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Hedging of swaptions in a Lévy driven Heath-Jarrow-Morton framework

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Aim

- A compact representation for the pricing formula by using the Jamshidian decomposition
- Hedging strategies with default-free zero coupon bonds (delta-hedging $\leftrightarrow$ quadratic hedging)
- Numerical implementation and results

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Outline

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Outline

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Zero-coupon bond

$B(t, T)$

- $B(T, T) = 1$
- No coupons, No default
- $B(t, T) < 1$ for every $t < T$
- $f(t, u)$ instantaneous forward rate:
  $B(t, T) = \exp(-\int_t^T f(t, u)du)$

Original HJM model

Dynamics of forward interest rate

$$df(t,T) = \alpha(t,T)dt + \sigma'(t,T)dW_t$$

with W standard d-dimensional Brownian motion under $\mathbb{P}$

α and σ adapted stochastic processes in $\mathbb{R}$, resp $\mathbb{R}^d$

$'$ denotes transpose

Original HJM model

Dynamics of zero coupon bonds

$$dB(t,T) = B(t,T)(a(t,T)dt - \sigma^{*\prime}(t,T)dW_t)$$

with

$$\begin{aligned} a(t,T) =& f(t,t) - \alpha^*(t,T) + \frac{1}{2}|\sigma^*(t,T)|^2 \\ \alpha^*(t,T) =& \int_t^T \alpha(t,u)du \\ \sigma^*(t,T) =& \int_t^T \sigma(t,u)du. \end{aligned}$$

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Lévy driven HJM model

Dynamics of forward interest rate

$$df(t,T) = \alpha(t,T)dt - \sigma(t,T)dL_t$$

with L: one-dimensional time-inhomogeneous Lévy process

The law of L_t is characterized by the characteristic function

$$E[e^{izL_t}] = e^{\int_0^t \theta_s(iz)ds}, \quad \forall t \in [0,T^*]$$

with θ_s cumulant associated with L by the Lévy-Khintchine triplet (b_s, c_s, F_s):

$$\theta_s(z) := b_s z + \frac{1}{2}c_s z^2 + \int_{\mathbb{R}} (e^{xz} - 1 - xz)F_s(dx)$$

with $b_t, c_t \in \mathbb{R}$, $c_t \geq 0$, F_t Lévy measure

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Lévy driven HJM model

Integrability assumptions:

- $$\int_0^{T^*}\left(|b_s| + |c_s| + \int_{\mathbb{R}}(x^2 \wedge 1)F_s(dx)\right) ds < \infty$$
- There are constants $M, \epsilon > 0$ such that for every $u \in [-(1+\epsilon)M, (1+\epsilon)M]$:

$$\int_0^{T^*}\int_{\{|x|>1\}} \exp(ux)F_s(dx)ds < \infty$$

$\Rightarrow$ L is an exponential special semimartingale

Interest rate products

Savings account and default-free zero coupon bond prices:

$$B_t = B(0,t)\exp\left(\int_0^t A(s,t)ds - \int_0^t \Sigma(s,t)dL_s\right)$$

$$B(t,T) = B(0,T)B_t\exp\left(-\int_0^t A(s,T)ds + \int_0^t \Sigma(s,T)dL_s\right)$$

with for $s \wedge T = \min(s,T)$ and $s \in [0,T^*]$

$$A(s,T) = \int_{s\wedge T}^{T} \alpha(s,u)du \quad \text{and} \quad \Sigma(s,T) = \int_{s\wedge T}^{T} \sigma(s,u)du,$$

Heath-Jarrow-Morton

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Unique martingale measure=spot measure

$$A(s,T) = \theta_s(\Sigma(s,T))$$

with θ the cumulant associated with L by (b_s, c_s, F_s)

$$\theta_s(z) = b_s z + \frac{1}{2} c_s z^2 + \int_{\mathbb{R}} (e^{xz} - 1 - xz) F_s(dx)$$

