Fields-MITACS
Mathematics of Medical Imaging
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Frequency Domain Photoacoustics: Specifics of Signal Processing and Image Reconstruction

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Photoacoustic Tomography: Objectives and Methods

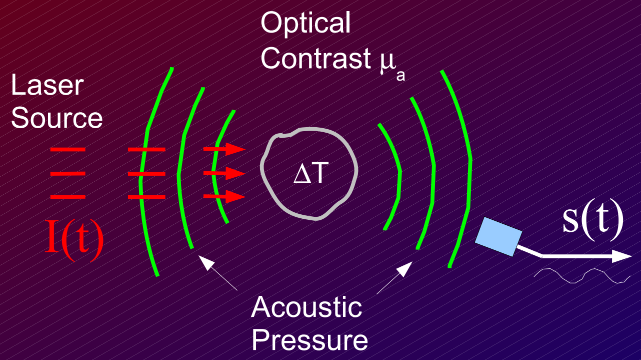
Imaging Objectives:

1. Positions and dimensions of photoacoustic sources.
2. Characteristics of PA sources: absorption coefficient, chemical composition, blood flow rate etc.

Standard Methods:

1. Short (nanosecond) laser irradiation and broadband detection.
2. Photoacoustic microscopy with high frequency (> 30 MHz) sources.
3. Photoacoustic spectroscopy with narrow band tunable sources.

Photoacoustic Imaging with Intensity Modulated CW Laser Source (Frequency Domain PA)


Optical Contrast μ_a

Laser Source

ΔT

I(t)

s(t)

Acoustic Pressure

$$\nabla^2 \tilde{p}(\vec{r},\omega)+k^2\tilde{p}(\vec{r},\omega)=\frac{-i\omega\beta}{C_p}\tilde{q}(\vec{r},\omega)$$

$$\tilde{q}(\vec{r},\omega)=\mu_a(\vec{r})I(\vec{r})\tilde{F}(\omega)$$

Confinement conditions in FD:

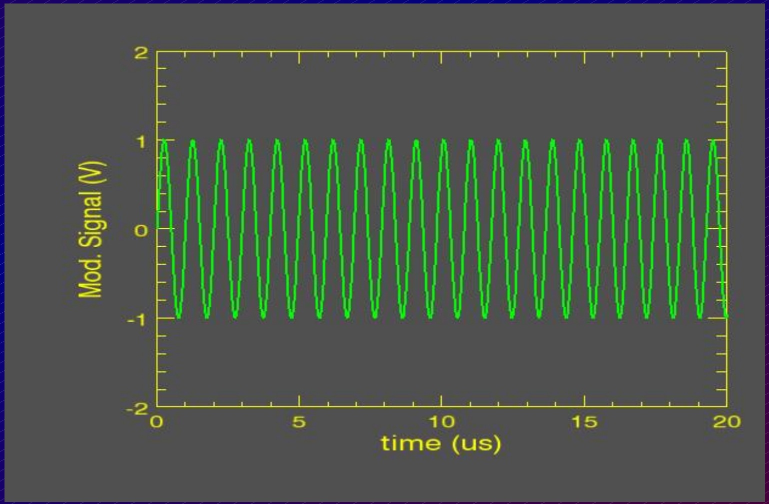
$$L_T=\left(\frac{D_T}{\omega}\right)^{1/2}\ll\mu_a^{-1}$$

