Prospect Theory, Partial Liquidation and the Disposition Effect

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The Problem

• Consider an agent with prospect theory preferences who seeks to liquidate a portfolio of (divisible) claims -

* how does the agent sell-off claims over time?

* how does prospect theory alter the agent's strategy vs (rational) expected utility?

* is the strategy consistent with observed behavior eg. disposition effect?

• Examples of claims might include stocks, executive stock options, real estate, managerial projects,...

Prospect Theory (Kahneman and Tversky (1979))

- Utility defined over gains and losses relative to a *reference point*, rather than final wealth
- Utility function exhibits concavity in the domain of gains and convexity in the domain of losses (“S shaped”)
- Steeper for losses than for gains, a feature known as loss aversion
- Non-linear probability transformation whereby small probabilities are overweighted

• The agent has prospect theory preferences denoted by the function $U(z); z \in \mathbb{R}$

(I) Piecewise exponentials: (Kyle, Ou-Yang and Xiong (2006))

$$U(z) = \begin{cases} \phi_1(1 - e^{-\gamma_1 z}) & z \geq 0 \\ \phi_2(e^{\gamma_2 z} - 1) & z < 0 \end{cases} \tag{1}$$

where $\phi_1, \phi_2, \gamma_1, \gamma_2 > 0$.

Assume $\phi_1\gamma_1 < \phi_2\gamma_2$ so that $U'(0-) > U'(0+)$

(II) Piecewise power: (Tversky and Kahneman (1992))

$$U(z) = \begin{cases} z^{\alpha_1} & z \geq 0 \\ -\lambda(-z)^{\alpha_2} & z < 0 \end{cases} \tag{2}$$

where $\alpha_1, \alpha_2 \in (0, 1)$ and $\lambda > 1$.

Locally infinite risk aversion, $U'(0-) = U'(0+) = \infty$.

The Disposition Effect

- Many studies find that investors are reluctant to sell assets trading at a loss relative to the price at which they were purchased
- For large datasets of share trades of individual investors, Odean (1998) (and others) *"finds the proportion of gains realized is greater than the proportion of realized losses"*
- Disposition effects have also been found in other markets - real estate, traded options and executive stock options
- Reluctance of managers to abandon losing projects "throwing good money after bad"

- Prospect theory has long been recognized as one potential way of understanding the disposition effect
- Intuition that more likely to sell when *ahead* (concave) and wait/gamble when *behind* (convex)
- Shefrin and Statman (1985) give intuition and one period numerical eg., we provide mathematical model
- Other recent models include Kyle, Ou-Yang and Xiong (2006), Barberis and Xiong (2008, 2008) but each of these results in a "strong" disposition effect whereby the agent *never* sells at a loss

Price Dynamics

- Let Y_t denote the asset price. Work on a filtration $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, \mathbb{P})$ supporting a BM $W = \{W_t, t \geq 0\}$ and assume Y_t follows a time-homogeneous diffusion process with state space $\mathcal{I} \subseteq \mathbb{R}$ and

$$dY_t = \mu(Y_t)dt + \sigma(Y_t)dW_t \quad Y_0 = y_0$$

with Borel functions $\mu : \mathcal{I} \to \mathbb{R}$ and $\sigma : \mathcal{I} \to (0, \infty)$.

We assume $\mathcal{I}$ is an interval with endpoints $-\infty \leq a_{\mathcal{I}} < b_{\mathcal{I}} \leq \infty$ and that Y is regular in $(a_{\mathcal{I}}, b_{\mathcal{I}})$.

The Optimal Stopping Problem - Indivisible Claims

• Agent chooses when to receive payoff $h(Y_\tau)$, h non-decreasing.

Let y_R denote the reference level. Interpret y_R as price paid, hence "breakeven" level.

