A History of Options

From the Middle Ages to Harrison and Kreps

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Risk Instruments in the Middle Ages

- Casualty, credit, and market risks associated with shipments of goods, notably by ships on the Mediterranean.
- Insurance contracts were common for casualty and credit risks, as early as 1350 in Palermo, with the two kinds written separately.
- A popular contract was a conditional sale (sort of a put option) where insurer agreed to purchase ship or cargo if it failed to arrive.
- For market risk a merchant could hedge two ways: forward transactions (sometimes with advance payment) or derivative contracts.
- In 1400s the Cerchi bank of Florence bought and sold call options.
- In 1500s a derivative called a *premium transaction* was popular: at settlement either buyer or seller could cancel by paying a premium.
- Trading in derivatives took place in context of widespread gambling.
- See “Risk Instruments in the Medieval and Early Modern Economy” by Meir Kohn, Working Paper 99-07, Economics Dept., Dartmouth.

The Economist 1885

"The Virtues and Vices Of Options"

policy would arouse the opposition not only of the Home Rulers, but of a large body of English Radicals. The silent unanimity with which the House of Commons responded to Mr Gladstone's appeal has completely dispelled these delusions, and no Continental statesman any longer believes that England is ready to pay for peace whatever price Russia sees fit to demand.

In one sense it is true that Mr. Gladstone's speech has made war more probable. It has publicly defined the issue between England and Russia, and defined it in such terms as to show that the choice between peace and war hangs upon its determination. But we cannot help feeling that a similar declaration, equally frank in its explanations, and equally energetic in its tone, if made a month or six weeks ago, might have had an eminently pacific tendency. The source of our present troubles is in a large measure to be traced to the inveterate belief of Russia that we had no serious purpose in the matter of the Afghan frontier. This belief has certainly been encouraged by the manner in which our Government has treated each successive stage of the controversy. It was probably a mistake, in the first instance, to have dispatched the English Commissioners before any working basis had been laid down for the operations of the Commission. It was clearly a mistake to have allowed Sir Peter Lumsden to be kept waiting for months for his fellow-commissioner, while a special envoy was sent from St. Petersburg to this country, and received at the Foreign Office. It was, again, an act of weakness not to have insisted on the withdrawal of the Russian outposts from the positions which they had taken up in dangerous contiguity to the Afghans, as a condition precedent to the continuance or resumption of negotiations. All these symptoms of apparent irresolution and half-heartedness were duly noted at St Petersburg and taken advantage of. We welcome, therefore, Mr Gladstone's speech, as proving conclusively that a new attitude has been adopted, which, though it may be too late to avert the impending catastrophe of war, is at least worthy of the dignity of a great nation in an hour of difficulty and peril.

THE VIRTUES AND VICES OF OPTIONS.

A RATHER marked feature in the Stock Exchange recently has been the revival of "option" dealing. In years gone by, a considerable amount of business was habitually transacted in "options," especially in Consols, but more recently this species of speculation had dwindled down to very restricted dimensions. But at no period has it ever been as popular as it is on the continental bourses, and on the stock exchanges across the Atlantic. At Paris, and on all the German bourses, there is a vast amount of speculation constantly carried on by means of options, not separate from, but ancillary to direct operations for the rise or fall. In New York "options" or "privileges" are also a very favourite form of speculation, and that the means for indulging in it have been abundant, is evidenced by the fact that Mr Russell Sage, the well-known associate of Mr Jay Gould, who was, until the collapse of May, 1884, one of the wealthiest and most powerful manipulators in Wall Street, has always been a great dealer in "stock privileges." It is difficult to understand why options have so far not been acclimatised in England, but in view of their becoming more popular, it may be well to refer to their advantages and disadvantages from an outside standpoint.