$\Rightarrow$ Discounted zero-coupon bonds are martingales

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Forward MM

$$\frac{d\mathbb{P}_T}{d\mathbb{P}^*} = \frac{1}{B_T B(0,T)} = \exp(-\int_0^T \theta_s(\Sigma(s,T)ds + \int_0^T \Sigma(s,T)dL_s)$$

$$\left.\frac{d\mathbb{P}_T}{d\mathbb{P}^*}\right|_{\mathcal{F}_t} = \frac{B(t,T)}{B_t B(0,T)} = \exp(-\int_0^t \theta_s(\Sigma(s,T))ds + \int_0^t \Sigma(s,T)dL_s)$$

L: time-inhomogeneous Lévy process under $\mathbb{P}_T$ and special with characteristics $(b_s^{\mathbb{P}_T}, c_s^{\mathbb{P}_T}, F_s^{\mathbb{P}_T})$:

$$b_s^{\mathbb{P}_T} = b_s + c_s\Sigma(s,T) + \int_{\mathbb{R}} x(e^{\Sigma(s,T)x} - 1)F_s(dx)$$

$$c_s^{\mathbb{P}_T} = c_s$$

$$F_s^{\mathbb{P}_T}(dx) = e^{\Sigma(s,T)x}F_s(dx)$$

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Interest rate derivative

Swaption: option granting its owner the right but not the obligation to enter into an underlying interest rate swap.

- Interest rate swap: contract to exchange different interest rate payments, typically a fixed rate payment is exchanged with a floating one.
- A: Payer swaption
- B: Receiver swaption

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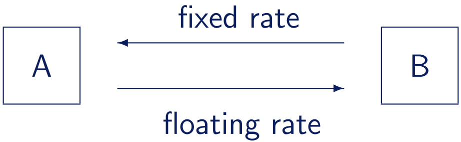

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Jamshidian

- Closed-form expression for European option price on coupon-bearing bond
- $P(r,t,s)$: Price at time t of a pure discount bond maturing at time s, given that $r(t)=r$ and $R_{r,t,s}$ is a normal random variable

$$\left(\sum a_j P(R_{r,t,T},T,s_j)-K\right)^+ = \sum a_j \left(P(R_{r,t,T},T,s_j)-K_j\right)^+$$

with $K_j = P(r^*,T,s_j)$
and r^* is solution to equation $\sum a_j P(r^*,T,s_j)=K$

- Holds for any short rate model as long as zero coupon bond prices are all decreasing (comonotone) functions of interest rate

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Fourier transformation

Theorem Eberlein, Glau, Papapantoleon (2009)

If the following conditions are satisfied:

(C1) The dampened function $g = e^{-Rx} f(x)$ is a bounded, continuous function in $L^1(\mathbb{R})$.

(C2) The moment generating function $M_{X_T}(R)$ of rv X_T exists.

(C3) The (extended) Fourier transform $\hat{g}$ belongs to $L^1(\mathbb{R})$,

$$\Rightarrow E[f(X_T - s)] = \frac{e^{-Rs}}{2\pi} \int_{\mathbb{R}} e^{ius} \varphi_{X_T}(-u - iR) \hat{f}(u + iR) du,$$

with φ_{X_T} characteristic function of the random variable X_T.

Outline

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Pricing of swaption

Assumptions on volatility structure

- Volatility structure σ: bounded and deterministic. For $0 \leq s$ and $T \leq T^*$:

$$0 \leq \Sigma(s, T) = \int_{s \wedge T}^{T} \sigma(s, u) du \leq M' < M,$$

- For all $T \in [0, T^*]$ we assume that $\sigma(\cdot\, , T) \not\equiv 0$ and

$$\sigma(s, T) = \sigma_1(s)\sigma_2(T) \quad 0 \leq s \leq T,$$

  where $\sigma_1 : [0, T^*] \rightarrow \mathbb{R}^+$ and $\sigma_2 : [0, T^*] \rightarrow \mathbb{R}^+$ are continuously differentiable.
- $\inf_{s \in [0, T^*]} \sigma_1(s) \geq \underline{\sigma}_1 > 0$