$$\omega<\omega_a=\mu_a c_a$$

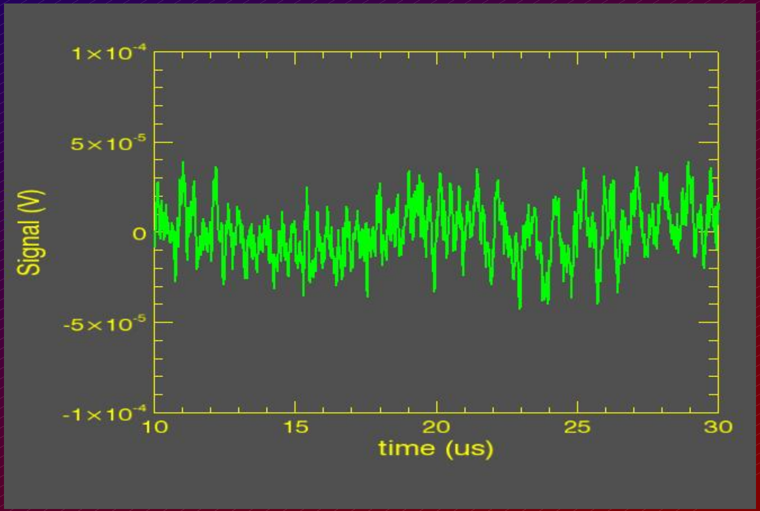
Difficulties of FD photoacoustics:

1. Low optical power (0.1 – 1 W) → Low SNR
2. Long pulse duration (> 1 ms) → Poor spatial resolution

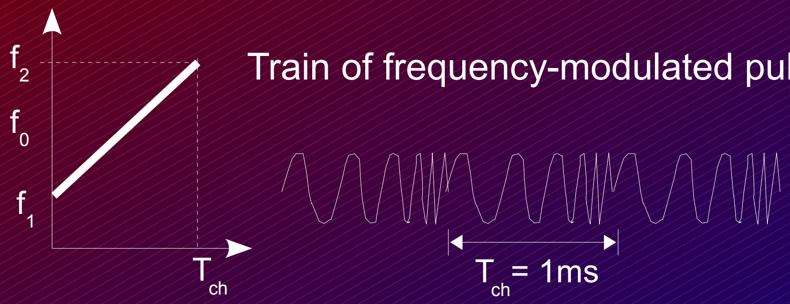
Modulation waveform



Raw signal after 1000 averages



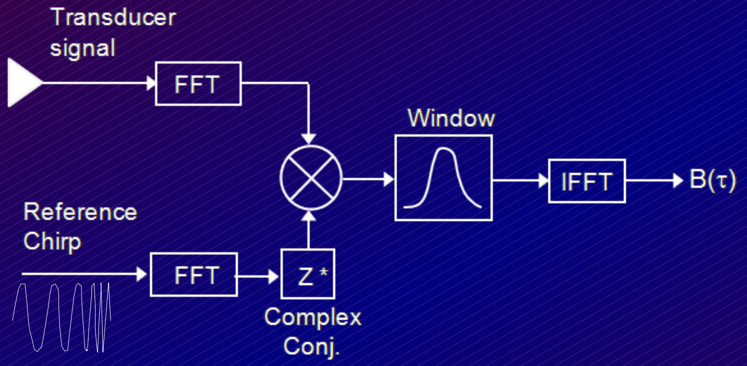
Spatially-Resolved PA Imaging with Chirped Waveforms



Train of frequency-modulated pulses

Correlation Processor

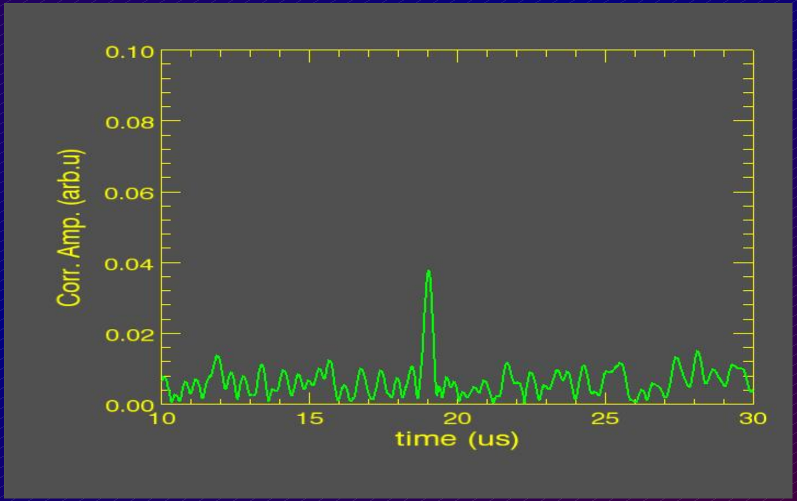
$$B(\tau)=\int_{-\infty}^{\infty} r(t+\tau)\,s(t)\,dt=\frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{R}^{*}(\omega)\,\tilde{S}(\omega)\,e^{i\omega\tau}\,d\omega$$


