Optimal control of trading algorithms: a general impulse control approach

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6th World Congress of the Bachelier Finance Society
June 24 2010

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Outline

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$$\eta(v;\tau,\tau+\delta) = \alpha \cdot \underbrace{\psi_{BA}(\tau,\tau+\delta)}_{\text{Bid-Ask spread}} + \kappa \cdot \sigma(\tau,\tau+\delta) \underbrace{\left(\frac{v}{V(\tau,\tau+\delta)}\right)^{\gamma}}_{\text{asked volume ratio}}.$$

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- Remark: η introduced to take into account quantitatively the aggregated effet on price.

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Classical approaches

- Original framework: a priori discretization of the trading phases [Almgren and Chriss, 2000], taking into account real time analytics came from continuous time models [Almgren, 2009].
- Modified framework: inclusion of new effect
  - statistical effects [Lehalle, 2008],
  - specific properties of the price diffusion process (mean reverting) [Lehalle, 2009],
  - information at an orderbook level [Hewlett, 2007],
  - Bayesian estimation of the market trend [Almgren and Lorenz, 2008].

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- Offer the capability to model the market underlying moves via time-continuous models.
- Algorithmic trading process is the one of switching between those states:
  - the passive regime: no slice in market, price formation process will continue to take place without interaction with the controlled order,
  - the active regime: parametrized slice in market, duration bounded from below by a constant $\underline{\delta}$, characteristics of a slice are chosen just before its launch, cannot be modified until it ends.

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- Also to control *meta trading algorithms*: optimal launch of *sequence of traditional algorithms* (the first to proposed this possibility).
- Continuous-time allows the use of traditional models and tools from quantitative finance.

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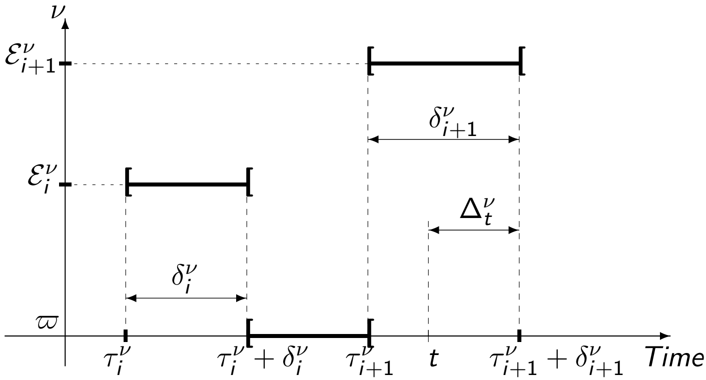
- $(\delta_i, \mathcal{E}_i)$ is $\mathcal{F}_{\tau_i}$-measurable , $i \geq 1$,
- $\nu : t \in [\tau_i, \tau_i + \delta_i) \mapsto \nu_t$: value of the parameter at t, $\nu_t = \varpi \in \mathbb{R}^d \setminus E$ for $t \in A((\tau_i, \delta_i)_{i\geq 1})$, defined as

$$A((\tau_i, \delta_i)_{i\geq 1}) := \mathbb{R}_+ \setminus \left(\bigcup_{i\geq 1} [\tau_i, \tau_i + \delta_i) \right) .$$

Control policy



- $\nu_t = \varpi \mathbf{1}_{t \in A((\tau_i,\delta_i)_{i\geq 1})} + \sum_{i\geq 1} \mathbf{1}_{t\in[\tau_i,\tau_i+\delta_i)} \mathcal{E}_i \ , \ t \in [0,T] \ ,$
- $\Delta^\nu_t := \sum_{i\geq 1} [\tau^\nu_i + \delta^\nu_i - t]^+ \mathbf{1}_{t \geq \tau^\nu_i} \ , \ t \in [0,T] \ ,$
- $\bar{E} := E \cup \{\varpi\} \ .$

Dynamics - Objective functional

- $(t, x) \in [0, T] \times \mathbb{R}^d$, $X^{\nu}_{t,x}$ strong solution of

$$X^{\nu}_{t,x}(s) = x + \mathbf{1}_{s \geq t} \Big(\int_t^s b(X^{\nu}_{t,x}(r), \nu_r) dr + \int_t^s a(X^{\nu}_{t,x}(r), \nu_r) dW_r + \sum_{i \geq 1} \beta(X^{\nu}_{t,x}(\tau^{\nu}_i -), \mathcal{E}^{\nu}_i, \delta^{\nu}_i) \mathbf{1}_{t < \tau^{\nu}_i \leq s} \Big) .$$

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- Controller maximizes the functional

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- over the admissible set

$$\mathcal{S}_{t,\delta,e} := \left\{ \nu \in \mathcal{S} \ : \ \nu_s = e \text{ for } s \in [t, t+\delta) \text{ and } \Delta^{\nu}_{t+\delta} = 0 \right\},$$

$(\delta, e) \in \mathbb{R}_+ \times \bar{E}$: initial state (remaining latency, parameters).