• Agent's objective is:

$$V_1(y) = \sup_{\tau} \mathbb{E}[U(h(Y_\tau) - y_R) | Y_0 = y], \quad y \in \mathcal{I} \tag{3}$$

where $U(.)$ is increasing

Heuristics

• Approach is to consider stopping times of the form "stop when price Y exits an interval" and choose the "best" interval.

• The key is to transform into natural scale via $\Theta_t = s(Y_t)$ where scale function $s(.)$ is such that the scaled price Θ_t is a (local) martingale.

Define

$$g_1(\theta) := U(h(s^{-1}(\theta)) - y_R)$$

...value of the game if the asset is sold immediately

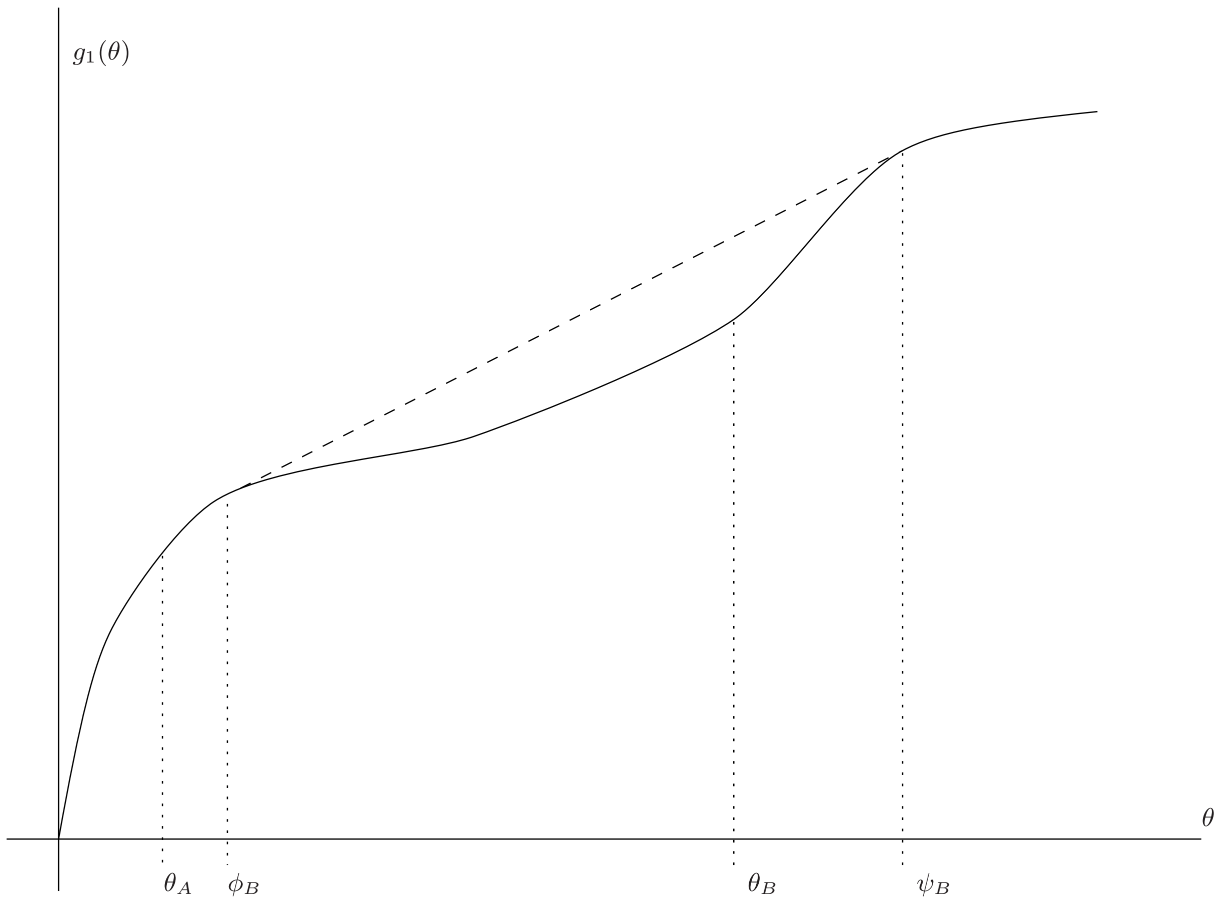

Figure 1: *Stylized representation of the function $g_1(\theta)$ as a function of transformed price θ, where $\theta = s(y)$.*

