What does compressive sensing mean for X-ray CT and comparisons with its MRI application

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University of Chicago- Dept. of Radiology

work supported by the NIH

Outline

* CT and image reconstruction background

* Application: mammography

* Compressive sensing in CT versus MRI

* Some results with real CT data

* Ongoing studies:
  - extremely small objects — real data
  - sparsity-based sampling sufficiency — theoretical study

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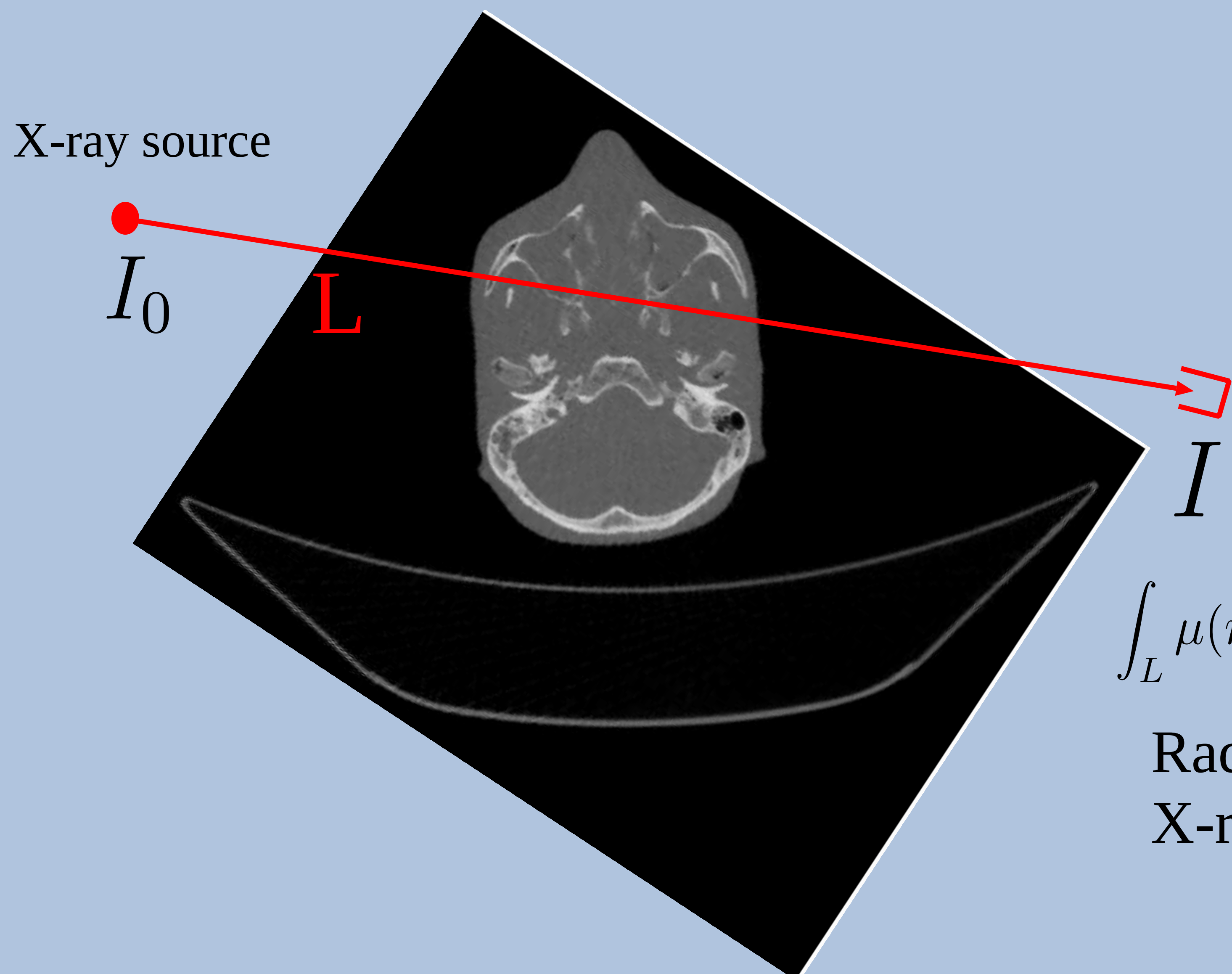
Standard introduction to CT



$$\int_L \mu(\vec{r})d\ell = -\ln(I/I_0)$$

Radon transform
X-ray transform

Reality of CT

* object function is simplified:

$$\mu(\vec{r}) \longrightarrow \mu(\vec{r}, E, t)$$

* data model also simplified:
  - X-ray scatter
  - X-ray source beam-spectrum
  - detector physics
  - random processes

  ....

* CT is a digital instrument:
  finite number of samples

Overview of image reconstruction algorithms

* An algorithm consists only of a number of data processing steps

* Data/imaging models and their methods of solution help guide their design

* Trade-off (see Foundations of Image Science by Barrett and Myers)

simple model		complex model
easy to solve	⟷	hard to solve
model error is large		model error is small

* Practical I.R. algorithms evaluated on imaging task
  Theoretical I.R. research based on model solution

Implicit v. Explicit image reconstruction

$$g = X(f)$$

(example: compressive sensing)
solved iteratively
non-linear
complex models can be devised
zoology of data models
need to reconstruct whole image

$$f = X^{-1}(g)$$

(example: FBP)
one-shot processing
usually linear
modeling limited
models more uniform
can reconstruct point-by-point

Model zoology

$$\vec{g} = X\vec{f}$$

$$g(\theta_i, \xi_i) = \int_{L(\theta_i,\xi_i)} d\ell\, f(\vec{r}) \longrightarrow \text{Radon/X-ray}$$

Implicit / Iterative / CS

type of expansion elements:
- pixels, blobs, wavelets

number of expansion elements

ray sampling

measurement model
- line integration
  - Siddon's method, ray-tracing
- area-weighted integration

Explicit / FBP/ FDK

continuous object function

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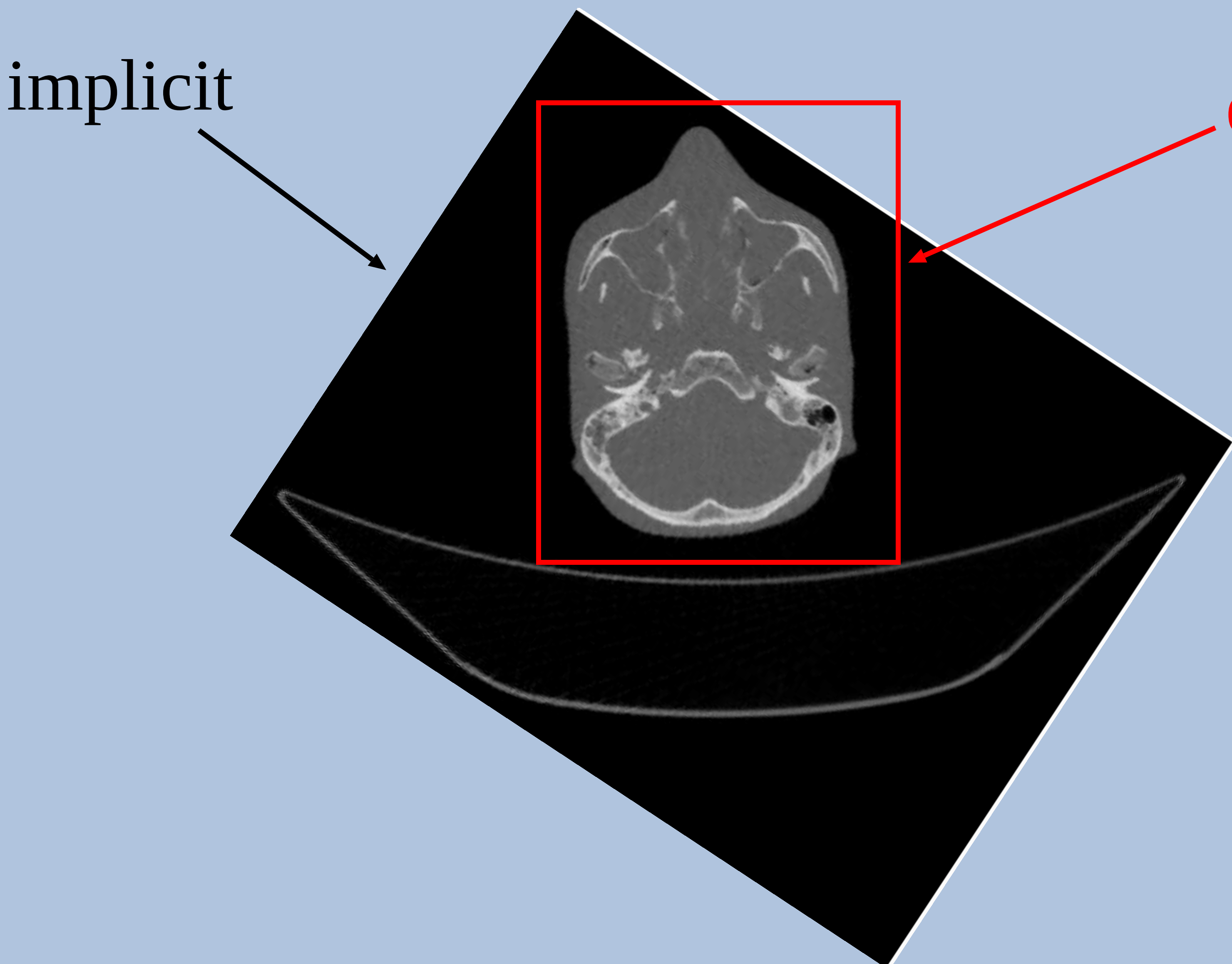
continuous data function

measurement model
- line integration

Full solution v. point-by-point



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X-ray Imaging for Breast Cancer Screening

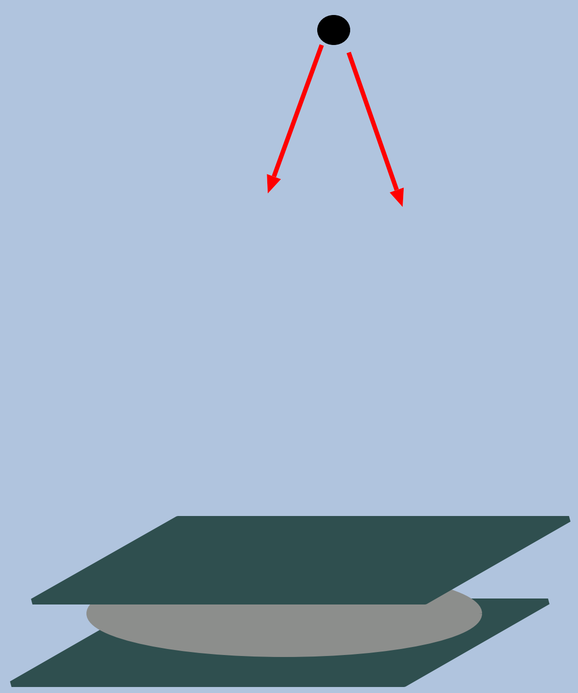
Goal: Early detection

Task: image asymptomatic women and decide to recall or not

Imaging: suspicious mass (tumor) or micro-calcification cluster (DCIS)

X-ray Imaging for Breast Cancer Screening

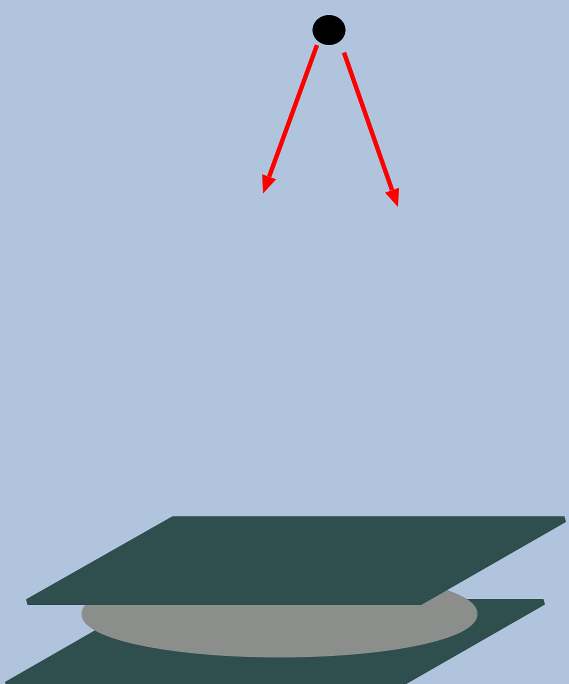
Digital mammography



resolution
depth: 6.0 cm
in-plane: 0.1 mm

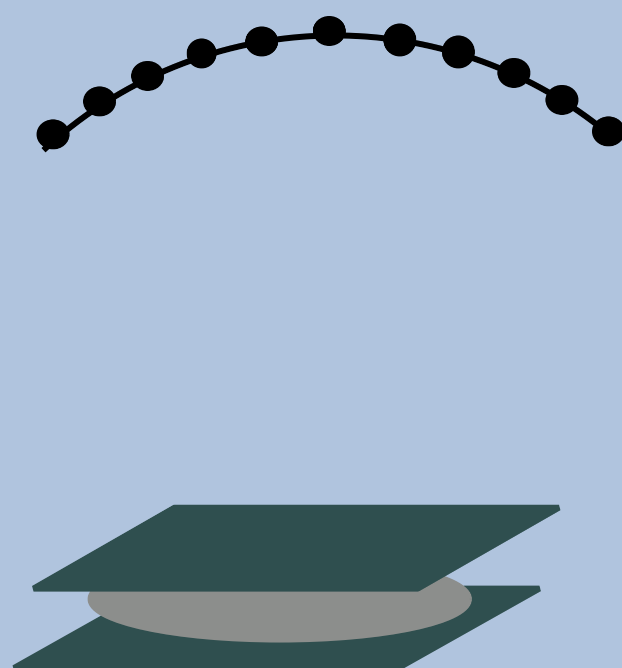
X-ray Imaging for Breast Cancer Screening