An "option" is the price paid for the right to demand or to deliver a certain amount of stock at a given price within a certain definite period. The prices given for this "option" may, of course, range infinitely, according to the supposed value of the elements of which it is composed. The right to demand a stock is termed the "call," and the right to deliver it the "put." For instance, one may pay to-day, say, 2 per cent. for the "call" a month hence of 1,000*l* Russian 1873, which right may or may not be exercised. And a "put" would be exactly the converse of this. It is possible to buy the double privilege of both "put" and "call," but the price asked is usually so heavy as to be practically prohibitive. Now, the idea of the speculator who dabbles a little in options is simply to buy the "put" or "call," according to whether he thinks the market will fall or rise; whereas their real *raison d'etre* is something altogether different. They should always serve as a protection to other operations. For instance, a speculator becomes a "bear" of, say, 10,000*l* Russian 1873, and buys the "call" of the same amount of stock. If the price falls, as he anticipates, the profits which he realises are reduced by the amount paid for the "call." On the other hand, if the stock rises, no matter how much, he can "call" the same amount of stock as that sold at presumably the same price, which liquidates the stock sold, leaving him only the premiums paid for the "call" out of pocket. It is, of course, evident that an "option" often affords protection not to one, but to a series of operations. Moreover, the holder of an "option," using it this way, may finally find it to his advantage to close all operations for which it acted as protection, and using, say, the "call" in a direct manner, turn over from the "bear" to the "bull" side of the market. An "option" used properly therefore affords ample scope for skilful speculation, while no loss can be incurred beyond the premium paid in the first instance. But when a speculator who dabbles a little in this sort of business just buys the "put" or "call," and, as it is termed, "sits upon it," he simply plays a losing game, for his operations for the fall or rise, which would be sufficiently weighted in the case of a purchase or sale by his own inexperience, and by the expenses of commission &c., are now burdened by the heavy prices paid for the option itself. In fact, the charges are probably multiplied ten fold against him. It is true that the loss is limited, but then the prospect of a profit is reduced almost to the vanishing point. On the other hand, "options" capably used not only limit the loss, but offer a fair chance of making a profit. They are, in fact, an excellent medium for clever, yet cautious operators. From what we have said, it will be seen that those who advise people to buy "options," without taking any other measures, are simply considering their own interests, the more especially as the securities so often recommended are those which are extremely unlikely to fluctuate to the extent of the given premium—the latter frequently remaining in the hands of the broker, or so-called "broker," as something of a much more satisfactory nature than any commission.

From the standpoint of business morality, two things may be adduced in connection with "options," one for and one against. In the first place, they foster a form of speculation which already flourishes too abundantly. They do this not only directly, but also indirectly, as, owing to the way in which they limit loss, they encourage people to speculate in stocks and shares who otherwise would be restrained, not so much by a positive prudence as by a negative timidity. But it is evident that one can be as effectually destroyed by a poison taken in regular and known quantities, as by a large draught taken heedlessly. It is only a question of time—both methods are equally certain. On the other hand, used by experienced speculators, "options" are generally great safeguards against unexpected and violent movements in prices, and hence in times like the present (speculation being a fact which must simply be acknowledged and dealt with) they are entitled to some commendation. As a matter of fact, speculation in stocks and shares at the present time is for most people gambling of an ultra-violent character, and is only tolerable when protected in the way described.

INDIAN CHIPS.

(FROM OUR SPECIAL CORRESPONDENT.)

THE heading of this letter affords an explanation for my protracted silence. I have nothing before me but fragments. I commenced the new year by promising a series of letters on Indian railway projects. I had hardly finished my account of the Katni line, and was about to commence a treatise on the Nagpur-Bengal line, when railway enterprise began to stagger under the bare rumour of a shock of arms on the further frontier of Afghanistan. Within the last few weeks the prosecution of the railway works on the Bilaspur-Etawah line has been suddenly stopped, and parties of Indian platelayers have actually

Excerpts from the 1885 Economist Article

- “At Paris, and on all the German bourses, there is a vast amount of speculation constantly carried on by means of options… In New York options or ‘privileges’ are also a favourite form of speculation…”
- “From the standpoint of business morality, two things may be adduced in connection with options, one for and one against. In the first place, they foster a form of speculation which already flourishes too abundantly. … On the other hand, used by experienced speculators, options are generally great safeguards against unexpected and violent movements in prices…”

Louis Bachelier

- Born in 1879 Le Havre
- Sorbonne 1892
- Thesis defense 1900
- Sorbonne lecturer
- World War I army
- 1919-1937 professor at Besancon, Dijon, and Rennes
- Died in 1946
- Bachelier Finance Society founded in 1996

LOUIS BACHELIER ON THE CENTENARY OF
THÉORIE DE LA SPÉCULATION

The photo was taken in Le Havre, France, on August 17, 1858 by professional photographer M. Caccia, 126 Blvd. de Strasbourg.