Proposition 1 *On the interval* $(s(a_{\mathcal{I}}), s(b_{\mathcal{I}}))$*, let* $\bar{g}_1(\theta)$ *be the smallest concave majorant of* $g_1(\theta) := U(h(s^{-1}(\theta)) - y_R)$*.*
(i) Suppose $s(a_{\mathcal{I}}) = -\infty$*. Then*

$$V_1(y) = U(h(b_{\mathcal{I}}) - y_R); \quad y \in (a_{\mathcal{I}}, b_{\mathcal{I}})$$

(ii) Suppose $s(a_{\mathcal{I}}) > -\infty$*. Then*

$$V_1(y) = \bar{g}_1(s(y)); \quad y \in (a_{\mathcal{I}}, b_{\mathcal{I}})$$

Model 1: Piecewise Exponential S-shaped utility and Brownian motion (cf. Kyle, Ou-Yang, Xiong (2006))

Proposition 2 *The solution to problem (3) with* $h(y) = y$, $dY = \mu dt + \sigma dW$*, and* $U(z)$ *is given by piecewise exponential* S*-shape, consists of four cases:*

(I): If $\mu \geq 0$*, the agent waits indefinitely*

(II) If $\mu < 0$ *and* $\mu/\sigma^2 > -\frac{1}{2}\gamma_2$ *and* $|\mu|/\sigma^2 < \frac{1}{2}\frac{\phi_1}{\phi_2}\gamma_1$*, the agent stops at and above a level* $\bar{y}_u^{(1)} > y_R$ *given by:*

$\bar{y}_u^{(1)} = y_R - \frac{1}{\gamma_1} \ln\left(\left(\frac{2\mu}{2\mu - \gamma_1\sigma^2}\right)\left(\frac{\phi_1+\phi_2}{\phi_1}\right)\right)$

(III) If $\mu < 0$ *and* $\mu/\sigma^2 > -\frac{1}{2}\gamma_2$ *and* $|\mu|/\sigma^2 \geq \frac{1}{2}\frac{\phi_1}{\phi_2}\gamma_1$*, the agent stops everywhere at and above the break-even point* y_R*, but waits below the break-even point. Thus if the agent sells, she exactly breaks even*

(IV) If $\mu/\sigma^2 \leq -\frac{1}{2}\gamma_2$*, the agent sells immediately at all price levels*