Value function

- $J(t, x; \nu) := \mathbb{E}[\Pi_{t,x}(\nu)]$ is well defined for all $\nu \in \mathcal{S}$ and admits at most polynomial growth.
- Technical reason related to the DPP, consider only admissible trading strategies $\nu \in \mathcal{S}_{t,\delta,e}$ such that ν is independent on $\mathcal{F}_t$, denote by $\mathcal{S}^a_{t,\delta,e}$ (see [Bouchard and Touzi, 2009]).
- Value function

$$V(t, x, \delta, e) := \sup_{\nu \in \mathcal{S}^a_{t,\delta,e}} J(t, x; \nu) .$$

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Remaining shares $Q_t^\nu = Q_0 - \int_0^t \nu_s \mathbf{1}_{\nu_s \neq \varpi} V_s ds$.

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$$
\min_\nu \mathbb{E}\left[\ell\big(\underbrace{0 + \int_0^T \tilde{S}_t q(\nu_t) V_t dt}_{\text{running cost}} + \underbrace{(S_T + c(Q_T^\nu, S_T, V_T))\,(Q_T^\nu)^+}_{\text{final cost}}\big)\right]
$$

for ℓ convex, polynomial growth, $\tilde{S}_t = S_t + \eta(\nu_t, S_t, V_t)$.

Domain of definition of V

- Natural domain

$$D := \left\{(t, x, \delta, e) \in [0, T) \times \mathbb{R}^d \times (((0, \infty) \times E) \cup \{(0, \varpi)\}) \right.$$
$$\left. : t + \delta \in [\underline{\delta}, T) \text{ or } e = \varpi \right\},$$

- which can be decomposed in two main regions:
  - *active region* : $\delta > 0$ and $e \neq \varpi$

    $$D_{E,>0} := \left\{(t, x, \delta, e) \in [0, T) \times \mathbb{R}^d \times (0, \infty) \times E : t + \delta \in [\underline{\delta}, T)\right\} .$$

  - *passive region*: $e = \varpi$, and therefore $\delta = 0$
    $D_{\varpi} := [0, T) \times \mathbb{R}^d \times \{0, \varpi\}$.

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- and can be completed by boundary regions
  - Boundary of active region $\delta \to 0$ and $t+\delta \to T$:

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  - Time boundary: $D_T := \{T\} \times \mathbb{R}^d \times \mathbb{R}_+ \times \bar{E}$.
- Closure of domain $\bar{D} :=$
$\Big\{(t,x,\delta,e) \in [0,T] \times \mathbb{R}^d \times \mathbb{R}_+ \times \bar{E} : t+\delta \in [\underline{\delta}, T) \text{ or } e = \varpi\Big\}$.

Special value of V

- For $\delta = T - t$ and $e \in E$ (i.e., keep e until maturity):

$$V(t, x, T-t, e) = \mathcal{V}(t, x, e) := \mathbb{E}\left[g(\mathcal{X}^e_{t,x}(T)) + f(\mathcal{X}^e_{t,x}(T), e)\right] ,$$

$$\mathcal{X}^e_{t,x}(s) = x + \int_t^s b(\mathcal{X}^e_{t,x}(r), e)dr + \int_t^s a(\mathcal{X}^e_{t,x}(r), e)dW_r] .$$

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- Standard arguments imply that $\mathcal{V}$ continuous, and for each $e \in E$, it is a viscosity solution of

$$-\mathcal{L}^e\varphi(t,x) = 0 \text{ on } [0,T)\times\mathbb{R}^d , \ \varphi(T,x) = g(x)+f(x,e) \text{ on } \mathbb{R}^d ,$$

where the Dynkin operator $\mathcal{L}^e\varphi$ defined for $e \in \bar{E}$, and a smooth function φ .