Digital mammography



resolution
depth: 6.0 cm
in-plane: 0.1 mm

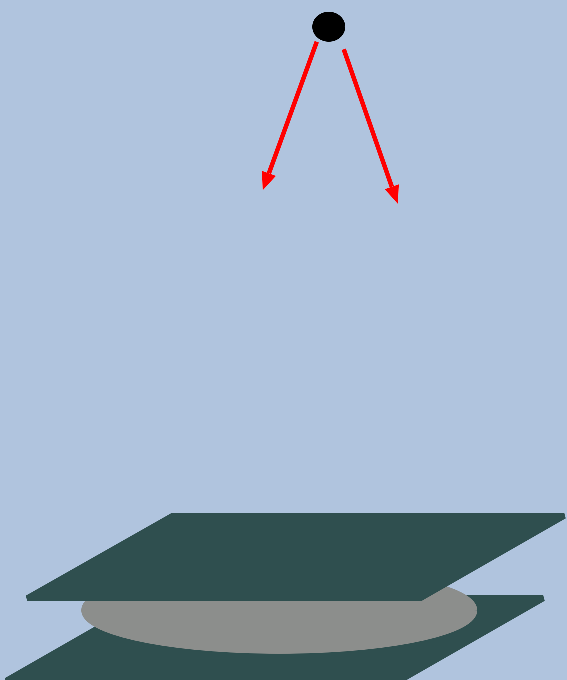
Digital breast tomosynthesis



resolution
depth: 1.0 mm
in-plane: 0.1 mm

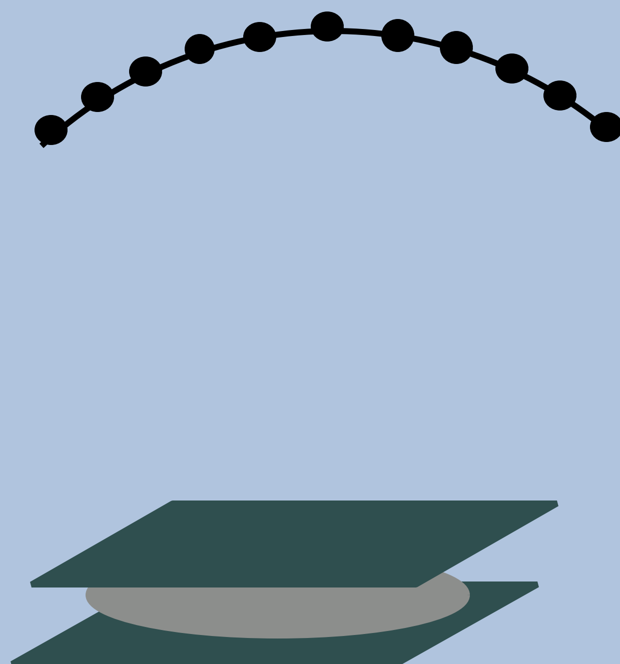
X-ray Imaging for Breast Cancer Screening

Digital mammography



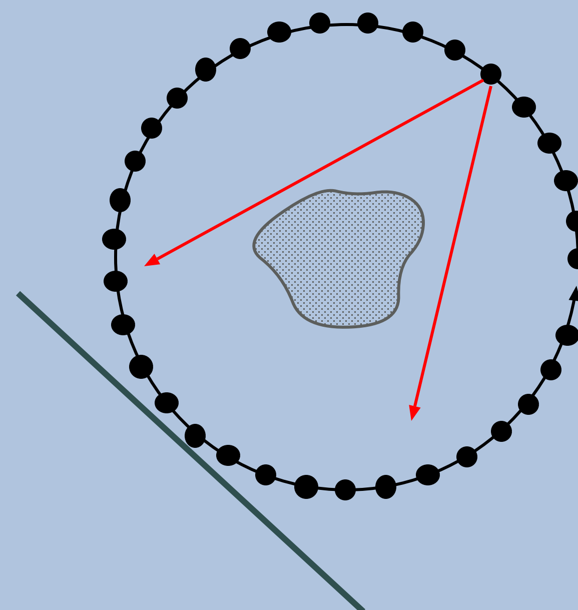
resolution
depth: 6.0 cm
in-plane: 0.1 mm

Digital breast tomosynthesis



resolution
depth: 1.0 mm
in-plane: 0.1 mm

Computed Tomography

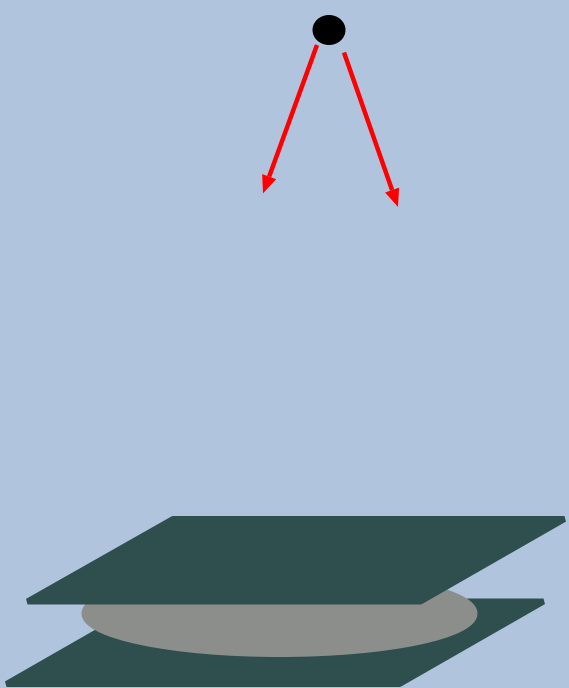


resolution
depth: 0.3 mm
in-plane: 0.3 mm

X-ray Imaging for Breast Cancer Screening

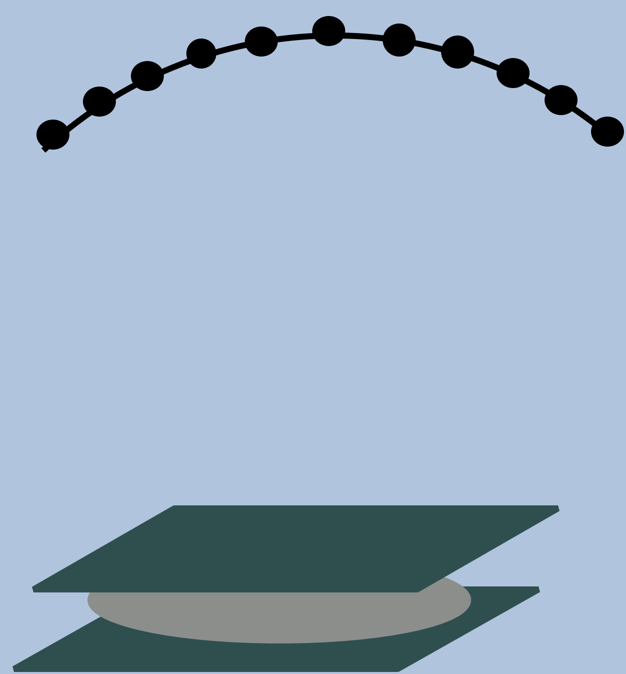
design constraint: Equal X-ray dose

Digital mammography



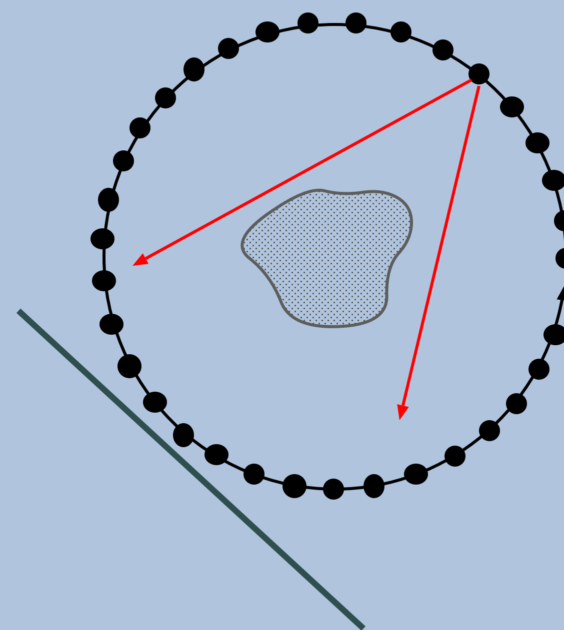
resolution
depth: 6.0 cm
in-plane: 0.1 mm

Digital breast tomosynthesis



resolution
depth: 1.0 mm
in-plane: 0.1 mm

Computed Tomography

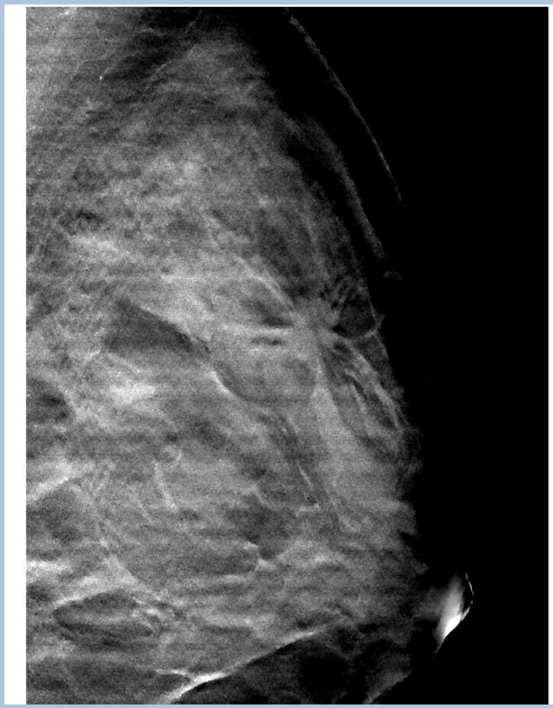


resolution
depth: 0.3 mm
in-plane: 0.3 mm

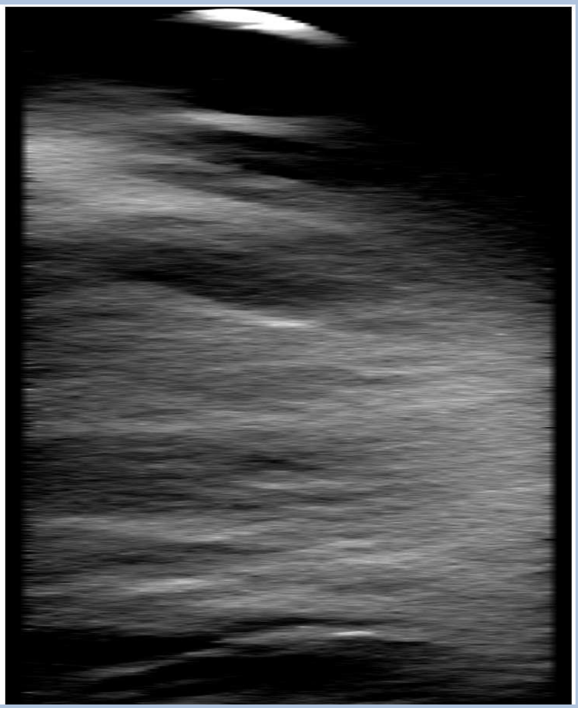
Mass imaging

Projection image

Digital breast tomosynthesis



in-plane

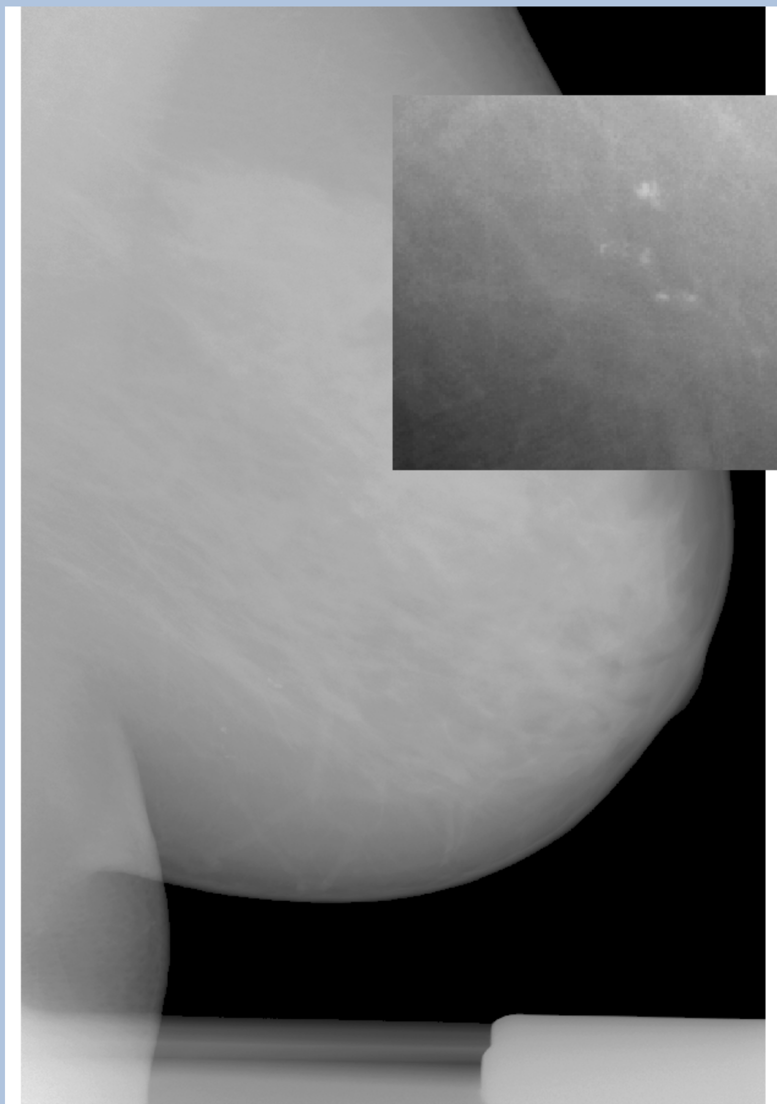


depth

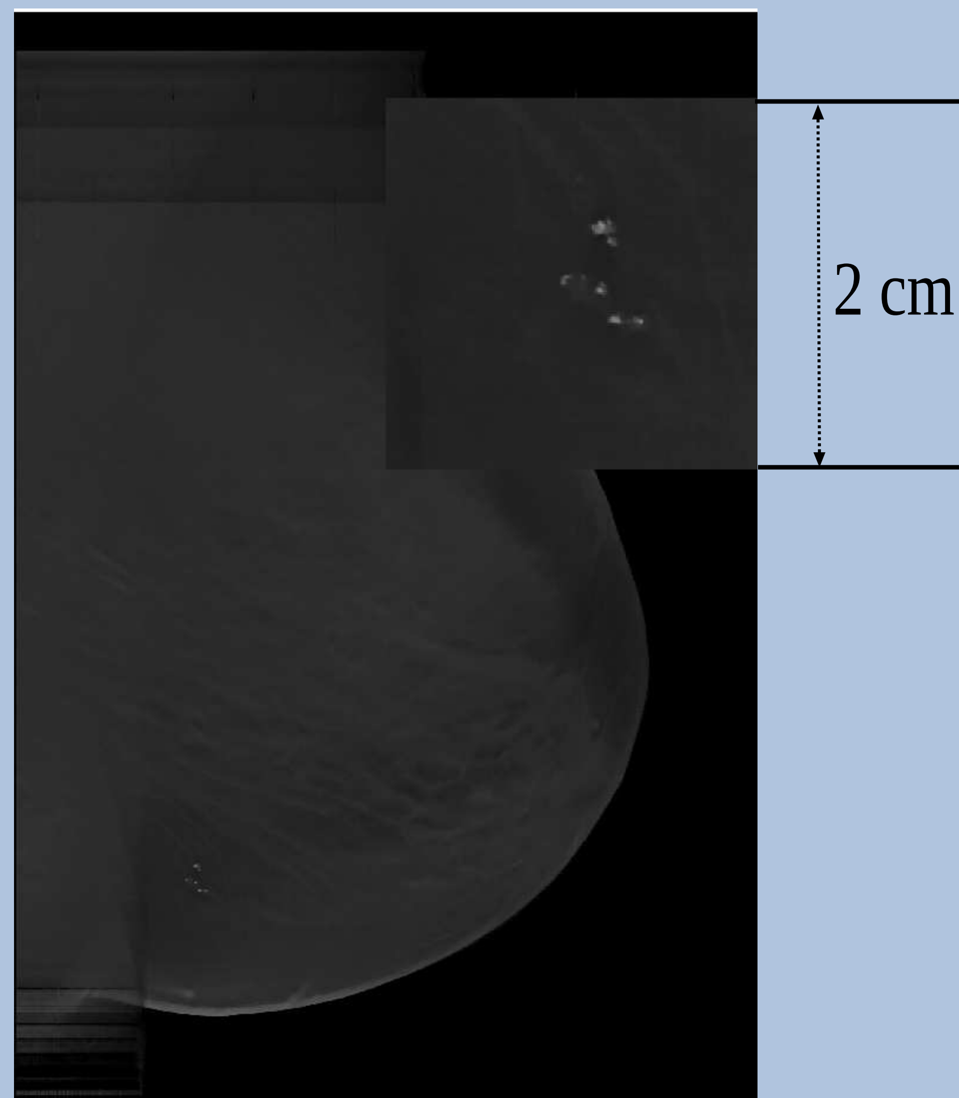
Courtesy: Massachusetts General Hospital
GE prototype DBT scanner