Transducer signal
FFT
Window
IFFT
B(τ)
Reference Chirp
FFT
Z *
Complex Conj.

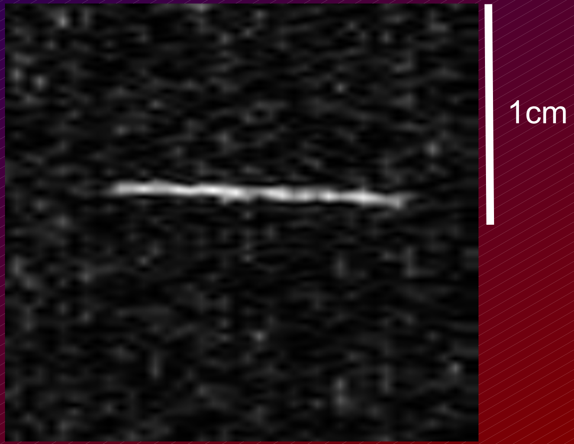
$$B(\tau)=\underbrace{\frac{A^{2}T_{ch}}{2}}_{\text{Peak Amplitude}}\;\underbrace{\frac{\sin\left[\frac{\pi m\tau}{T_{ch}}\left(1-\frac{\tau}{T_{ch}}\right)\right]}{\pi m\tau/T_{ch}}}_{\text{Side lobes}}\;\underbrace{\cos(\omega_{0}\tau)}_{\text{Harmonic carrier}}$$

$$m = T_{ch}\Delta f$$

Correlation function of chirped PA signal



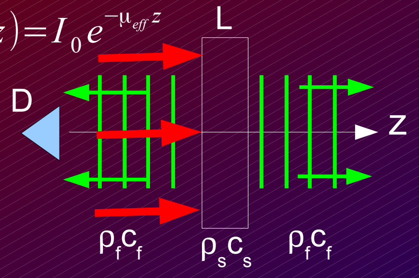
Correlation image of optical contrast in scattering medium



Analytical Model of PA Generation

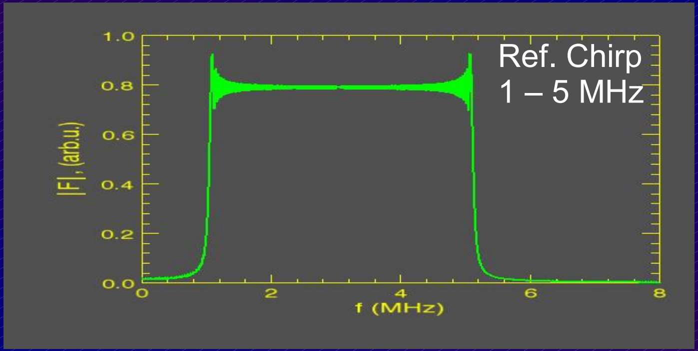
1-D Model with Acoustic Impedance Discontinuity