THÉORIE

DE

LA SPÉCULATION,

Par M. L. BACHELIER.

INTRODUCTION.

Les influences qui déterminent les mouvements de la Bourse sont innombrables, des événements passés, actuels ou même escomptables, ne présentant souvent aucun rapport apparent avec ses variations, se répercutent sur son cours.

A côté des causes en quelque sorte naturelles des variations, interviennent aussi des causes factices : la Bourse agit sur elle-même et le mouvement actuel est fonction, non seulement des mouvements antérieurs, mais aussi de la position de place.

La détermination de ces mouvements se subordonne à un nombre infini de facteurs : il est dès lors impossible d'en espérer la prévision mathématique. Les opinions contradictoires relatives à ces variations se partagent si bien qu'au même instant les acheteurs croient à la hausse et les vendeurs à la baisse.

Le Calcul des probabilités ne pourra sans doute jamais s'appliquer aux mouvements de la cote et la dynamique de la Bourse ne sera jamais une science exacte.

Mais il est possible d'étudier mathématiquement l'état statique du marché à un instant donné, c'est-à-dire d'établir la loi de probabilité des variations de cours qu'admet à cet instant le marché. Si le marché, en effet, ne prévoit pas les mouvements, il les considère comme étant

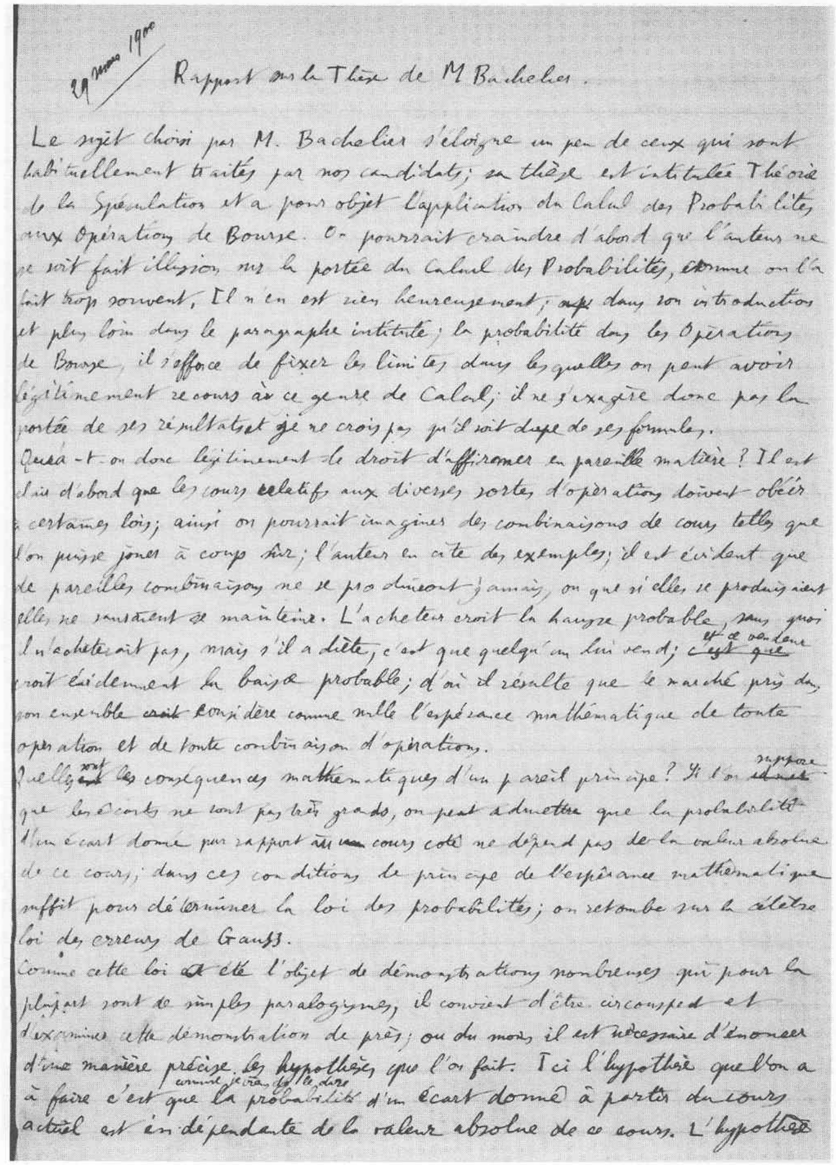
Poincare's Report

Thesis Committee

Paul Appell

Joseph Boussinesq

Henri Poincare


29 mars 1900 Rapport sur la Thèse de M Bachelier.