Microcalcification imaging

Projection image



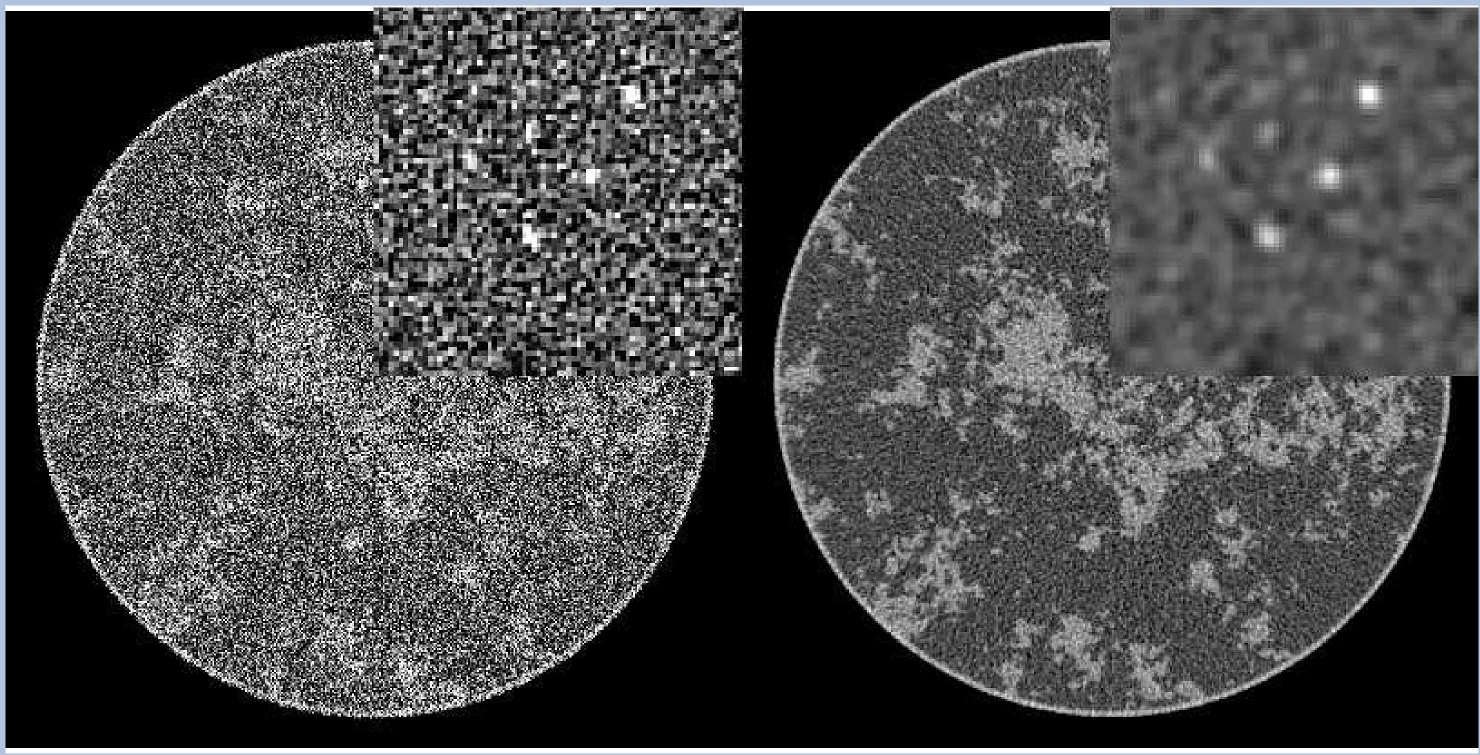
Digital breast tomosynthesis



Courtesy: Massachusetts General Hospital
GE prototype DBT scanner

Breast computed tomography (bCT)

512-view, bCT simulation
FBP reconstruction



unregularized Gaussian filtered

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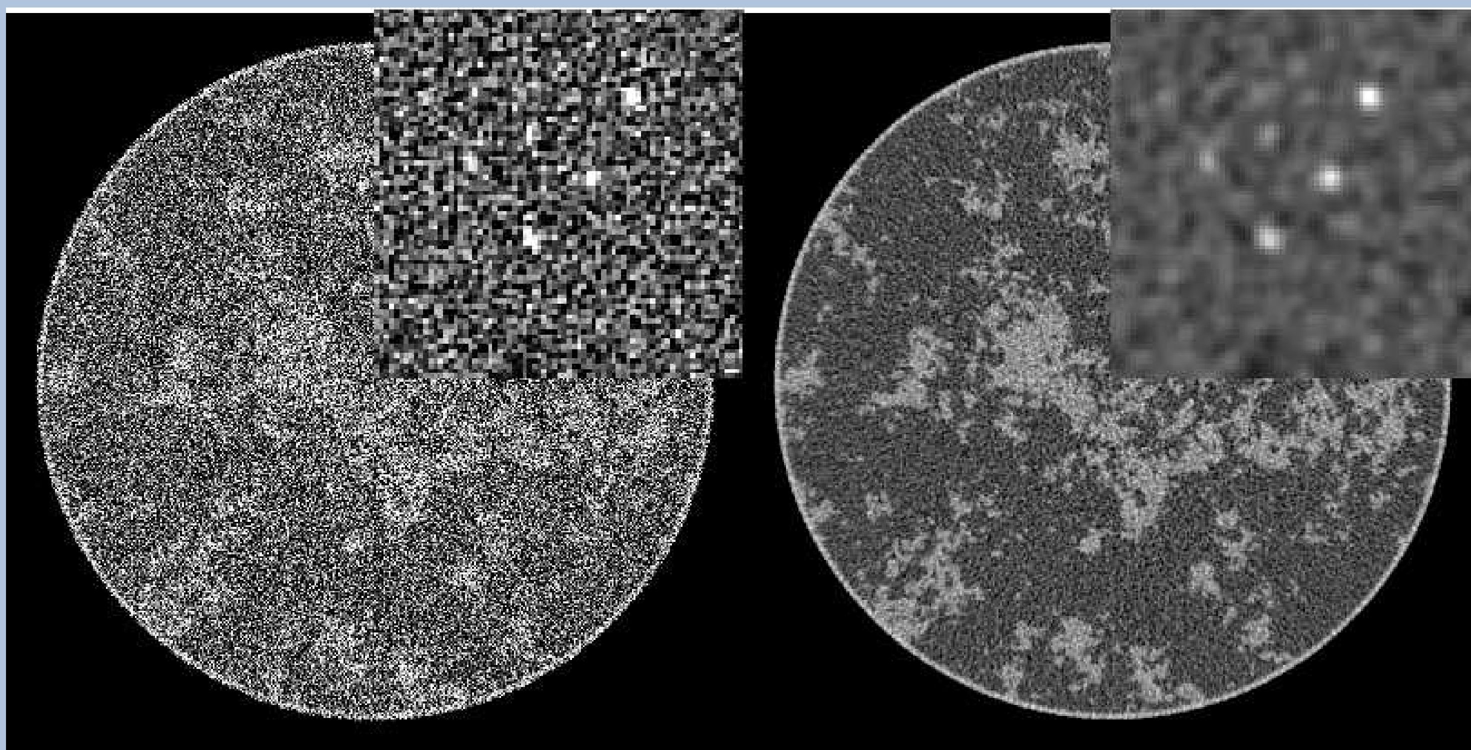

unregularized

Gaussian filtered

Can CS help?

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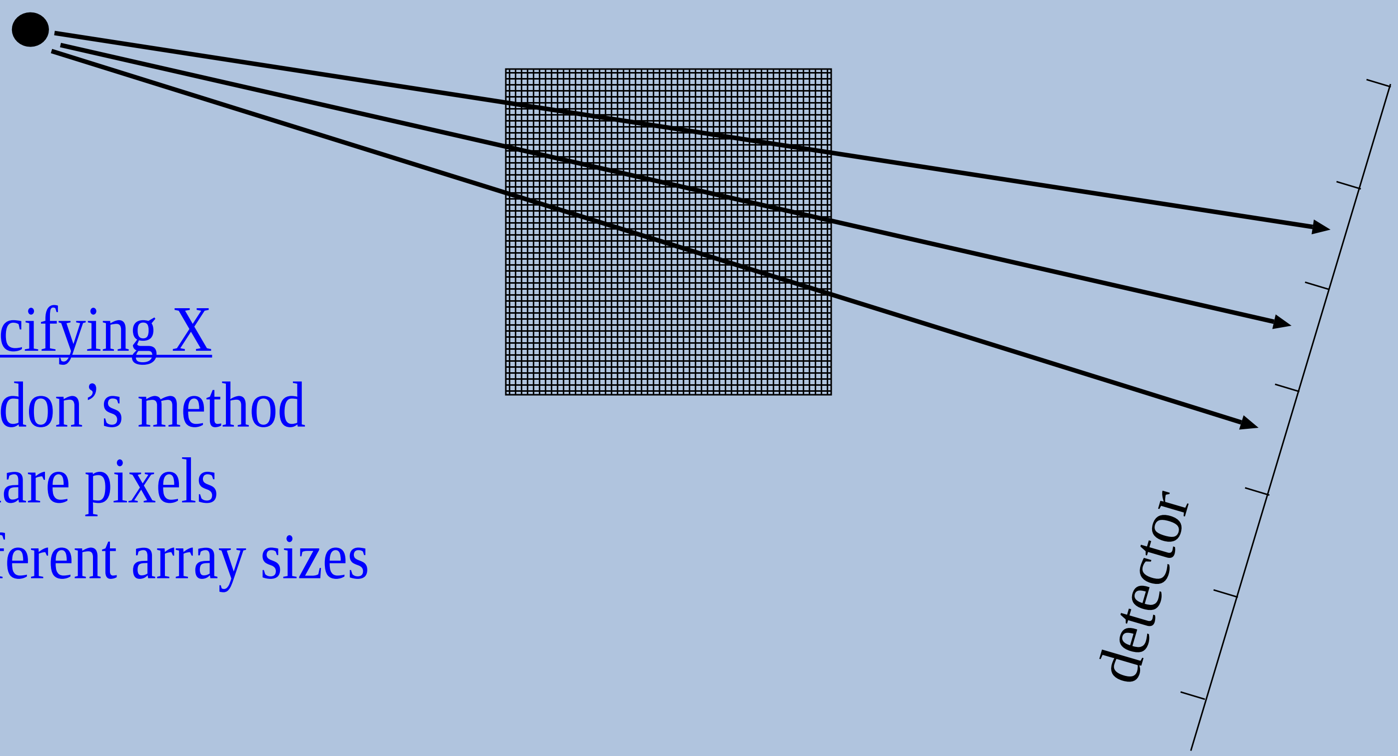
Compressive sensing for CT with gradient magnitude sparseness

$$\vec{f}^* = \text{argmin} \|\vec{f}\|_{TV} \;\; \text{such that} \;\; X\vec{f} = \vec{g}$$

$$\|\vec{f}\|_{TV} = \sum_i |\vec{\nabla} f_i|$$



specifying X
Siddon's method
square pixels
different array sizes

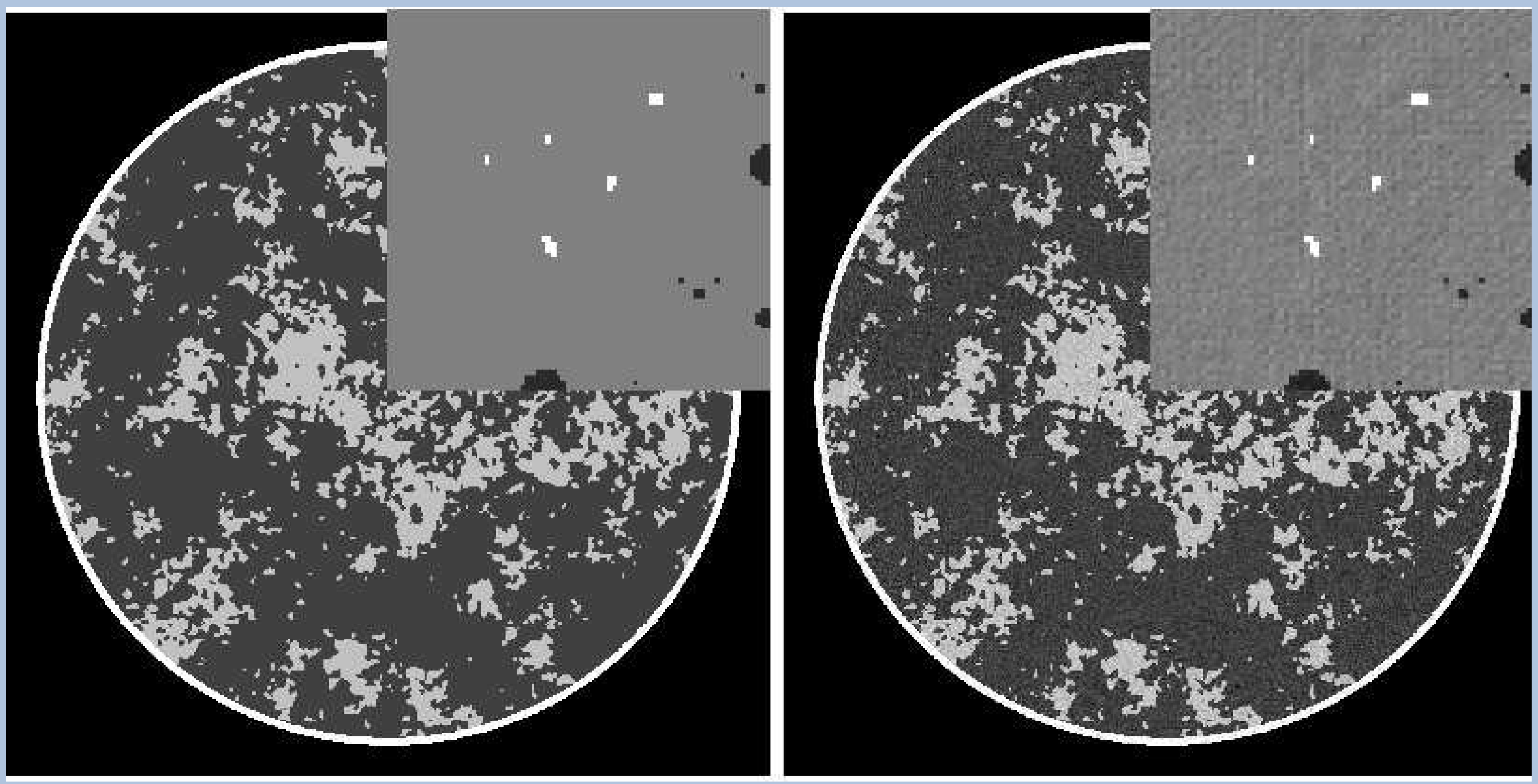
Compressive sensing for CT with
gradient magnitude sparseness
(comparison with FT/MRI image model)