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Pricing of swaption

- Payer swaption can be seen as a put option with strike price 1 on a coupon-bearing bond.
- Payer swaption's payoff at T_0:

$$\left(1 - \sum_{j=1}^{n} c_j B(T_0, T_j)\right)^+,$$

  - $T_1 < T_2 < \ldots < T_n$: payment dates of the swap with $T_1 > T_0$
  - $\delta_j := T_j - T_{j-1}$: length of the accrual periods $[T_{j-1}, T_j]$
  - κ: fixed interest rate of the swap
  - coupons $c_i = \kappa\delta_i$ for $i = 1, \ldots, n-1$ and $c_n = 1 + \kappa\delta_n$

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Pricing of swaption

- Start from

$$\mathrm{PS}_t = B_t E[\frac{1}{B_{T_0}}(1 - \sum_{j=1}^{n} c_j B(T_0, T_j))^+ \mid \mathcal{F}_t] \qquad t \in [0, T_0]$$

with expectation under risk-neutral measure $\mathbb{P}^*$

- Change to forward measure $\mathbb{P}_{T_0}$ eliminating instantaneous interest rate B_{T_0} under expectation

$$\mathrm{PS}_t = B(t, T_0) E^{\mathbb{P}_{T_0}}[(1 - \sum_{j=1}^{n} c_j B(T_0, T_j))^+ \mid \mathcal{F}_t] \quad t \in [0, T_0]$$

Pricing of swaption

Define:

- $g(s,t,x) = \widetilde{D}_s^t e^{\widetilde{\Sigma}_s^t x} \qquad \forall 0 \leq s \leq t \leq T^*$
- $\widetilde{D}_s^t = \frac{B(0,t)}{B(0,s)} \exp\left(\int_0^s \left[\theta_u(\Sigma(u,s)) - \theta_u(\Sigma(u,t)) \right] du \right)$
- $\widetilde{\Sigma}_s^t = \int_s^t \sigma_2(u) du \quad$ and $\quad X_s = \int_0^s \sigma_1(u) dL_u$

$$\Rightarrow g(s,t,X_s) = B(s,t) \qquad \forall 0 \leq s \leq t \leq T^*$$

and price payer swaption

$$\mathrm{PS}_t = B(t,T_0) E^{\mathbb{P}_{T_0}}[(1 - \sum_{j=1}^{n} c_j g(T_j, T_0, X_{T_0}))_+ \mid \mathcal{F}_t]$$

by volatility structure assumptions functions $x \mapsto g(T_0, T_j, x)$ are non-decreasing functions for $j = 1, \ldots, n$

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$$= B(t, T_0) \sum_{j=1}^{n} c_j E^{\mathbb{P}_{T_0}}[(b_j - g(T_0, T_j, X_{T_0}))^+ | \mathcal{F}_t]$$

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$$= B(t,T_0)\sum_{j=1}^{n} c_j E^{\mathbb{P}_{T_0}}[(b_j - B(T_0,T_j))^+|\mathcal{F}_t]$$

weighted sum of put options with different strikes on bonds with different maturities

with b_j such that $\widetilde{D}_{T_0}^{T_j} e^{\widetilde{\Sigma}_{T_0}^{T_j} z^*} = g(T_0,T_j,z^*) = b_j$ and z^* is the solution to the equation

$$\sum_{j=1}^{n} c_j g(T_0,T_j,z^*) = 1$$

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Pricing of swaption

PS_t

$$B(t, T_0)\sum_{j=1}^{n} c_j \frac{e^{-RX_t}}{2\pi}\int_{\mathbb{R}} e^{iuX_t}\varphi^{\mathbb{P}_{T_0}}_{X_{T_0}-X_t}(u+iR)\hat{v}^j(-u-iR)du$$