Le sujet choisi par M. Bachelier s'éloigne un peu de ceux qui sont habituellement traités par nos candidats; sa thèse est intitulée Théorie de la Spéculation et a pour objet l'application du Calcul des Probabilités aux Opérations de Bourse. On pourrait craindre d'abord que l'auteur ne se soit fait illusion sur la portée du Calcul des Probabilités, comme on l'a fait trop souvent. Il n'en est rien heureusement; dans son introduction et plus loin dans le paragraphe intitulé : la probabilité dans les Opérations de Bourse, il s'efforce de fixer les limites dans lesquelles on peut avoir légitimement recours à ce genre de Calcul; il ne s'exagère donc pas la portée de ses résultats et je ne crois pas qu'il soit dupe de ses formules.

Quel est donc légitimement le droit d'affirmer en pareille matière ? Il est clair d'abord que les cours relatifs aux diverses sortes d'opérations doivent obéir à certaines lois; ainsi on pourrait imaginer des combinaisons de cours telles que l'on puisse jouer à coup sûr; l'auteur en cite des exemples; il est évident que de pareilles combinaisons ne se produisent jamais, ou que si elles se produisaient elles ne sauraient se maintenir. L'acheteur croit la hausse probable, sans quoi il n'achèterait pas, mais s'il achète, c'est que quelqu'un lui vend; et ce vendeur croit évidemment la baisse probable; d'où il résulte que le marché pris dans son ensemble considère comme nulle l'espérance mathématique de toute opération et de toute combinaison d'opérations.

Quelles sont les conséquences mathématiques d'un pareil principe ? Si l'on suppose que les écarts ne sont pas très grands, on peut admettre que la probabilité d'un écart donné par rapport au cours coté ne dépend pas de la valeur absolue de ce cours; dans ces conditions le principe de l'espérance mathématique suffit pour déterminer la loi des probabilités; on retombe sur la célèbre loi des erreurs de Gauss.

Comme cette loi a été l'objet de démonstrations nombreuses qui pour la plupart sont de simples paralogismes, il convient d'être circonspect et d'examiner cette démonstration de près; ou du moins il est nécessaire d'énoncer d'une manière précise les hypothèses que l'on fait. Ici l'hypothèse que l'on a à faire c'est que la probabilité (comme je viens de le dire) d'un écart donné à partir du cours actuel est indépendante de la valeur absolue de ce cours. L'hypothèse

The original document is held at the Centre historique des Archives Nationales in Paris, classification number AJ/16/5537.

Accomplishments in Thesis

- Assumed price fluctuations over small time intervals are independent of present and past price levels
- Applied central limit theorem to deduce price increments are independent and normally distributed (so the price process is Brownian motion as the diffusion limit of a random walk!)
- Used lack-of-memory (Markov) property to derive (what is now called) the Chapman-Kolmogorov equation
- Established connection with heat equation
- Simple formula for the price of at the money calls
- Recognized concept of arbitrage
- Work was cited and used by Kolmogorov in 1931 and by Doob (the “father” of martingales)