$$\vec{g} = X\vec{f}$$

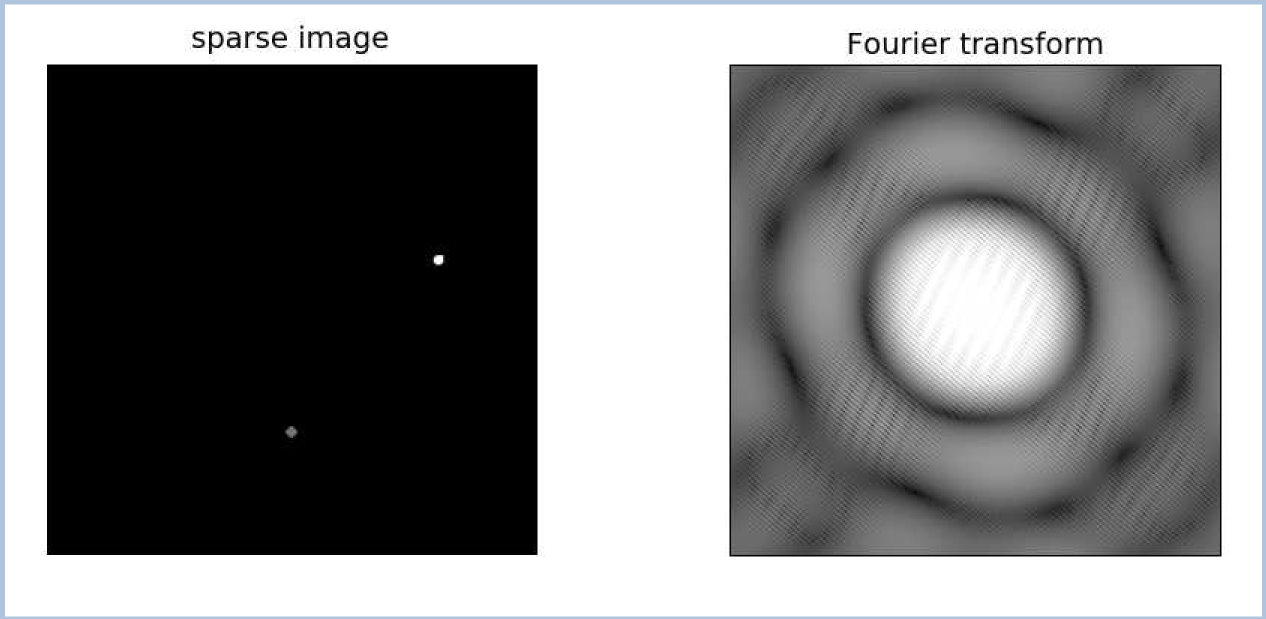
discrete Cartesian FT	discrete X-ray transform
consistent	may be inconsistent
discrete inverse	no known direct discrete inverse
need NxN samples	need 2Nx2N samples?
incoherence	partial incoherence

Inverse of the discrete X-ray transform?

1024x1024 discrete phantom FBP applied to 2048x2048 data set



Incoherence



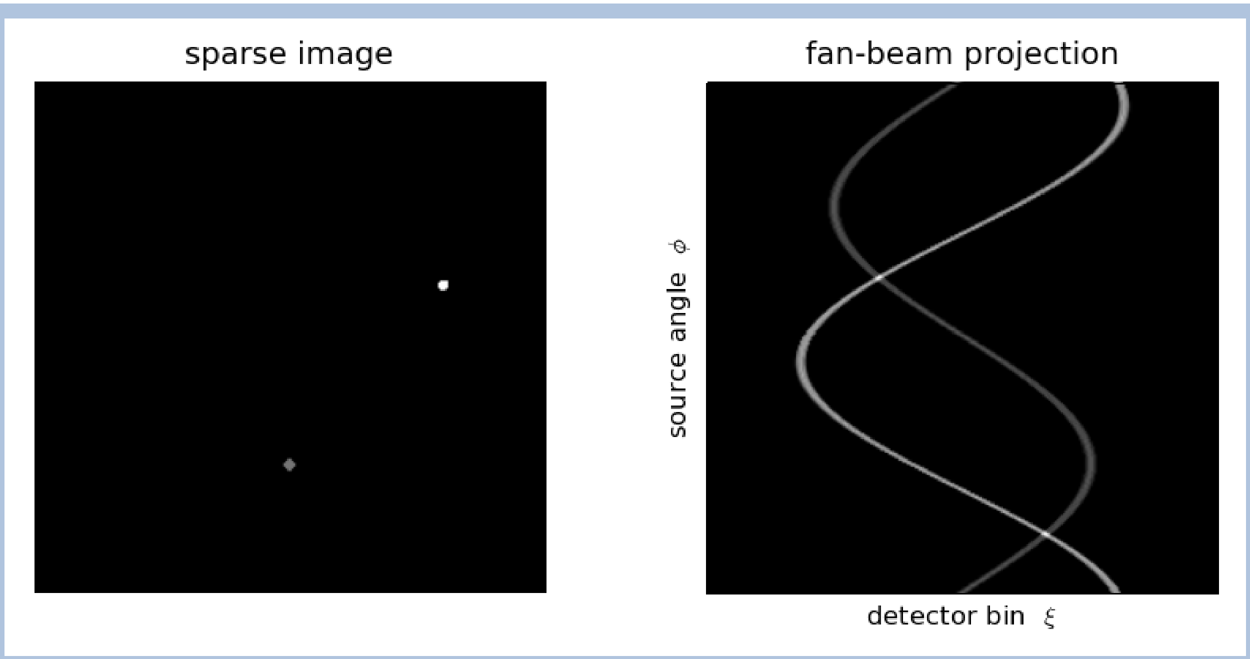


image gradient CS for CT

$$\vec{f}^* = \mathrm{argmin} \|\vec{f}\|_{TV} \;\; | \;\; |X\vec{f} - \vec{g}|^2 \leq \epsilon^2 \;\; \text{and} \;\; f_{max} > \vec{f} > 0$$

$$\|\vec{f}\|_{TV} = \sum_i |\vec{\nabla} f_i|$$

* data inconsistency --> ε>0
* no discrete inverse --> challenge for algorithm development
* partial incoherence --> no exact recovery theorems, RIP, NSP
  (we have performed extensive tests...)

Algorithm alternates POCS with TV-steepest descent
PMB 2008 - Sidky and Pan

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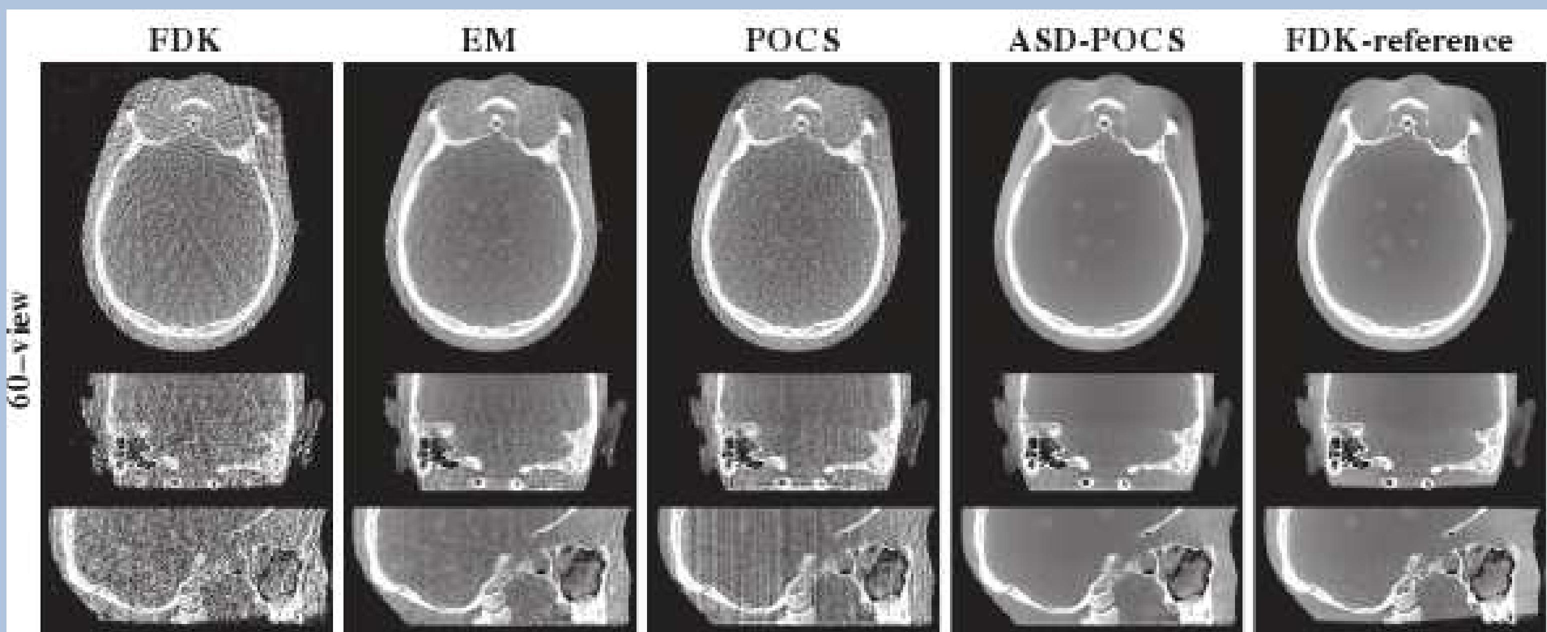
Evaluation of sparse-view reconstruction from flat-panel-detector cone-beam CT

Junguo Bian[1], Jeffrey H Siewerdsen[3], Xiao Han[1], Emil Y Sidky[1], Jerry L Prince[4], Charles A Pelizzari[2] and Xiaochuan Pan[1,2]

[1] Department of Radiology, The University of Chicago, Chicago, IL, USA
[2] Department of Radiation & Cellular Oncology, The University of Chicago, Chicago, IL, USA
[3] Departments of Biomedical Engineering, Johns Hopkins University, Baltimore, MD, USA
[4] Department of Electrical & Computer Engineering, Johns Hopkins University, Baltimore, MD, USA

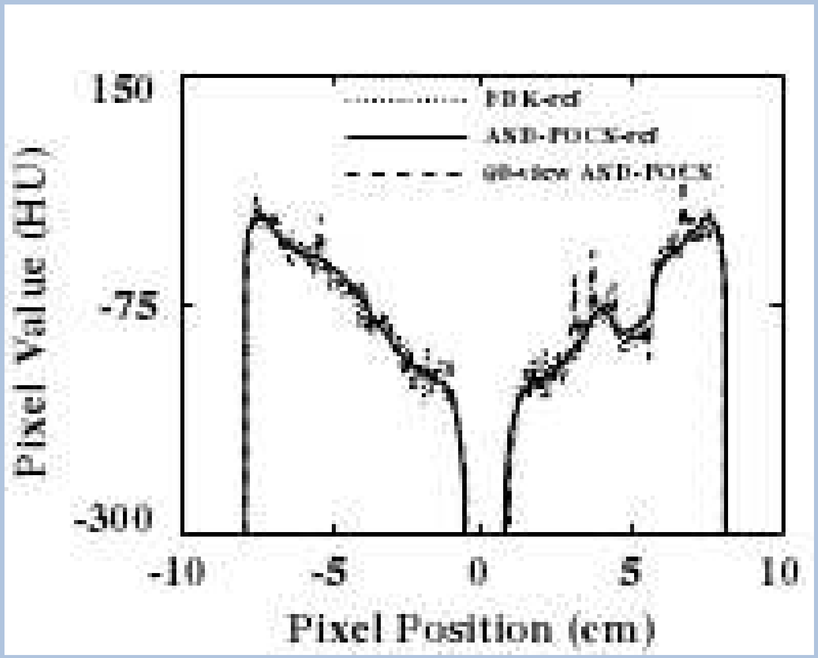


CS algorithm

960-views

PMB 2010

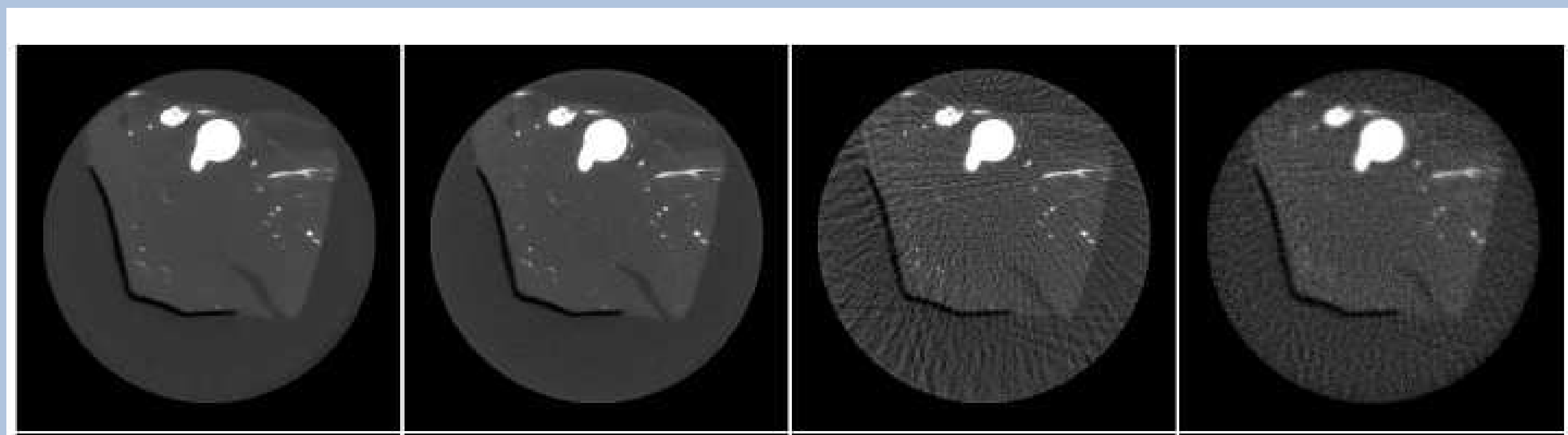
Robustness to model error



PMB 2010

Algorithm-enabled Low-dose Micro-CT Imaging

Xiao Han, *Student Member, IEEE*, Junguo Bian, *Student Member, IEEE*, Diane R. Eaker,
Timothy L. Kline, *Student Member, IEEE*, Emil Y. Sidky, Erik L. Ritman, and Xiaochuan Pan, *Fellow, IEEE*



CS
60-views

FDK
360-views

FDK
60-views

POCS
60-views

TMI 2011

Is CS really new?