with

$$\varphi^{\mathbb{P}_{T_0}}_{X_{T_0}-X_t}(z) = \exp\int_t^{T_0}[\theta_s(\Sigma(s, T_0)+iz\sigma_1(s)) - \theta_s(\Sigma(s, T_0))]ds$$

and where

$$\hat{v}^j(-u-iR) = \frac{b_j e^{(-iu+R)z^*\widetilde{\Sigma}^{T_j}_{T_0}}}{(-iu+R)(-iu+\widetilde{\Sigma}^{T_j}_{T_0}+R)}$$

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Outline

Assumptions

- Integrability assumptions
- Volatility structure assumptions
- $|\sigma_1| < \overline{\sigma}_1$ for a certain $\overline{\sigma}_1 \in \mathbb{R}$
- $|u| \cdot |\varphi^{\mathbb{P}_{T_0}}_{X_{T_0}-X_t}(u+iR)|$ is integrable

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Delta-hedging of PS

Theorem

The optimal amount, denoted by Δ_t^j, to invest in the zero coupon bond with maturity T_j to delta-hedge a short position in the forward payer swaption is given by:

$$\Delta_t^j = \frac{B(t,T_0)}{B(t,T_j)\widetilde{\Sigma}_t^{T_j}} \sum_{k=1}^{n} c_k\left(\widetilde{\Sigma}_t^{T_0} H^k(t,X_t) + \frac{\partial}{\partial X_t} H^k(t,X_t)\right),$$

with for $\ell = 0,1$

$$\frac{\partial^\ell H^k(t,X_t)}{\partial X_t^\ell} = \frac{1}{2\pi}\int_{\mathbb{R}} e^{(-R+iu)X_t}\varphi^{\mathbb{P}_{T_0}}_{X_{T_0}-X_t}(u+iR)\hat{v}^k(-u-iR)(-R+iu)^\ell du.$$

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Self-financing delta

$B(t, T_0)$: bond used as cash account, depends also on X

$B(t, T_j)$: bond in which to invest, with $T_j \neq T_0$

solving system of equations for Δ_t^j and Δ_t^0 to obtain discrete hedging strategy:

$$\begin{cases} \frac{\partial V_t}{\partial X_t} = -\frac{\partial \mathrm{PS}_t}{\partial X_t} + \Delta_t^j \frac{\partial B(t, T_j)}{\partial X_t} + \Delta_t^0 \frac{\partial B(t, T_0)}{\partial X_t} = 0 \\ (\Delta_t^j - \Delta_{t-1}^j) B(t, T_j) + (\Delta_t^0 - \Delta_{t-1}^0) B(t, T_0) = 0 \end{cases}$$

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MVH strategy

- Quadratic hedge in terms of discounted assets $\widetilde{S}$
- MVH strategy is self-financing
  $\Longrightarrow$ optimal amount of discounted assets is sensible amount to invest in non-discounted assets
- Minimizing the mean squared hedging error defined as

$$E[(H - (v + (\xi \cdot \widetilde{S})_T))^2]$$

MVH strategy

- Unrealistic to hedge with risk-free interest rate product
  $\Longrightarrow$ choose bond $B(\cdot, T_0)$ as numéraire
- MVH strategy for payer swaption under forward measure $\mathbb{P}_{T_0}$ using numéraire $B(\cdot, T_0)$

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MVH strategy

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MVH strategy

- Self-financing strategy minimizing

$$E^{\mathbb{P}_{T_0}}[(\mathrm{PS}_{T_0} - \widetilde{V}_{T_0})^2] = E^{\mathbb{P}_{T_0}}[(\mathrm{PS}_{T_0} - (\widetilde{V}_0 + \int_0^{T_0} \xi_u^j d\widetilde{B}(u, T_j)))^2]$$

with $\mathrm{PS}_{T_0} = \dfrac{\mathrm{PS}_{T_0}}{B(T_0, T_0)}$: (discounted) price of PS at time T_0