Other Option Research Prior to 1950's

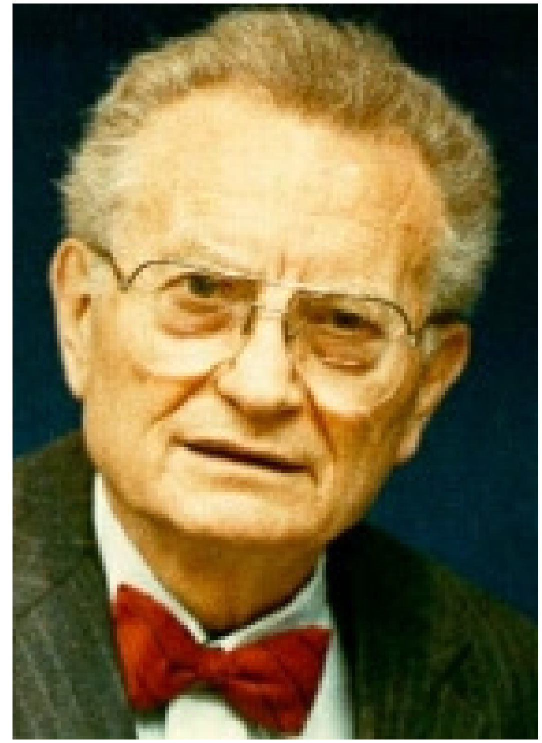
none

Bachelier “discovered” by Samuelson

In early 1950s Jimmy Savage sent postcards to various economists, including Samuelson, about Bachelier

Samuelson said, “In the early 1950s I was able to locate by chance this unknown book, rotting in the library of the University of Paris, and when I opened it up it was if a whole new world was laid out before me.”

Samuelson had been giving thought to option pricing, so he commissioned the translation by James Boness.



Inventor of the option terms “American” and “European”

Research in 1964 book edited by Paul Cootner *The Random Character of Stock Market Prices*

- By this time people were using geometric Brownian motion models of stock market prices
- People like Boness, Samuelson, and Sprenkle were calculating the expected discounted payoff of European puts and calls, but they were all using different choices for the discount factor and the stock's appreciation rate
- The mathematician McKean, in an appendix to Samuelson's paper, studied a free boundary problem pertaining to the pricing of an American put (optimal stopping time = optimal early exercise time)

Work by Sheen Kassouf and Edward Thorp

- Two young professors at University of California, Irvine
- Wrote *Beat the Market*, Random House, 1967
- Developed empirical formula
- Recognized and introduced concepts of *hedge ratio* and *dynamic hedging*
- Thorp learned of Cootner's book, and based on his empirical work he set the stock's appreciation rate equal to the riskless interest rate, arriving at the BS formula
- He used the Black-Scholes formula for profitable trading but he couldn't prove why it was correct

Fischer Black

Myron Scholes

Original Derivation of Black-Scholes Equation

Derivation of PDE was primarily due to Black. He focused on a portfolio of the form

$$V = QS + C,$$

where

V = portfolio value

Q = stock position

S = stock price

C = price of European call

Using a Taylor series expansion he figured out how to use dynamic hedging so that this portfolio will have <u>zero beta</u> at each point in time

Recall the Capital Asset Pricing Model (CAPM):

$$E[R_P] = R + \beta(E[R_M] - R),$$

where R_P = return of an arbitrary portfolio

R = return of a riskless investment

R_M = return of the market portfolio

β = beta of arbitrary portfolio with respect to market

Hence for Black's zero-beta portfolio, over any time period:

$$E[R_V] = R$$

More Taylor series calculations led to the famous Black-Scholes pde

In spite of Black's Harvard PhD in applied mathematics, it took a while to find a solution of the pde

Since the stock's appreciation rate μ did not appear in the pde, they set it equal to the riskless interest rate r, used an expression derived by Sprinkle (1961), and (voila!) they had the solution to the pde and its boundary condition.

The first draft working paper was dated October 1970

The paper was eventually published in the May/June 1973 issue of the *Journal of Political Economy*, but this was after an earlier rejection by this journal as well as a rejection by the *Review of Economics and Statistics*

Robert Merton made important subsequent contributions that were published in a 1973 issue of the *Bell Journal*. Merton's paper was accepted before the BS paper, and Merton asked the *Bell Journal* editor to hold up publication of his paper until a journal accepted and published the one by Black and Scholes

Black, Merton, Scholes and Samuelson were all together at MIT

Merton recognized how to use dynamic hedging to achieve a portfolio of the form

$$V = QS + C$$

that is actually riskless, not just zero-beta

This enabled him to derive the pde in a more rigorous fashion

Robert Merton

Merton's Derivation of the Black-Scholes PDE

Assumptions:

Stock price: $dS/S = \mu dt + \sigma dW$

Call price $C = c(S_t, t)$

Call's boundary condition: $c(S_T,T) = \max\{0, S_T - K\}$

Riskless interest rate: r (with continuous compounding)