* Edge-preserving TV regularization used since early 1990s
  Constrained, TV-minimization equivalent to
  TV-penalized unconstrained optimization

* Sparsity and L1-relaxation exploited for contrast-enhanced vessel imaging

INSTITUTE OF PHYSICS PUBLISHING PHYSICS IN MEDICINE AND BIOLOGY
Phys. Med. Biol. 47 (2002) 2599–2609 PII: S0031-9155(02)36445-5

PMB 2002

An accurate iterative reconstruction algorithm for sparse objects: application to 3D blood vessel reconstruction from a limited number of projections

Meihua Li[1], Haiquan Yang[2] and Hiroyuki Kudo[3]

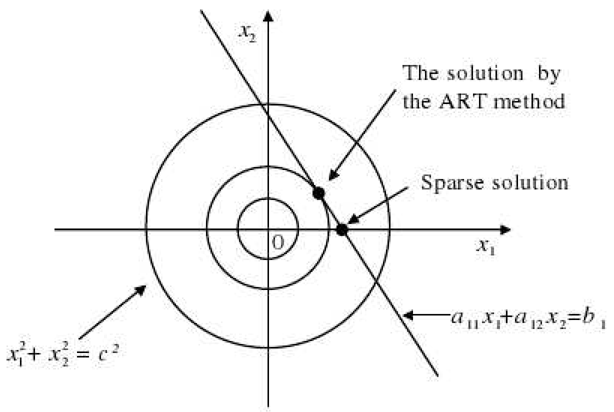


Figure 2. The cost function of ART method.

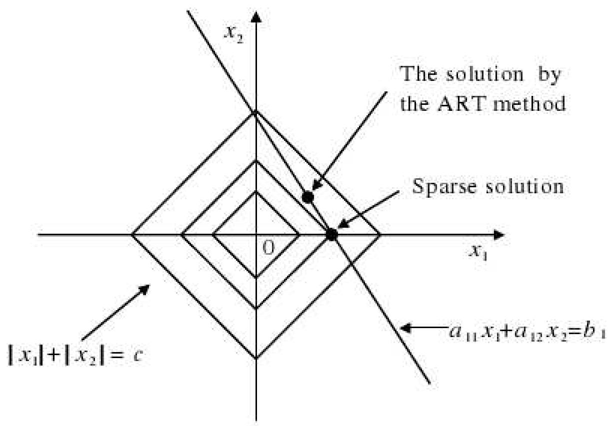
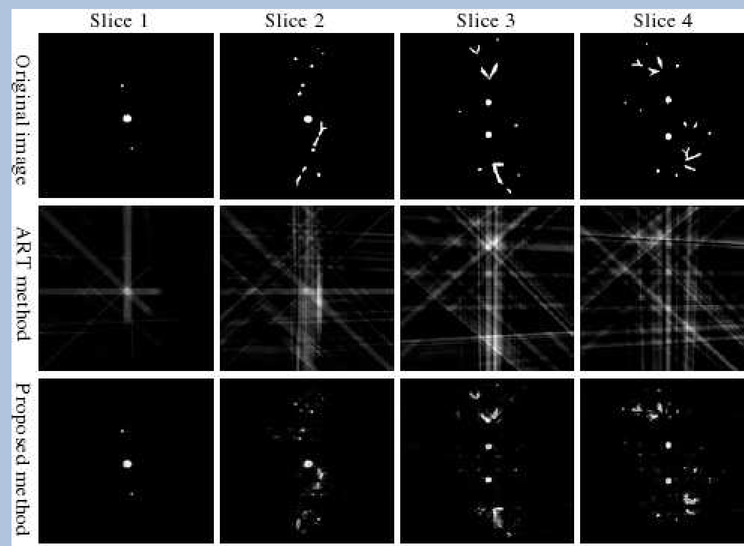


Figure 3. The L_1 norm cost function.



4-views!

L_1-relaxation

Contributions of CS

* Expanded thinking on optimization based image reconstruction
    Traditional iterative: minimize data fidelity + γ roughness penalty
    CS: Use penalty to break degeneracy of the solution space

* Novel rules for determining data sufficiency-based object sparsity

*Beating Nyquist Frequency??
  No.
    Nyquist is only one form of interpolation
    Use of interpolation, followed by FBP yields the continuous image
    CS yields only discrete representation of the image

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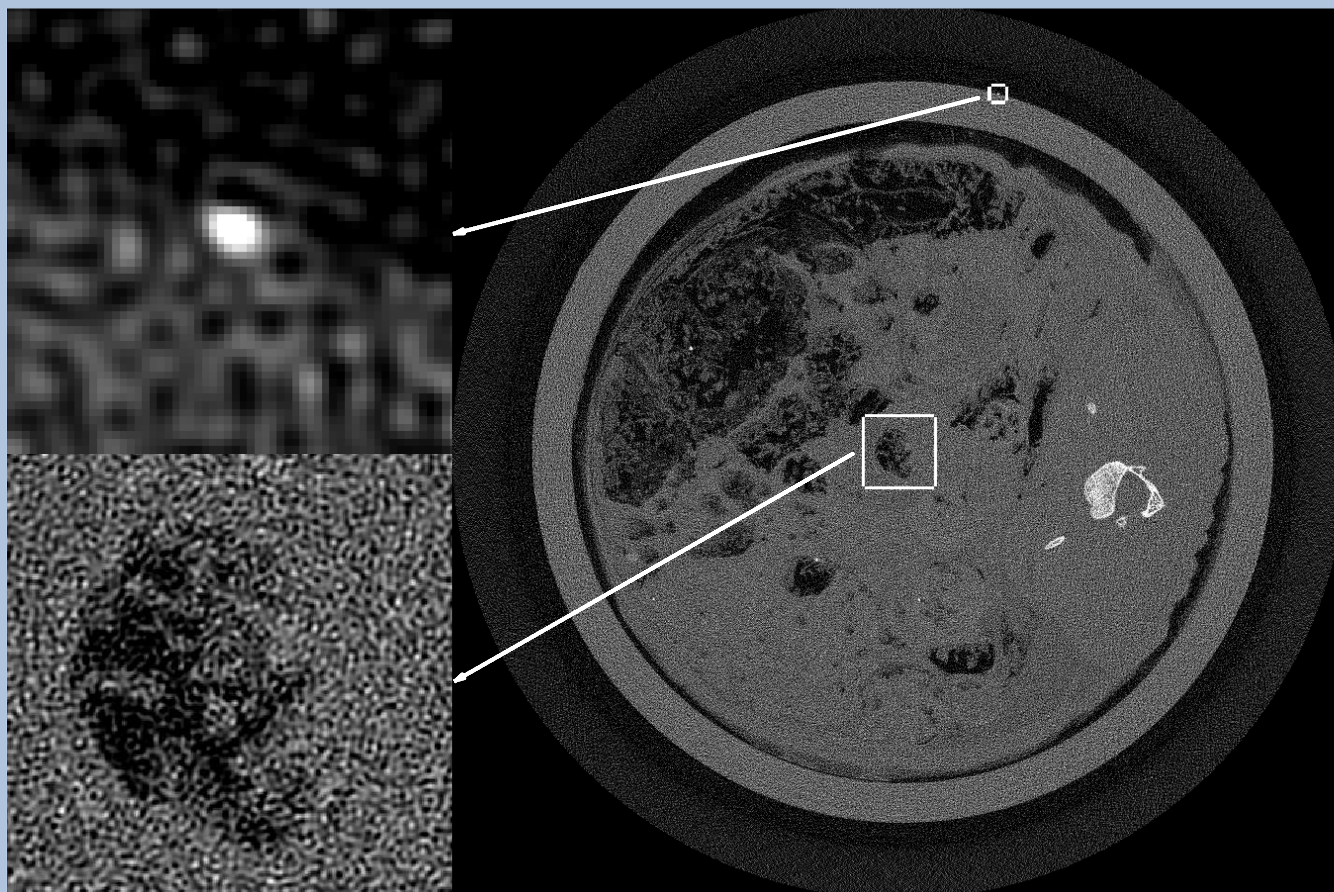
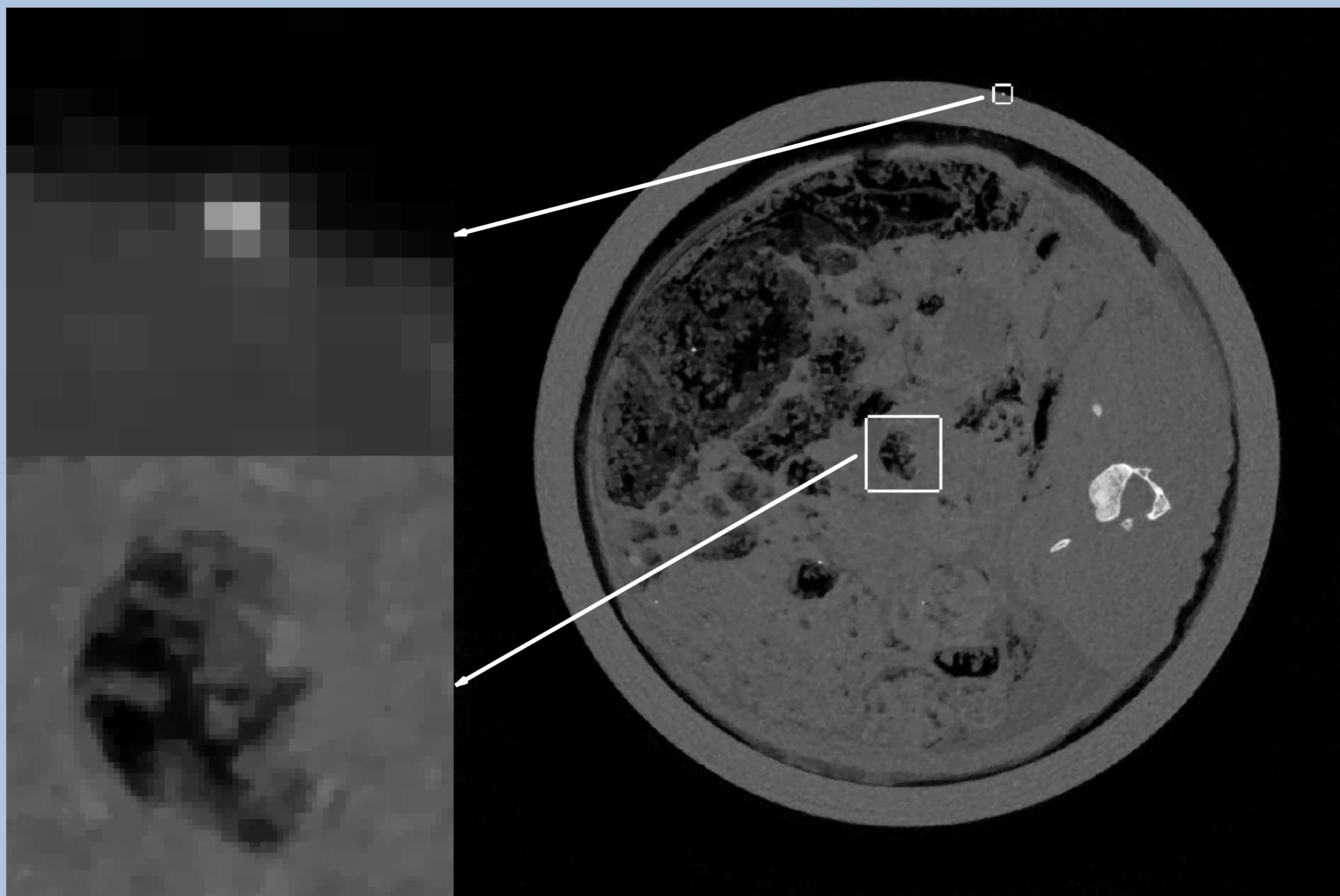
Focus on bCT:
tradeoff between view-number and noise-per-view