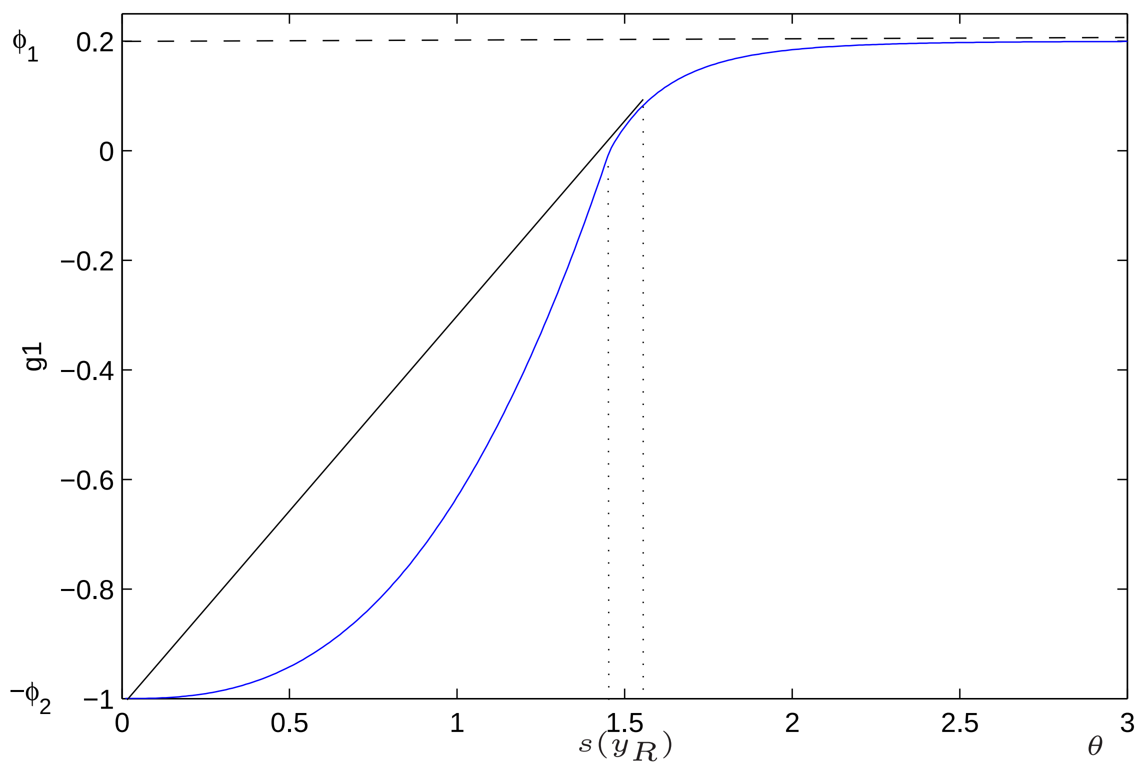

Figure 2: *(II).* $\mu = -0.03$, $s(y_R) = 1.455$. *The agent stops for* $\theta > 1.54$*; equivalently, for prices* $y > 1.15$*. Parameters are:* $\sigma = 0.4$, $\phi_1 = 0.2$, $\phi_2 = 1$, $\gamma_1 = 3$, $\gamma_2 = 1$ *and reference level,* $y_R = 1$*.*

Remarks

• Kyle et al (2006) study this eg. using variational techniques - non-differentiability implies cannot use smooth-pasting

• ...but agent *never* chooses to sell at a loss ... so ”strong” disposition effect!

Model 2: Piecewise Power S-shaped utility and Exponential BM

Proposition 3 *The solution to problem (3) with* $h(y) = y$, $dY = Y(\mu dt + \sigma dW)$, *and* $U(z)$ *is given by piecewise power* S*-shape, consists of three cases. Define* $\beta = 1 - \frac{2\mu}{\sigma^2}$.

(I): If $\beta \leq 0$*; or if* $0 < \beta < \alpha_1 < 1$*, the agent waits indefinitely and never liquidates*

(II) If $0 < \alpha_1 < \beta \leq 1$ *or* $\alpha_1 = \beta < 1$*, the agent stops at a level higher than the break-even point. If the agent liquidates, she does so at a gain*

(III) If $\beta > 1$*, the agent stops when the price reaches either of two levels. These two levels are on either side of the break-even point - liquidates either at a gain or at a loss*