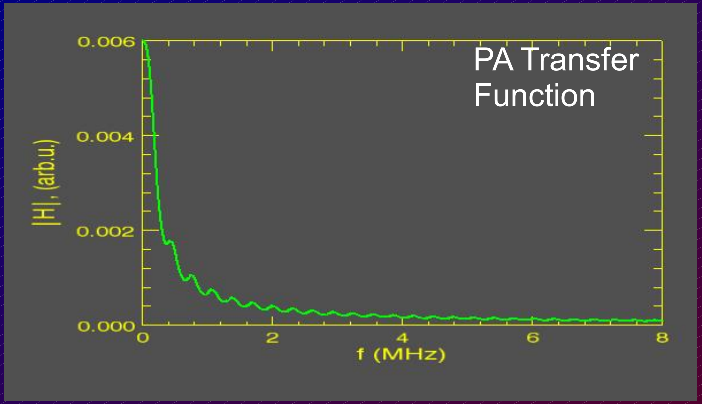
$$I(z)=I_0 e^{-\mu_{eff} z}$$

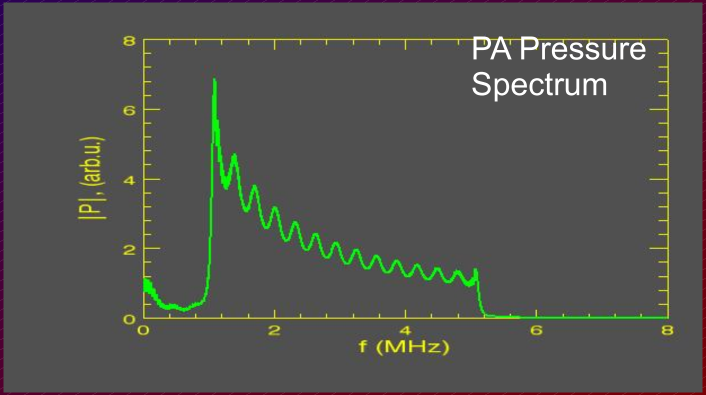


$$\tilde{p}_{zz}(z,\omega)+k^2\tilde{p}(z,\omega)=\frac{-i\omega\beta}{C_p}\tilde{q}(z,\omega)$$

$$\tilde{q}(z,\omega)=\mu_a I_0 e^{-\mu_a z}\tilde{F}(\omega),\quad \tilde{F}(\omega)$$ - Spectrum of the laser modulation waveform







Method of transfer functions:

$$\tilde{p}(\vec{r},\omega)=\tilde{H}_{PA}(\omega)\cdot I_0\cdot\tilde{F}(\omega)\cdot\Phi(\vec{r})$$

$$H_{PA}(\omega)=\frac{-i\beta\mu_a c_s}{C_p(\mu_a^2 c_s^2+\omega^2)}\cdot\frac{(\zeta k_f+i\mu_a)\cos(k_s L)+(ik_s-\zeta\mu_a c_s/c_f)\sin(k_s L)-(\zeta k_f+i\mu_a)e^{-\mu_a L}}{i(1/c_s^2+\zeta^2/c_f^2)\sin(k_s L)+(2\zeta/c_s c_f)\cos(k_s L)}$$

1-D solution for exponential source:

$$\tilde{p}(z,\omega)=\tilde{H}_{PA}(\omega)\cdot I_0\cdot\tilde{F}(\omega)\cdot e^{-ik_f z},\quad k_f=\omega/c_f$$

Correlation Processing of Chirped PA Signals

Simulation Results for a 1-D layer:

Layer thickness: 5 mm
$\rho_s c_s$ = 1.54 MRyals
$\rho_f c_f$ = 1.48 MRyals
Absorption: 4 cm^{-1}