1878-projections, 100 micron detector bins, low-intensity X-ray illumination



Courtesy XCounter

constrained, TV-minimization
First attempt: 100 micron pixel array



Second attempt: 25 micron pixel array

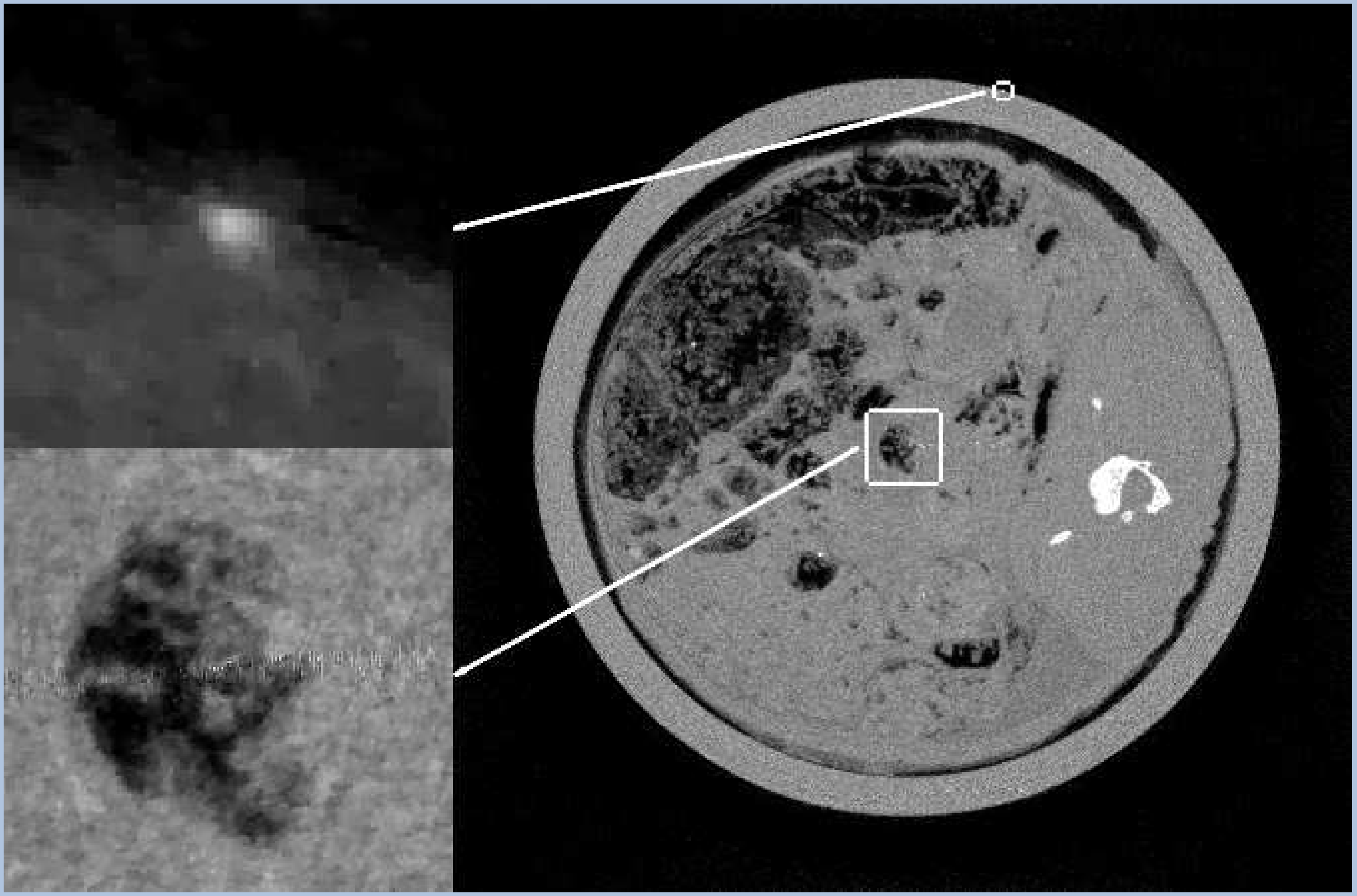
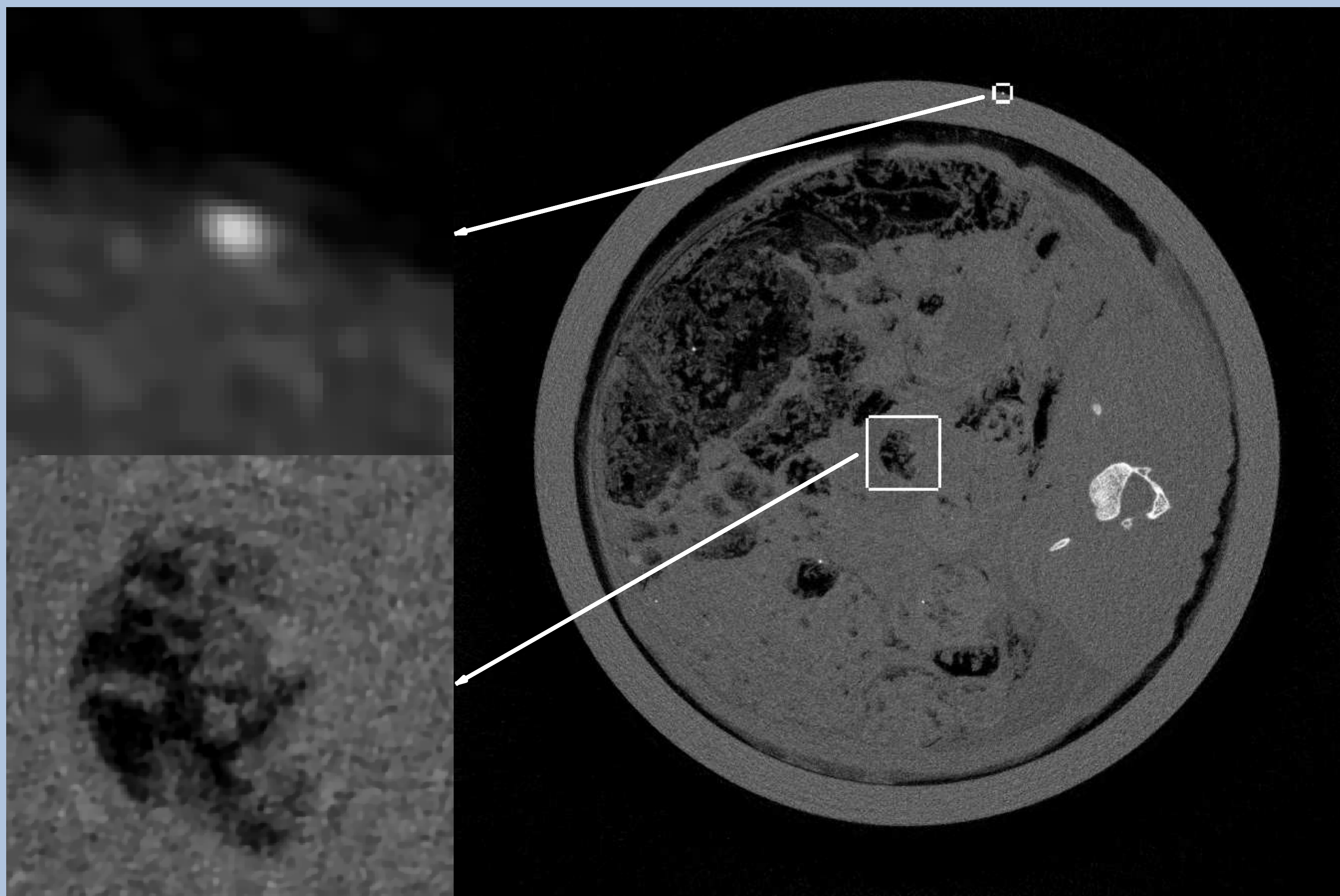


image array: 4096 x 4096
data samples: 1878 views x 1200 bins

<u>undersampled!!</u>

CS-algorithm modifications



detector-coordinate
Fourier upsampling

constrain image
spatial frequencies

Sidky et al. 2011- arxiv.org/abs/1104.0909

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Preliminary investigation on sparsity-based data sufficiency

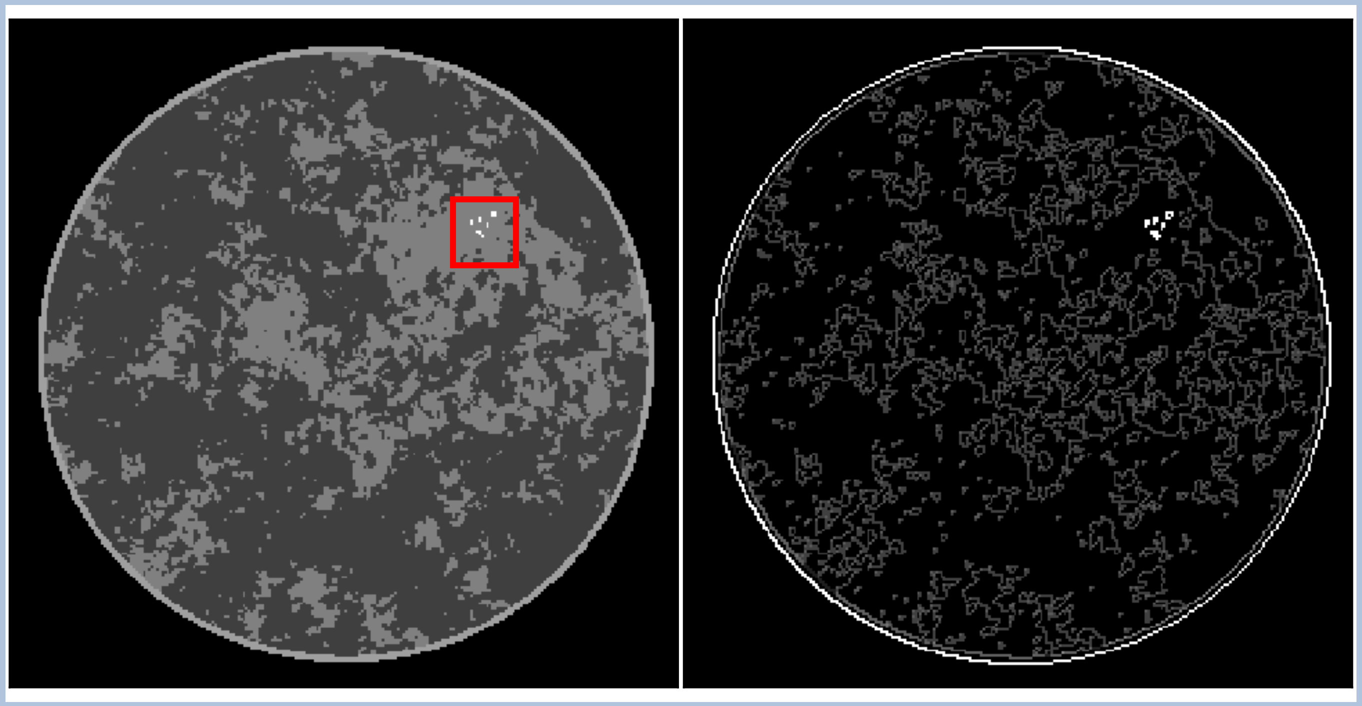
* Aiming for an empirical Donoho-Tanner type study

* Accurate, first order TV-minimization solver
  Jakob Joergensen - Danish Technical University
  T. Jensen et al. (arxiv.org/abs/1105.3723)

* Computer-generated breast phantom

Phantom

gradient magnitude



256x256 pixelized array
65536 unknowns

~10000 non-zero pixels

Sampling sufficiency study

objectives

$$|\vec{g} - X\vec{f}|^2 + \alpha|\vec{f}|^2$$ Tikhonov

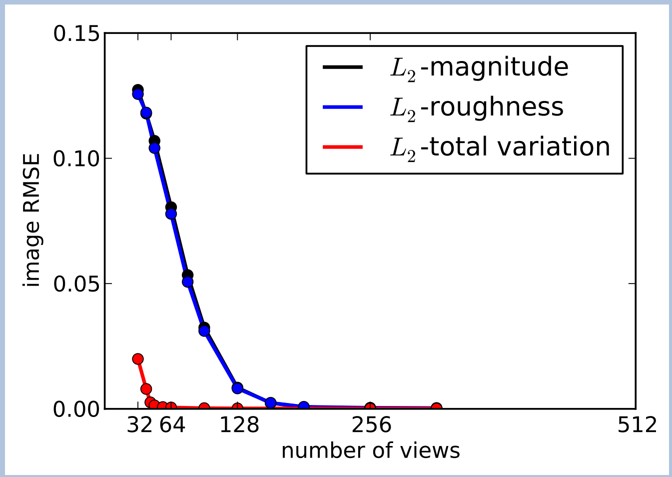
$$|\vec{g} - X\vec{f}|^2 + \alpha|\vec{\nabla} f|^2$$

$$|\vec{g} - X\vec{f}|^2 + \alpha\|\vec{f}\|_{TV}$$ CS

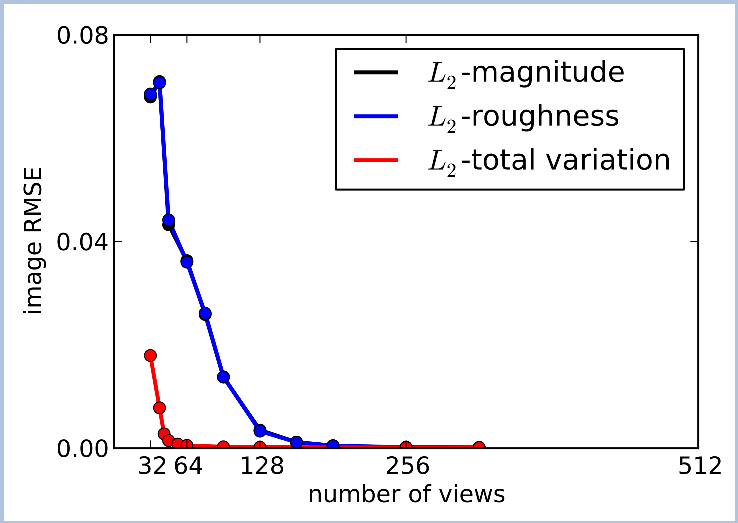
α extremely small -> data RMSE=10^{-5}

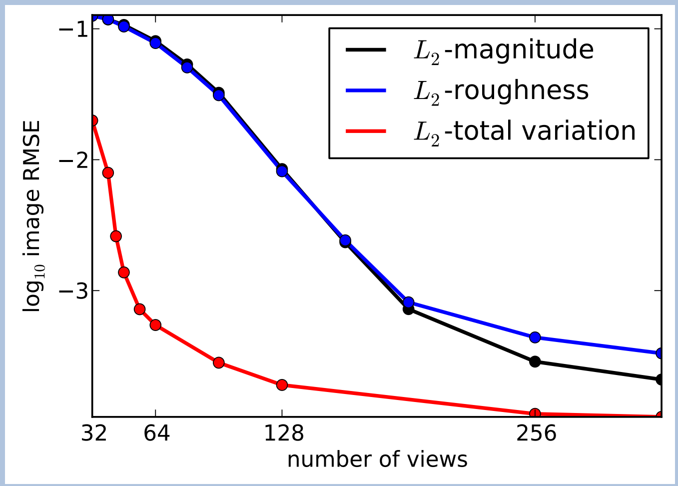
data: 32-512 views x 512 bins

ROI error



whole image error





necessary samples/sparsity ~ 2.5 ???

What is fully sampled?

the group working on CS in CT

University of Chicago

Xiaochuan Pan   Emil Sidky

Students:
Junguo Bian
Xiao Han
Eric Pearson
Zheng Zhang
Adrian Sanchez

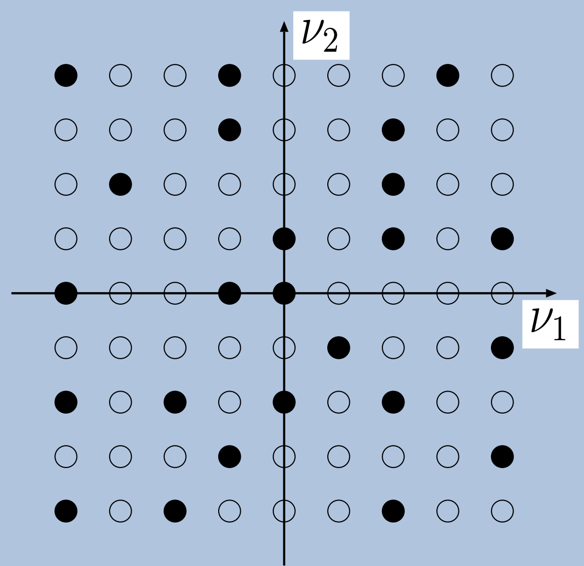
applied math experts:

Rick Chartrand
LANL

Jakob Joergensen
student at the Danish Technical Univ.