$\widetilde{V} = \dfrac{V}{B(\cdot, T_0)}$: (discounted) portfolio value process

- Value of self-financing portfolio V:

$$\begin{aligned} V_t &= \xi_t^0 B(t, T_0) + \xi_t^j B(t, T_j) \\ &= V_0 + (\xi^0 \cdot B(\cdot, T_0))_t + (\xi^j \cdot B(\cdot, T_j))_t \end{aligned}$$

MVH strategy

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Determination

- Ideas of

  Hubalek, Kallsen and Krawczyk (2006). Variance-optimal hedging for processes with stationary independent increments. *Annals of Applied Probability*, 16:853-885

  adapted to present setting
- GKW decomposition of special type of claims:

$$H(z) = \widetilde{B}(T_0, T_j)^z \quad \text{for a } z \in \mathbb{C}$$

- Express PS_{T_0} as $f(\widetilde{B}(T_0, T_j))$ with $f : (0, \infty) \to \mathbb{R}$ and

$$f(s) = \int s^z \Pi(dz)$$

  for some finite complex measure Π on a strip $\{z \in \mathbb{C} : R' \leq \mathrm{Re}(z) \leq R\}$

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Determination

- $(H_t(z))_{t\in[0,T_0]} := E^{\mathbb{P}_{T_0}}[\widetilde{B}(T_0,T_j)^z|\mathcal{F}_t]$
- Optimal number of risky assets related to claim $H_{T_0}(z)$ for every $t \in [0, T_0]$:

$$\xi_t^j(z) = \frac{d\langle H(z), \widetilde{B}(\cdot, T_j)\rangle_t^{\mathbb{P}_{T_0}}}{d\langle \widetilde{B}(\cdot, T_j), \widetilde{B}(\cdot, T_j)\rangle_t^{\mathbb{P}_{T_0}}} \implies \xi_t^j = \int \xi_t^j(z)\Pi(dz)$$

Determination

Lemma

$$PS_{T_0} = \sum_{k=1}^{n} c_k \frac{1}{2\pi} \int_{\mathbb{R}} e^{(iu-R)X_{T_0}} \hat{v}^k(-u - iR)du$$

can be expressed as

$$PS_{T_0} = \int_{\mathbb{R}} \widetilde{B}(T_0, T_j)^{\frac{iu-R}{\tilde{\Sigma}_{T_0}^{T_j}}} \Pi(du),$$

with

$$\Pi(du) = \sum_{k=1}^{n} \frac{c_k}{2\pi} (f_{T_0}^j)^{\frac{iu-R}{\tilde{\Sigma}_{T_0}^{T_j}}} \hat{v}^k(-u - iR)du,$$

$$f_{T_0}^j = \frac{B(0, T_0)}{B(0, T_j)} \exp\left(\int_0^{T_0} [\theta_s(\Sigma(s, T_j)) - \theta_s(\Sigma(s, T_0))]ds\right).$$

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Determination

Theorem

If additionally $3M' \leq M$ and if R is chosen in $]0, \frac{M}{2\sigma_1}] \Rightarrow$ GKW-decomposition of the PS exists.
Optimal number ξ_t^j to invest in $B(\cdot, T_j)$ is according to the MVH strategy given by

$$\int_{\mathbb{R}} e^{\int_t^{T_0} \kappa_s^{\widetilde{X}^j}\left(\frac{iu-R}{\widetilde{\Sigma}_{T_0}^{T_j}}\right)ds} \widetilde{B}(t-, T_j)^{\frac{iu-R}{\widetilde{\Sigma}_{T_0}^{T_j}}-1} \frac{\kappa_t^{\widetilde{X}^j}\left(\frac{iu-R}{\widetilde{\Sigma}_{T_0}^{T_j}}+1\right)-\kappa_t^{\widetilde{X}^j}\left(\frac{iu-R}{\widetilde{\Sigma}_{T_0}^{T_j}}\right)}{\kappa_t^{\widetilde{X}^j}(2)} \Pi(du),$$