Passive region

- Rely on dynamic programming principle: for any $[t, T]$-valued stopping time ϑ:

$$V(t,x,\delta,e) = \sup_{\nu \in S^a_{t,\delta,e}} \mathbb{E}\Big[V(\vartheta, X^\nu_{t,x}(\vartheta), \Delta^\nu_\vartheta, \nu_\vartheta) + \sum_{i \in \mathbb{I}^\nu_{t,\vartheta}} f(X^\nu_{t,x}(\tau^\nu_i + \delta^\nu_i), \mathcal{E}^\nu_i) \Big] .$$

- For $(t,x,\delta,e) \in D_\varpi$, possible to immediately launch algorithm with new set of parameters $(\delta', e') \in [\underline{\delta}, T-t] \times E$. Taking $\vartheta = t$: $V(t,x,0,\varpi) \geq \mathcal{M}[V](t,x)$,

$$\mathcal{M}[V](t,x) := \sup_{(\delta',e') \in [\underline{\delta}, T-t] \times E} V(t, x + \beta(x,e',\delta'), \delta', e') ,$$

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- Also able to wait before passing a new order, i.e. choose $\nu = \varpi$ on some time interval $[t, t+\delta')$ with $\delta' > 0$, by arbitrariness of $\vartheta < t + \delta'$:

$$-\mathcal{L}^{\varpi} V(t,x,0,\varpi) \geq 0 .$$

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- Dynamic programming principle:

$$\min\left\{ -\mathcal{L}^\varpi V(t,x,0,\varpi) \; ; \; V(t,x,0,\varpi) - \mathcal{M}[V](t,x) \right\} = 0 .$$

Active region

- Rely on dynamic programming principle: for any $[t, T]$-valued stopping time ϑ:

$$V(t,x,\delta,e) = \sup_{\nu \in S^a_{t,\delta,e}} \mathbb{E}\Bigg[V(\vartheta, X^{\nu}_{t,x}(\vartheta), \Delta^{\nu}_{\vartheta}, \nu_{\vartheta}) + \sum_{i \in \mathbb{I}^{\nu}_{t,\vartheta}} f(X^{\nu}_{t,x}(\tau^{\nu}_i + \delta^{\nu}_i), \mathcal{E}^{\nu}_i) \Bigg] .$$

- For $(t,x,\delta,e) \in D_{E,>0}$, cannot change the parameter before the end of the initial latency period $\delta > 0$. Choosing ϑ arbitrarily small:

$$\left(-\mathcal{L}^e + \frac{\partial}{\partial \delta} \right) V(t,x,\delta,e) = 0 .$$

Boundary conditions

- Active region:

$$V(t,x,\delta,e) = V(t,x,0,\varpi) + f(x,e)\ , \quad \text{if } (t,x,\delta,e) \in D_{E,0}\ ,$$
$$V(t,x,\delta,e) = \mathcal{V}(t,x,e)\ , \quad \text{if } (t,x,\delta,e) \in D_{E,T}\ .$$

- Terminal condition:

$$V(t,x,\delta,e) = g(x) + f(x,e)\ , \quad \text{if } (t,x,\delta,e) \in D_T\ .$$

Viscosity characterization

Define

$$\mathcal{H}\varphi := \begin{cases} \left(-\mathcal{L}^e + \frac{\partial}{\partial\delta}\right)\varphi(\cdot,\delta,e) & \text{on} \quad D_{E,>0} \ , \\ \varphi(\cdot,\delta,e) - \varphi(\cdot,0,\varpi) - f(x,e) & \text{on} \quad D_{E,0} \ , \\ \varphi(\cdot,\delta,e) - \mathcal{V}(\cdot,e) & \text{on} \quad D_{E,T} \ , \\ \min\left\{ -\mathcal{L}^{\varpi}\varphi(\cdot,\delta,e) \, ; \, \varphi(\cdot,\delta,e) - \mathcal{M}[\varphi](\cdot)\right\} & \text{on} \quad D_{\varpi} \ , \\ \varphi(\cdot,\delta,e) - g(\cdot) - f(\cdot,e) & \text{on} \quad D_T \ . \end{cases}$$

Also define for $(t,x,\delta,e) \in \bar{D}$

$$V^*(t,x,\delta,e) := \limsup_{(t',x',\delta',e')\in D \to (t,x,\delta,e)} V(t',x',\delta',e')$$

$$V_*(t,x,\delta,e) := \liminf_{(t',x',\delta',e')\in D \to (t,x,\delta,e)} V(t',x',\delta',e') \ .$$

Viscosity characterization (cont.)

Theorem

The function V_* (resp. V^*) is a vicosity supersolution (resp. subsolution) of $\mathcal{H}\varphi = 0$ on $\bar{D}$.

Proof and comparison theorem omitted.

What remain to do?

- Numerical resolution by finite difference method
- Convergence verified similarly in [Barles and Souganidis, 1991].