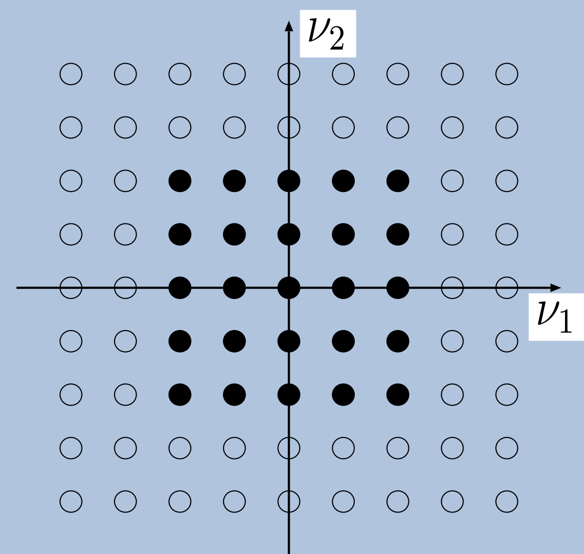
Fourier sampling problems

interpolation



"standard" CS

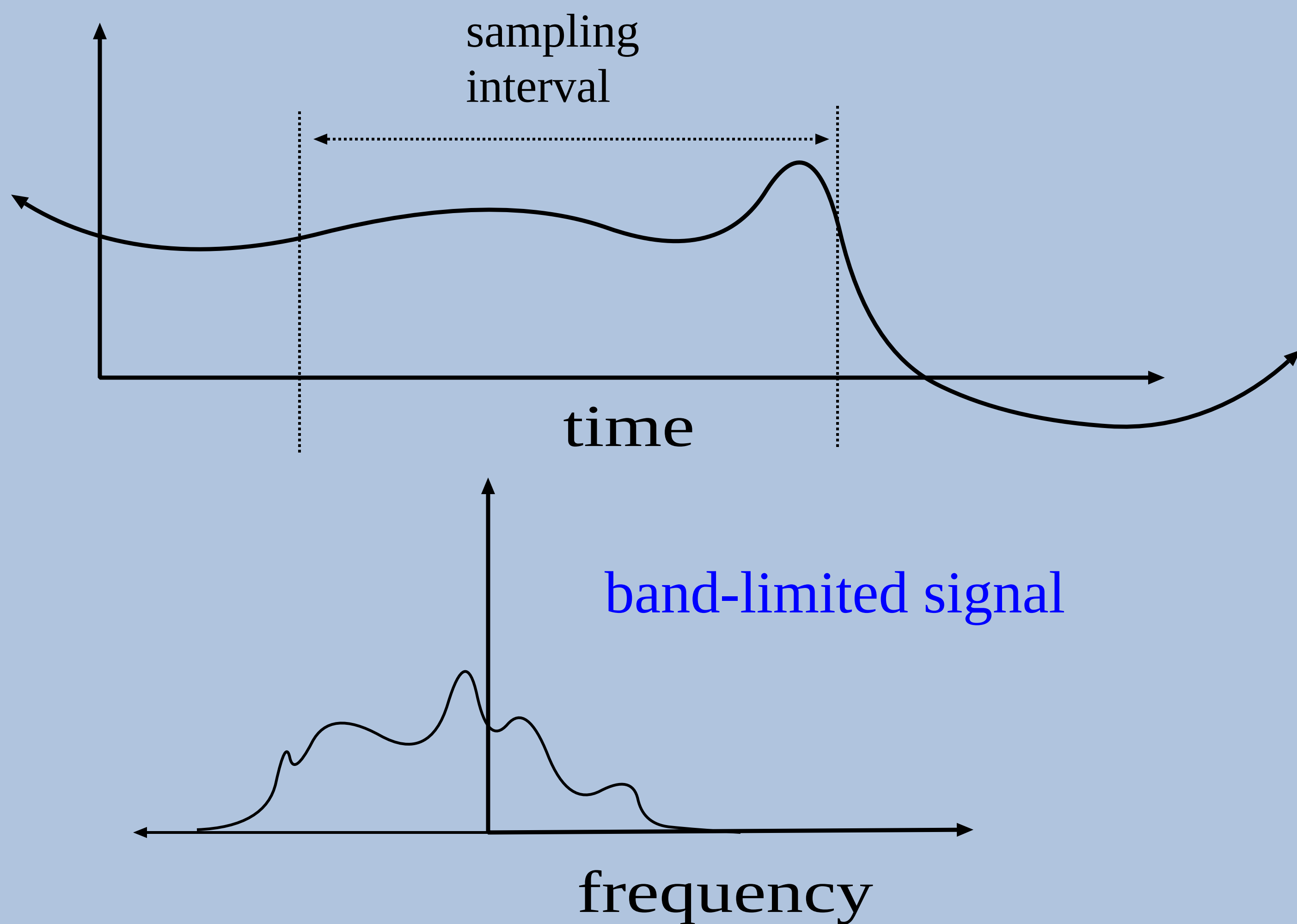
extrapolation



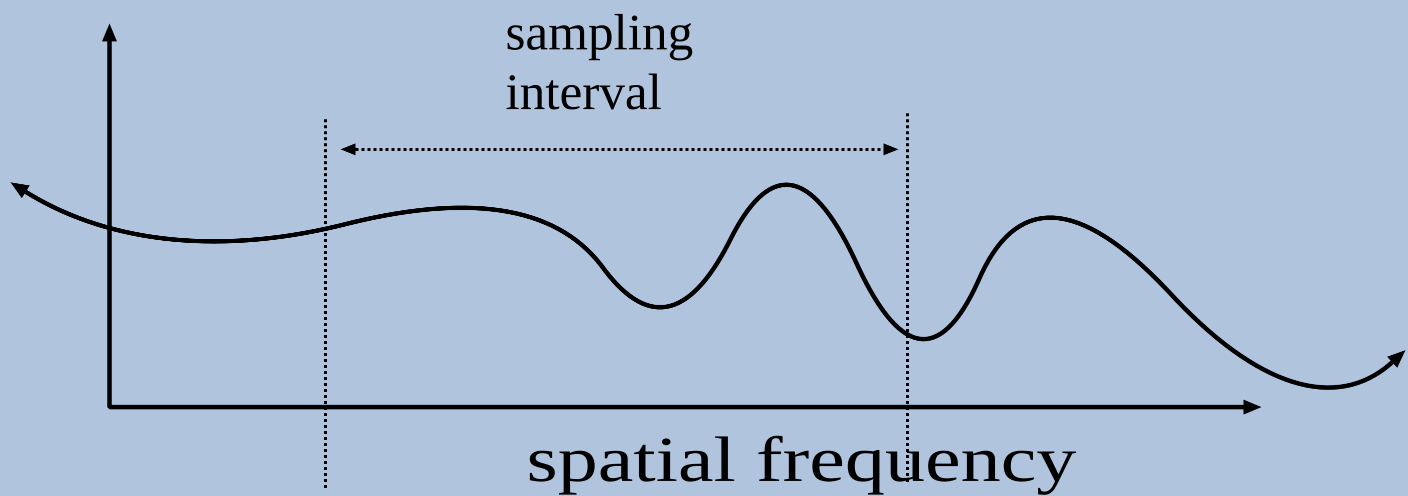
CS approach to an old problem

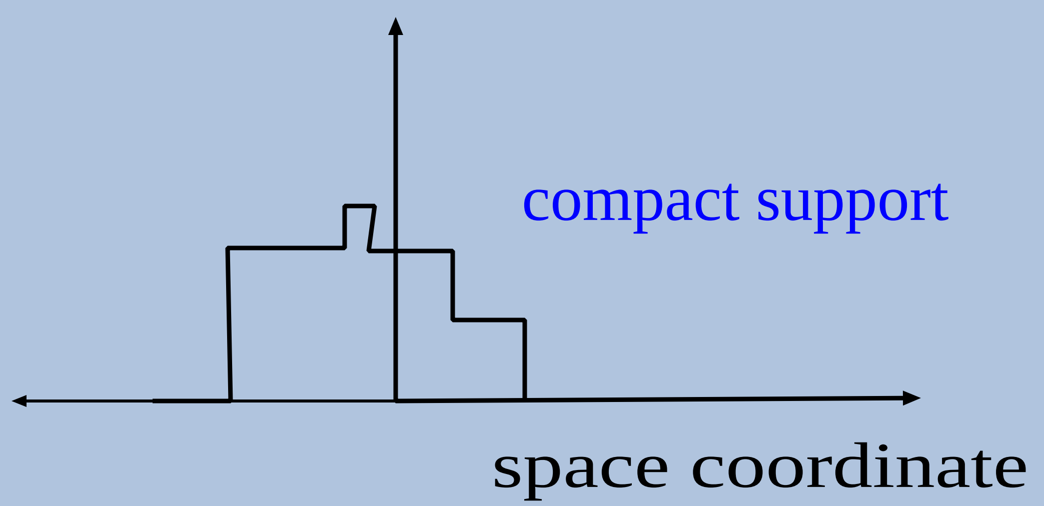
Chartrand, Sidky and Pan
math.lanl.gov/~rick/Publications/chartrand-2011-frequency.shtml

Papoulis-Gerchberg



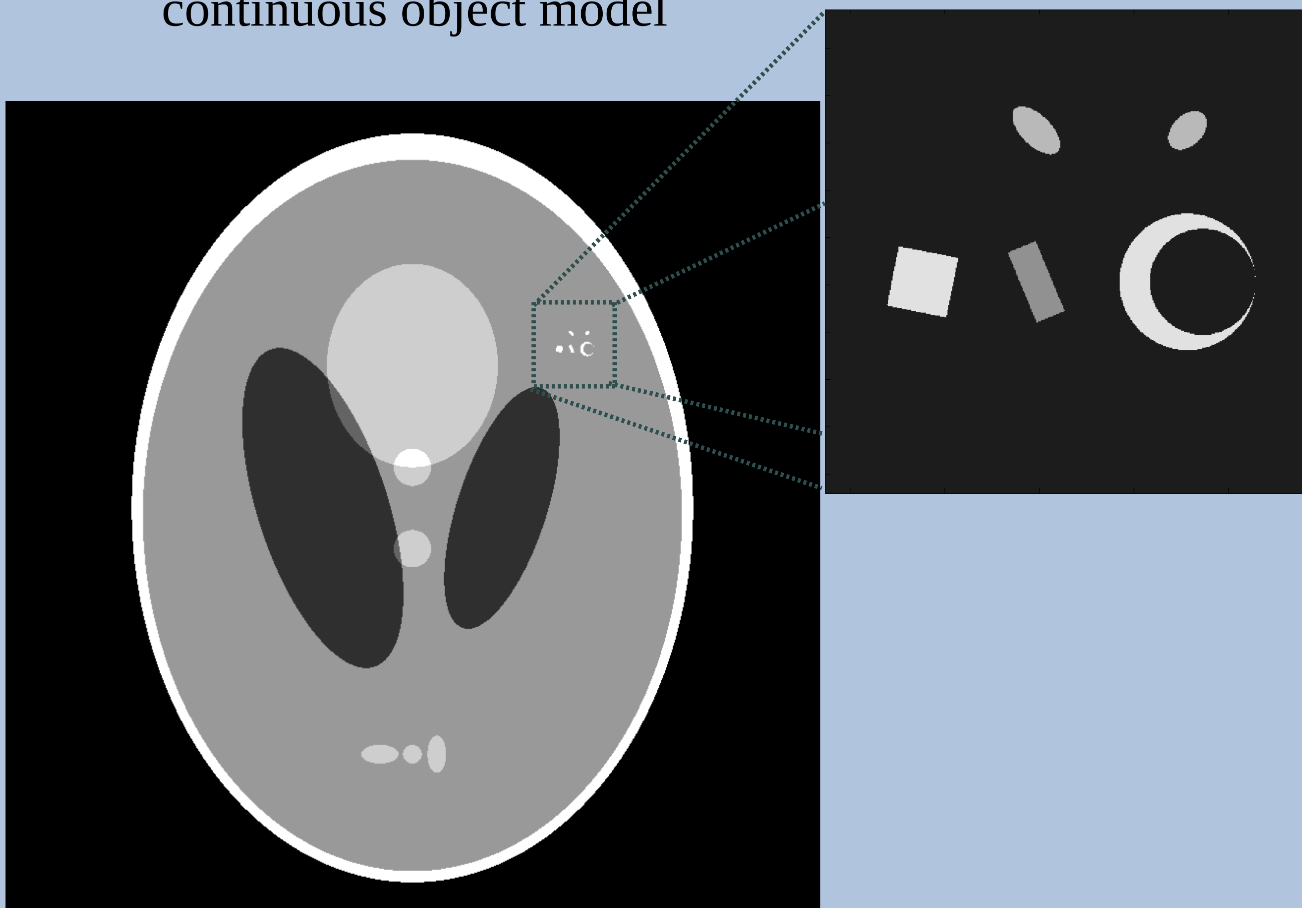
Papoulis-Gerchberg reversed



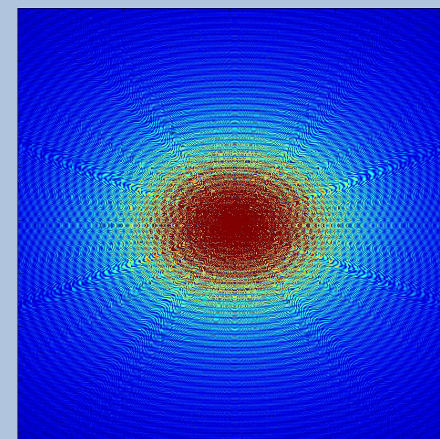


Frequency extrapolation experiment

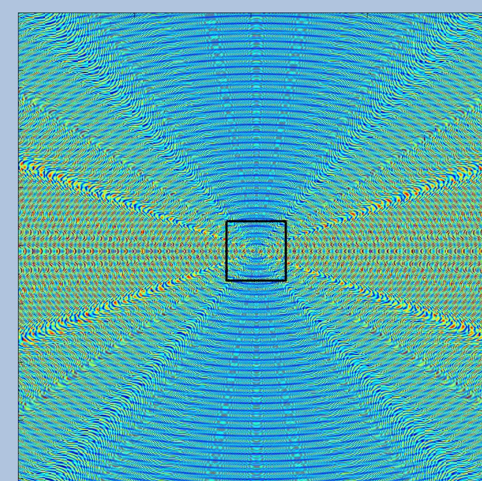
continuous object model



4Kx4K samples
of continuous FT



scaled FT samples



Problem: recover 4Kx4K FT sample grid
from central set of 512x512 samples.