with $\Pi(du)$ as in previous lemma and with for $w^c = 1 - w$

$$\kappa_s^{\widetilde{X}^j}(w) = \theta_s(w\Sigma(s,T_j) + w^c\Sigma(s,T_0)) - w\theta_s(\Sigma(s,T_j)) - w^c\theta_s(\Sigma(s,T_0)),$$

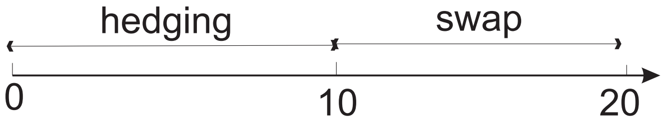
Outline

Numerical results

- Receiver swaption
- Normal Inverse Gaussian
- Vasiček volatility structure

$$\sigma(s, T) = \hat{\sigma} e^{-a(T-s)}$$

- Maturity in 10 years
- Tenor=10 years
- Two payments/year



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Hedging strategies

	$B(\cdot, T_1)$	$B(\cdot, T_{10})$	$B(\cdot, T_{20})$
Delta	9.51 (0.77)	3.02 (0.24)	−2.30 (0.22)
Delta-gamma	87.93 (5.78)	35.19 (2.63)	30.01 (2.64)
MVH	4.36 (0.40)	3.88 (0.39)	3.28 (0.38)

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Delta- and gamma

	$a = 0.02$	$a = 0.06$
$\delta = 0.1$	30.01 (2.64)	20.92 (1.80)
$\delta = 0.06$	17.68 (1.53)	12.32 (1.07)

Characteristic function of the NIG model

$$\phi(z) = \exp(-\delta(\sqrt{\alpha^2 - (\beta + iz)^2} - \sqrt{\alpha^2 - \beta^2})),$$

Vasiček volatility model

$$\sigma(s, T) = e^{-a(T-s)}$$

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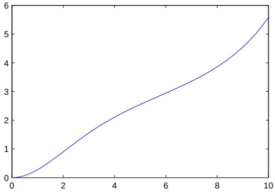
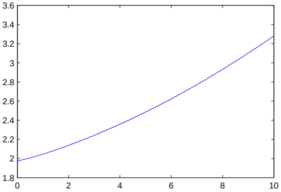
Intentions

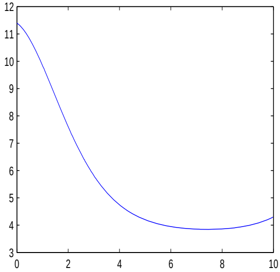


Delta-hedge

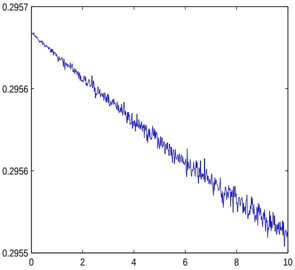


Mean-variance hedge

$B(\cdot, T_{20})$

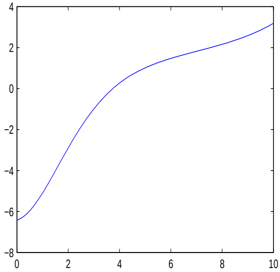


Delta-hedge on average

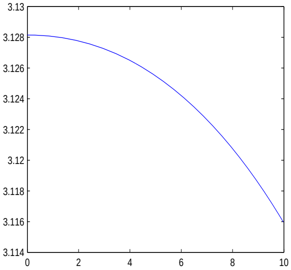


MVH on average

$B(\cdot, T_0)$



Delta-hedge on average



MVH on average

Thank you for your attention

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