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Parameters:

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- ℓ is the identity, simplifies the numerical resolution since linear in y-variable.

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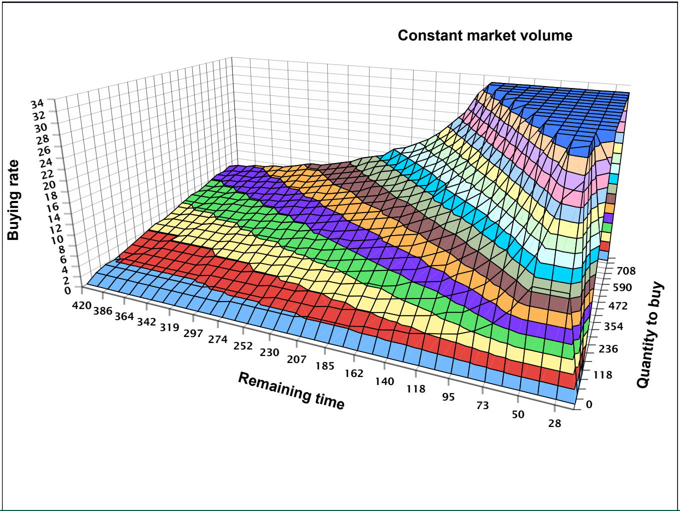
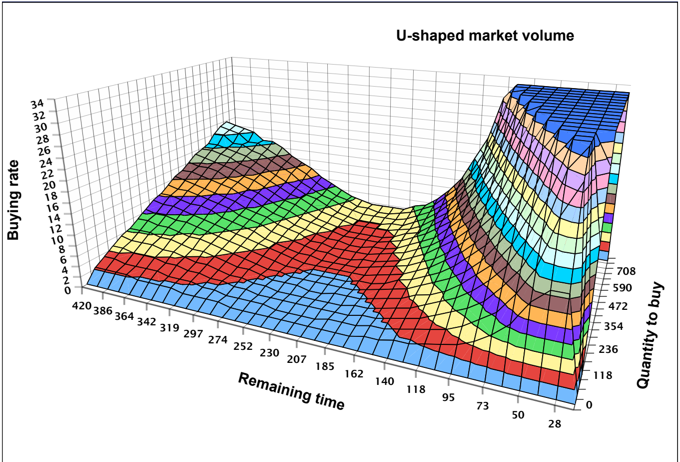
PoV algorithm - Case flat market volume $V_t \equiv 100$



PoV algorithm - Case U-shaped volume $V_t = 100(1.1 - \sin(\pi t/T))$



Percent of Volume Algorithm - Real case

Volume and volatility estimated from real data (France Telecom Jan. 2008 to Dec. 2008), normalized such that average daily volume = 2000.

$S_0 = 10$, $Q_0 = 50$, maximal rate $E_{max} = 0.1$.

Functions: $\eta(e, s, v) = 0.2e^{1.1}$, $c(q, s, v) = 0.3q^+$, $\ell(y) = (y^+)^2$.

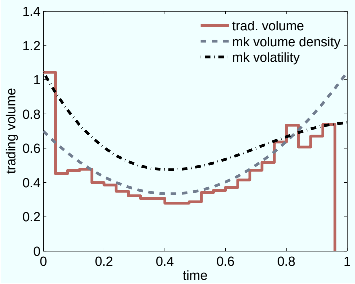


Figure: Volume - Volatility - Average trading curves

Some simulated trajectories

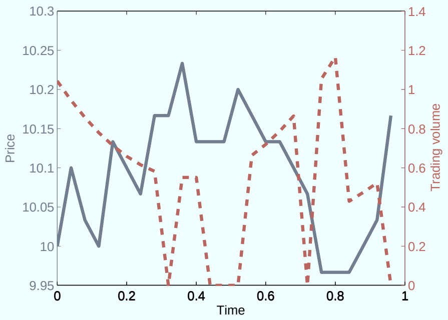


Figure: Simulated price and volume - 1

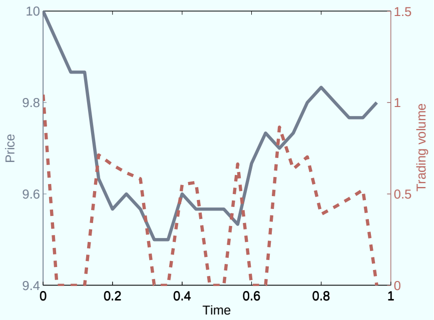


Figure: Simulated price and volume - 2

Thank you for your attention

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