Frequency extrapolation method

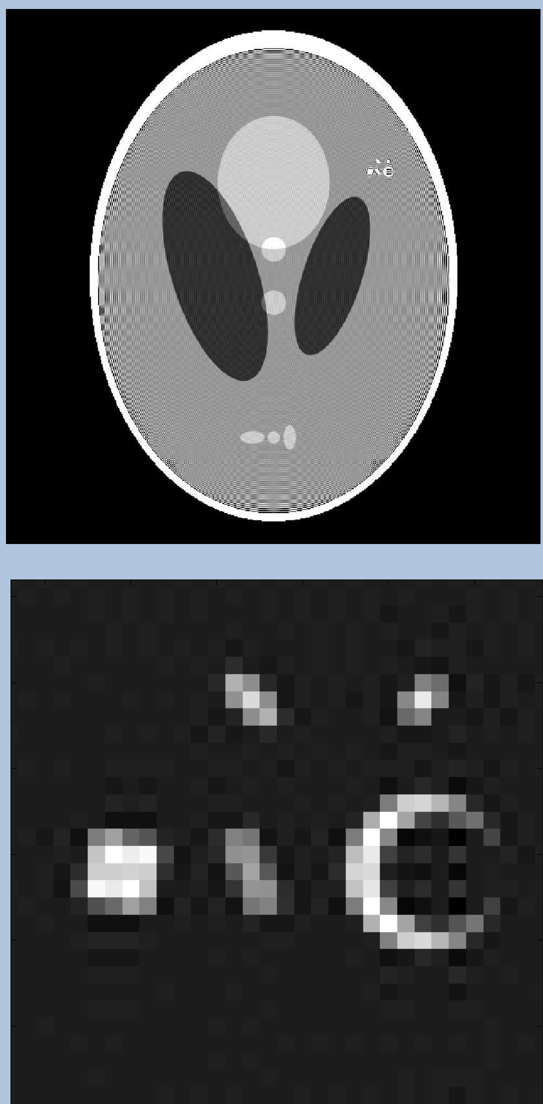
$$x^* = \operatorname{argmin} \sum_{i=1} \varphi_p(|\nabla x|_i) + \lambda \|Ax - b\|_2^2$$

$$\varphi_p(t) = \begin{cases} \gamma |t|^2 & \text{if } |t| \leq \alpha \\ \gamma |t|^p / p - \delta & \text{if } |t| > \alpha \end{cases}$$

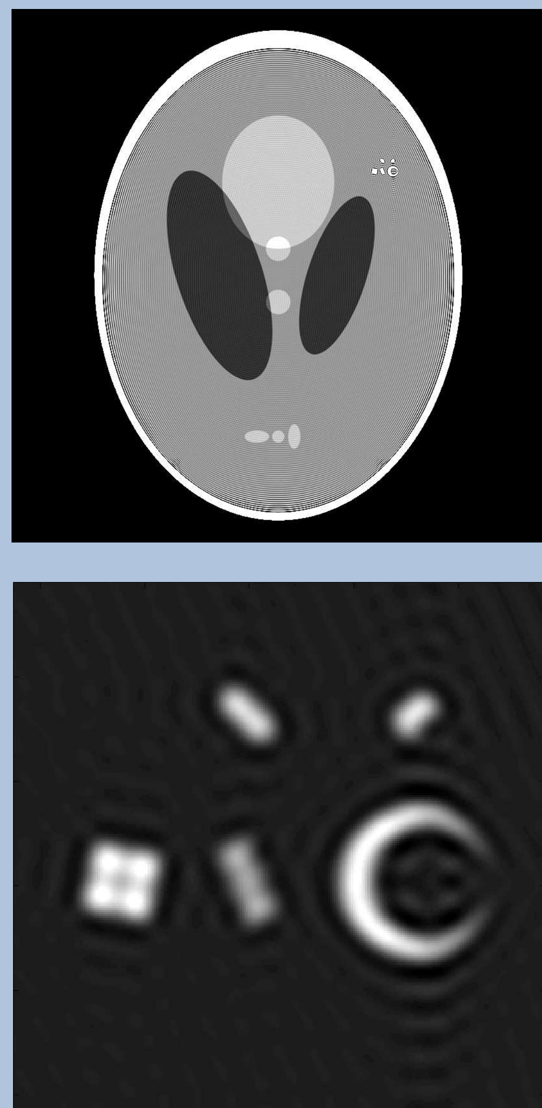
Chartrand ISBI 2009 for details
*efficient solver

Results: no frequency extrapolation

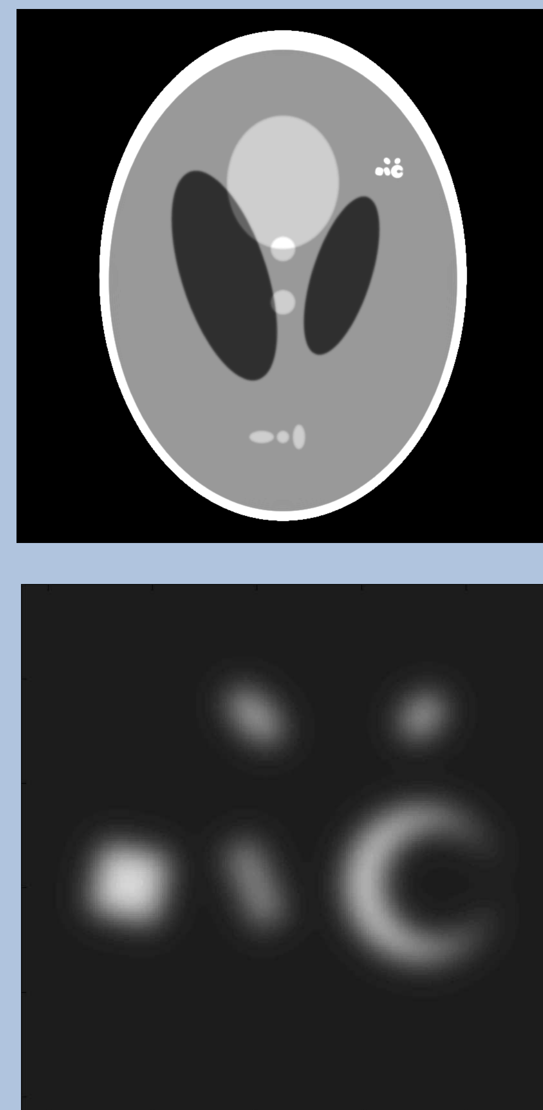
inverse DFT



zero pad

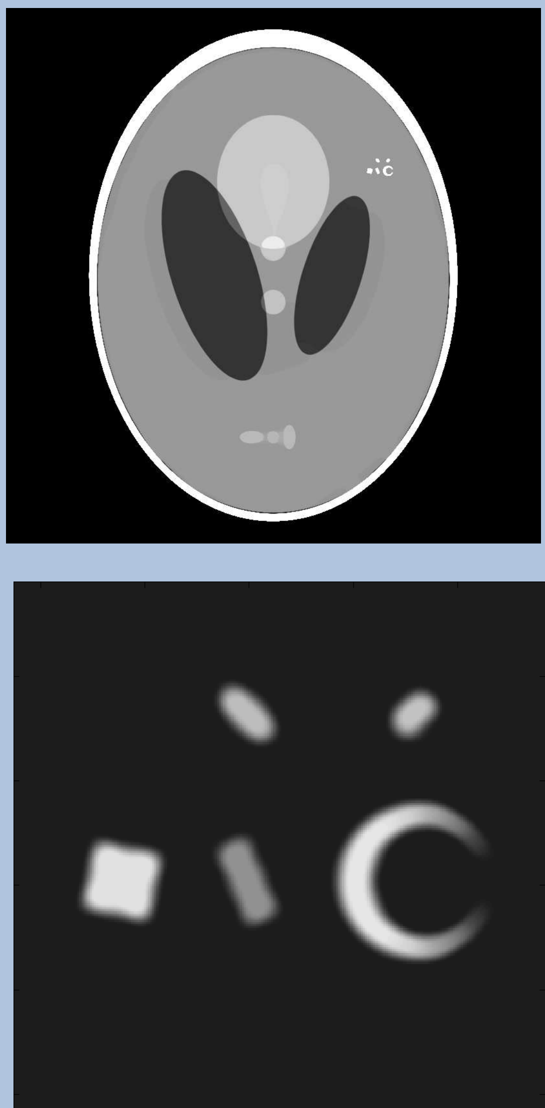


zero pad and filter

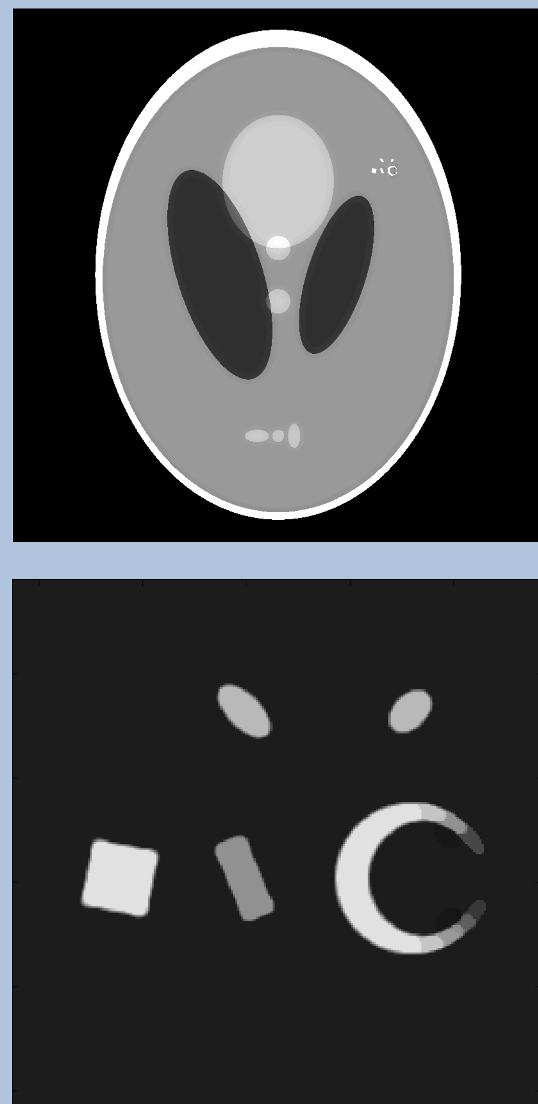


Results: non-convex frequency extrapolation

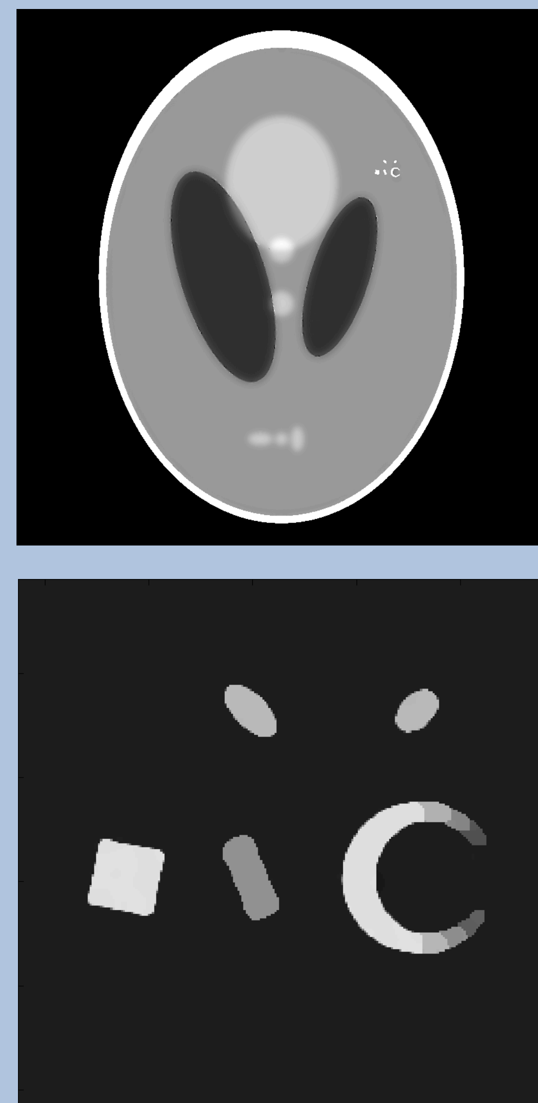
p=1



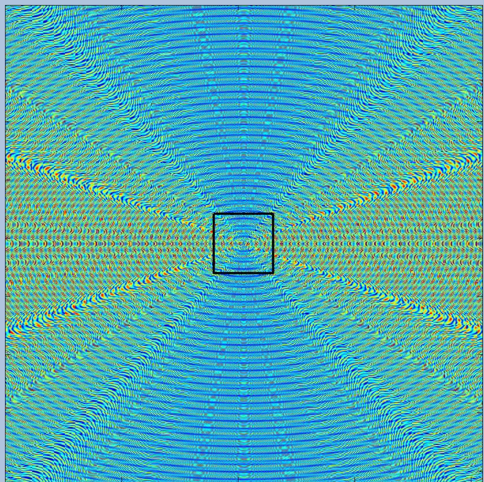
p=0.25



p=-0.5



Results: non-convex frequency extrapolation



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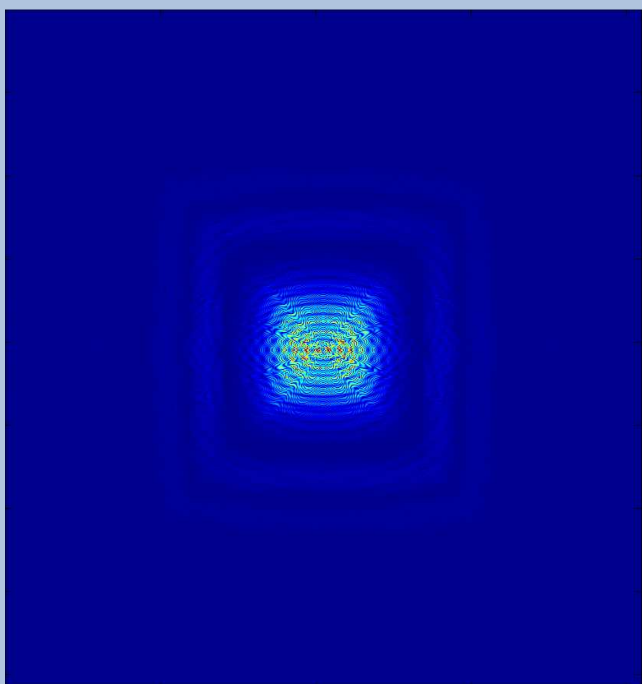

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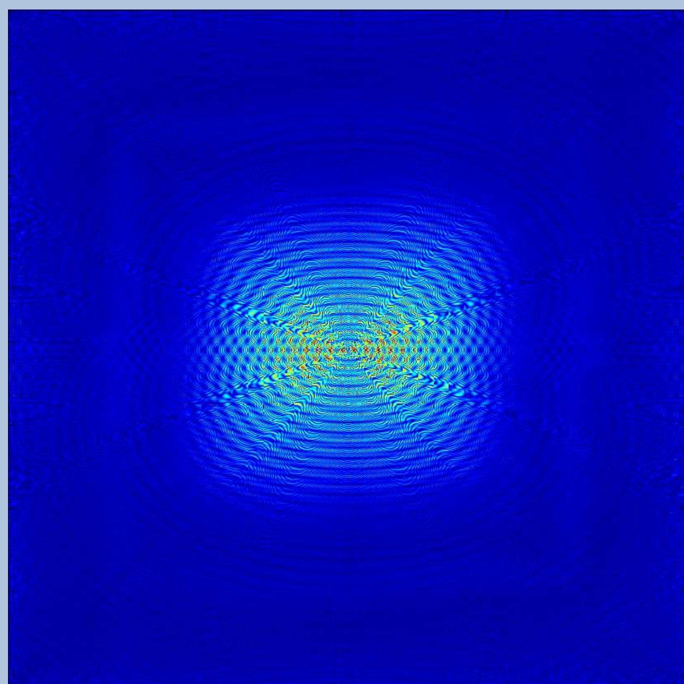

p=-0.5