On the Maximal Resolvability of Monotonically Normal Spaces

Menachem Magidor

February 9, 2010

(joint work with I. Juhasz)

A topological space X is called κ resolvablle if it contains κ many disjoint dense subsets. Trivially a space can not be more than $\Delta(X)$ resolvablle, where $\Delta(X)$ is the smallest cardinality of a non empty open subset of X. X is called maximally resolvable if it is $\Delta(X)$ resolvable. It is easy to construct in ZFC, for every cardinal Δ, a Hausdorf space X with $\Delta(X) = \Delta$ which is not even 2-resolvable.

But about spaces which are nicer? A natural generalization of metric spaces (and linearly ordered spaces) are the Monotonically Normal Spaces. A space X is called Monotonically Normal (NM) if there is an operation $H(x, U)$ defined on all the pairs (x, U) such that $x \in X$, U an open subset of X and $x \in U$. The requirements for H are that $x \in H(x, U) \subseteq U$ and if $H(x, U) \cap H(y, V) \neq \emptyset$ then either $x \in V$ or $y \in U$. It is not difficult to see that every crowded NM space is ω-resolvable.

Using previous work of Juhasz, Soukup and SzentMikolossy we prove that if there is no inner model with measurable cardinal then every NM space is maximally resolvable and if the existence of measurable cardinal is consistent then it is consistent that there is a NM space X with $|X| = \Delta(X) = \aleph_\omega$ which is not ω_1-resolvable. ($\aleph_\omega$ is the minimal cardinal for which such a result can be proven.)