} \int \widetilde{V}_t^{\mathbb{Q}^I}(l) q_t(L_t - l) P^L(dl)$$

where

$$\widetilde{V}_t^{\mathbb{Q}^I}(l) = R_t \mathbb{E}_{\mathbb{P}}\big[C R_T^{-1} F_{t,T}^I(u,l) + \int_t^T F_{t,\theta}^I(u,l) R_\theta^{-1} dG_\theta - \int_t^T R_\theta^{-1} Z_\theta dF_{t,\theta}^I(u,l) | \mathcal{F}_t \big]_{u=L_t},$$

$$F_{t,\theta}^I(u,l) = \mathcal{E}\Big(\int_t^\theta \int \rho_s^I(u+y) \mu_{t,\theta}(dy) dB_s \Big)^{-1} F_\theta^M(l).$$

OUTLINE

Binomial Model

- Let L be a $(\mathcal{F}_t)_{t\geq 0}$-independent random variable taking two values $l_i, l_s \in \mathbb{R}$, $l_i \leq l_s$ such that

$$\mathbb{P}(L = l_i) = \alpha, \quad \mathbb{P}(L = l_s) = 1 - \alpha \quad (0 < \alpha < 1).$$

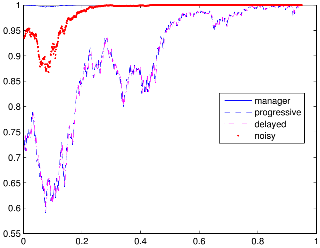
- We suppose that the asset values process X satisfies the Black Scholes model :

$$\frac{dX_t}{X_t} = \mu dt + \sigma dB_t, \quad t \geq 0.$$

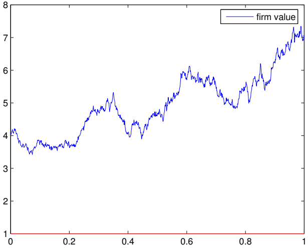
- We compute the value process of a defaultable bond.

VALUE PROCESS OF A DEFAULTABLE BOND

Numerical values in the following simulations : $l_i = 1, l_s = 3, \alpha = \frac{1}{2}$.

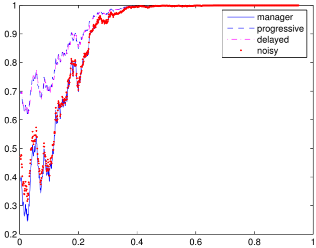


dynamic price of the defaultable bond

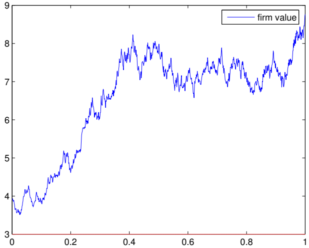


firm value

FIGURE: $L = l_i$



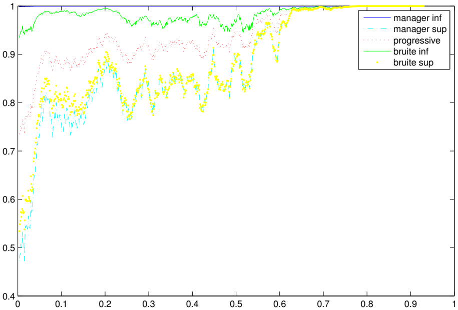
dynamic price of the defaultable bond



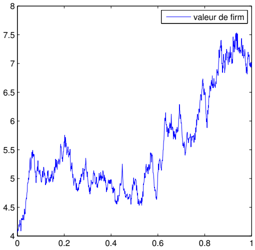
firm value

FIGURE: $L = l_s$

We compare the dynamic price of the defaultable bond, for the same scenario of the firm value but depending on the level of the threshold fixed by the manager.



dynamic price of the defaultable bond



firm value

FIGURE: $L = l_s$ or $L = l_i$

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