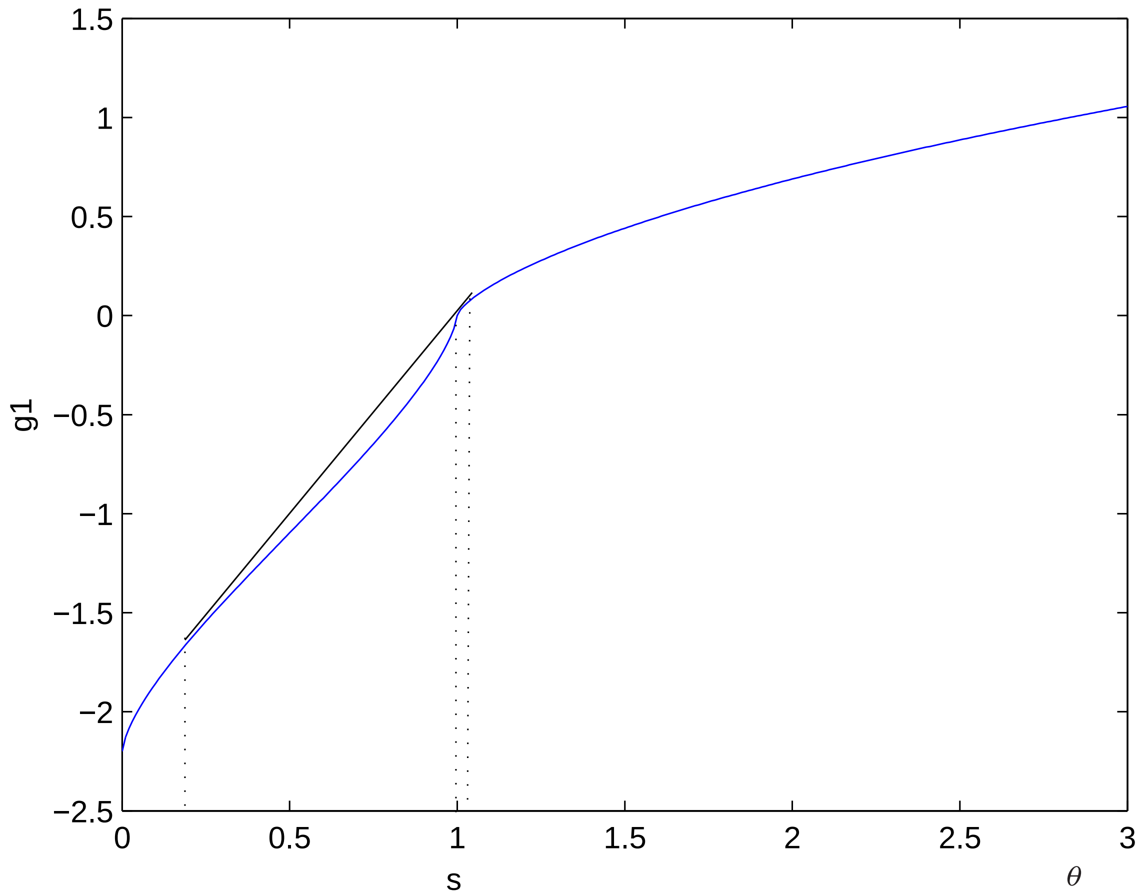

Figure 3: *(III).* $\beta = 1.5$, $\alpha_1 = 0.7$, $s(y_R) = 1$. *The agent waits for* $\theta \in (0.1723, 1.0105)$ *and stops otherwise. Equivalently, the agent waits for* $y \in (0.31, 1.007)$. *Parameters are:* $\lambda = 2.2$, $\alpha_2 = \alpha_1$ *and reference level* $y_R = 1$

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Remarks

- Conclusions (and findings of Kyle et al) not robust to changing the S-shaped function
- Piecewise power functions lead to situation where if odds are bad enough (price transient to zero, a.s), agent "gives up" and sells at a loss - consistent with eg. of Shefrin and Statman (1985)
- Is it consistent with the disposition effect? Is selling at a gain more likely than at a loss?