Portfolio value: $V = QS + C$

It follows that: $dV = QdS + dC$

Applying Ito's lemma to $c(S_t, t)$:

$$dC = \frac{\partial c}{\partial S} dS + \frac{\partial c}{\partial t} dt + \frac{1}{2} \frac{\partial^2 c}{\partial S^2} \sigma^2 S^2 dt$$

Substituting this in dV = QdS + dC:

$$dV = QdS + \frac{\partial c}{\partial S} dS + \frac{\partial c}{\partial t} dt + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 c}{\partial S^2} dt$$

$$= \left[Q + \frac{\partial c}{\partial S} \right] dS + \frac{\partial c}{\partial t} dt + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 c}{\partial S^2} dt$$

This becomes, after setting $Q = -\frac{\partial c}{\partial S}$:

$$dV = \frac{\partial c}{\partial t} dt + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 c}{\partial S^2} dt$$

This describes the dynamics of a deterministic, riskless portfolio, so its return, namely *dV/V,* must always equal the riskless interest *rdt*

In other words:

$$dV = rVdt = r(QS + C)dt = r\left(-\frac{\partial c}{\partial S}S + C\right)dt$$

$$= \frac{\partial c}{\partial t}dt + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 c}{\partial S^2}dt$$

Finally, dropping the common factor *dt* we get the Black-Scholes pde:

$$\frac{\partial c}{\partial t} = rc - rS\frac{\partial c}{\partial S} - \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 c}{\partial S^2}$$

This and the boundary condition $c(S_T,T) = \max\{0,S_T - K\}$ are solved to obtain the famous Black-Scholes formula for the call option price.

Next Major Development: Risk Neutral Valuation

- By working with the BS model and a stock price model featuring Poisson jumps, John Cox and Steve Ross introduced the concept of *risk neutrality*
- They hypothesized (but did not prove) that in some generality the price of an option can be computed with *preferences* (which we call *probabilities)* such that expected returns for both the stock and the option are equal to returns under the riskless rate.
- For stock the preferences should satisfy: $E[S_T/S_t|S_t] = exp\{r(T-t\}$
- For European option with price H satisfying $H_T = h(S_T)$ they said:

$$\mathrm{E}[H_T / H_t \mid S_t] = e^{r(T-t)} \Leftrightarrow H_t = e^{-r(T-t)}\mathrm{E}[h(S_T) \mid S_t]$$

Remarks About the Cox-Ross Results

- The generality and “why” are unclear
- There was no mention of martingales
- Harrison and Kreps were greatly stimulated by the Cox-Ross paper, for they said, “…Cox and Ross provide the following key observation. If a claim is redundant in a world with one stock and one bond, then its value can be found by first modifying the model so that the stock earns at the riskless rate, and then computing the expected (discounted) value of the claim. They analyze two examples, and in each case they determine the correct modification by the following procedure. First, using the technique of Black and Scholes, they derive an analytical expression (e.g., a pde) that the value of the claim must satisfy. Having observed that one model parameter (e.g., the geometric BM appreciation rate) does not appear in this relationship, they then adjust the value of the parameter so that the stock earns at the riskless rate.”

J. Michael Harrison

David Kreps

The Harrison-Kreps Results

- Recognizing that the Cox-Ross equation $E[S_T/S_t|S_t] = \mathit{exp}\{r(T-t)\}$ is the same as $E[e^{-rT}S_T|S_t] = e^{-rt}S_t$, they were led to the idea that the *risk neutral probabilities* (i.e., Cox-Ross preferences) must be such that the *discounted price processes* are *martingales*
- This led to the notion of *equivalent martingale measures*
- Another important notion is *viability*; this is approximately the same as the *absence of arbitrage opportunities*
- Key Result #1: the model is viable if and only if there exists an equivalent martingale measure
- Still another notion: a *redundant* claim is one which can be replicated by some portfolio involving the stocks and bank account
- Key Result #2: in a viable model a claim is redundant iff it has the same expectation under every equivalent martingale measure
- Key Result #3: if a claim is redundant, then its arbitrage value is that common expectation

Remarks About the Harrison-Kreps Results

- They greatly increased both the understanding and the generality of the risk neutral approach
- They opened the door to the relevance of martingale theory and stochastic integration
- They applied their results to the case where a collection of stock prices is a vector diffusion process, but did not proceed further
- Their assumptions about trading strategies were somewhat restrictive and thus limited the generality of their results

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