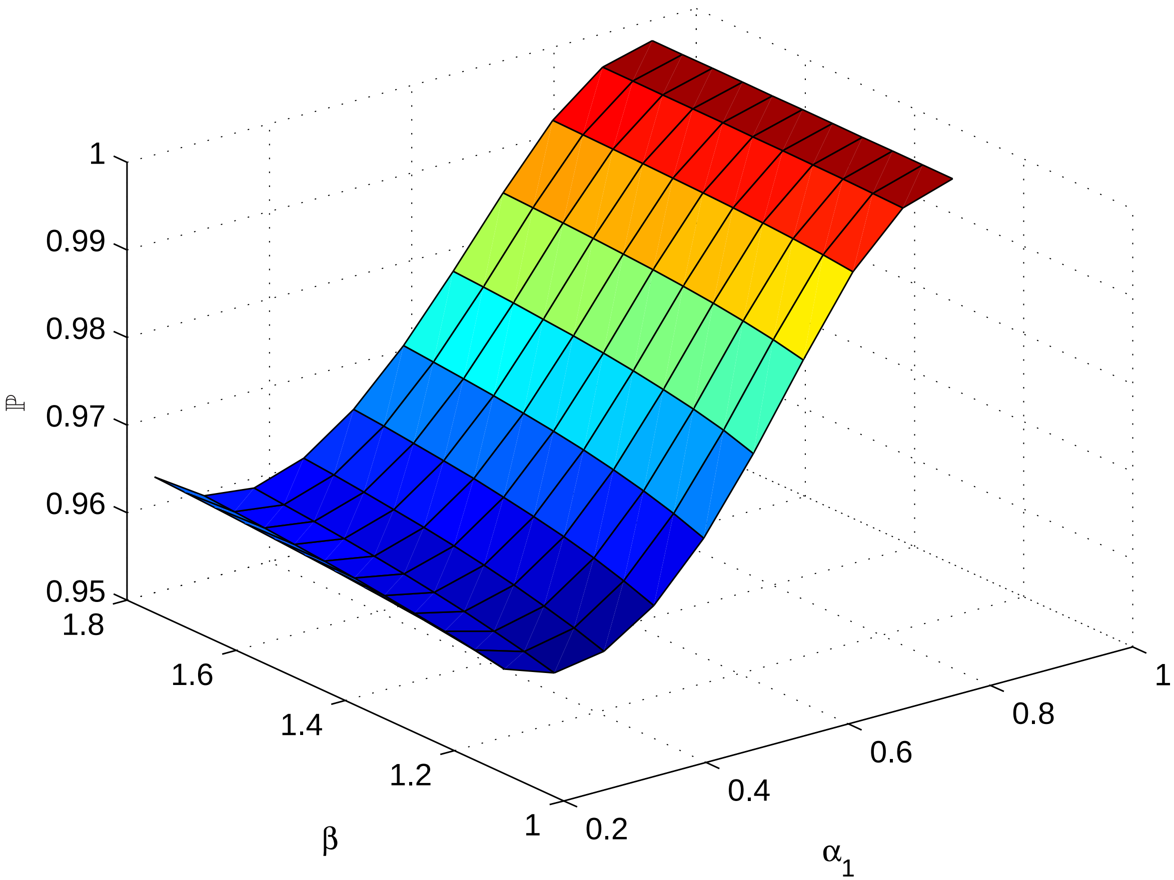

Figure 4: *Probability of liquidating at a gain in Case (III), as a function of* β *and* α_1*. The reference level is* $y_R = 1$ *and take* $y = 1$*;* $\lambda = 2.2$*.*

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Extension to Divisible Claims

• In both piecewise exponential and piecewise power models, agent follows "all-or-nothing" sales strategy

• ...in contrast to an agent with standard concave utility (over wealth) where units are sold-off over time (cf. Grasselli and Henderson (2006), Rogers and Scheinkman (2007), or Henderson and Hobson (2008))

Concluding Remarks

• In contrast to existing literature, we provide prospect theory optimal stopping model (with Tversky and Kahneman (1992) piecewise power functions) under which the agent will liquidate at a loss, enter the position ex-ante, and will be more likely to sell at a (small) gain than a (large) loss, consistent with *disposition effect.*

• Agent's strategy not robust to change in S-shaped function

• Extend to divisible positions and show prospect agent prefers to liquidate on an *"all-or-nothing"* basis