Optical Modulation:
Sine chirp: 1 – 5 MHz
Chirp duration: 1 ms

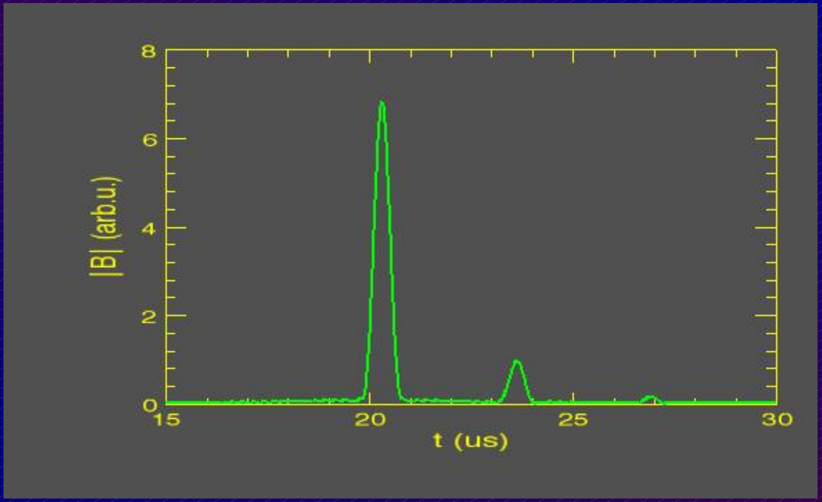
Zero-mean Gaussian noise:
Input SNR = - 40 dB

Coherent averaging of 1000 chirps

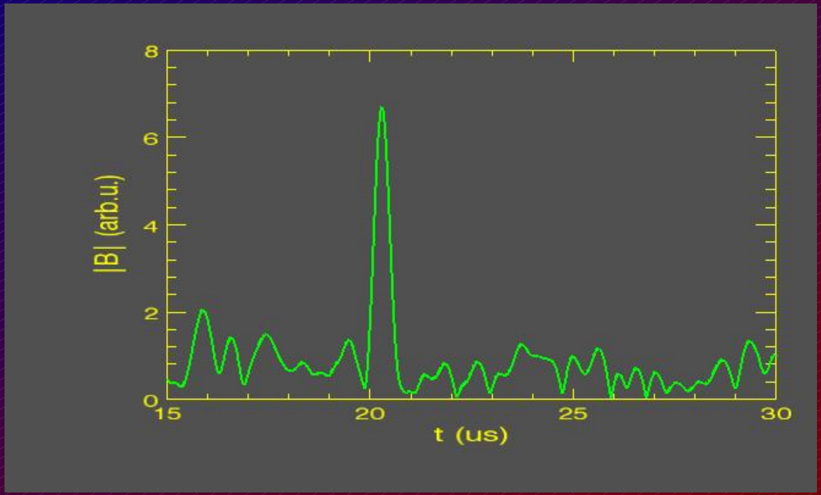
SNR Improvement ~ 56 dB

Axial Resolution: $c_a/\Delta f$ < 1 mm

Noise-free correlation function



Correlation function of noisy signal



Signal-to-Noise of Frequency Domain PA Measurements

Matched Filter (Correlation):

$$B(\tau)=\frac{1}{2\pi}\int \tilde{R}(\omega)\tilde{S}(\omega)e^{i\omega\tau}d\omega$$

$$|B(\tau)|=\sqrt{\Re^2 B(\tau)+\Im^2 B(\tau)}$$

Gaussian noise PSD:

$$N_0=\frac{\langle P_N\rangle}{f_s/2}=\frac{2\sigma^2}{f_s}$$

Noise of matched filter
(Rayleigh distribution):

$$PDF=\frac{A}{\sigma_A^2}\exp(-A^2/2\sigma_A^2)$$

$$E[A]=\sqrt{\frac{\pi}{2}}\sigma_A,\ \sigma_A^2=\frac{E_s\sigma^2}{f_s}$$

$$Var=0.43\,\sigma_A^2$$

SNR of Matched Filter (Single Chirp):

$$SNR_{MF}=\frac{B^2(0)}{\langle P_{NB}\rangle}=\frac{E_s f_s}{0.43\,\sigma^2}$$

Multiple Chirps: Coherent vs
Incoherent Averaging of N_p Chirps

1) Coherent Averaging (Phase retained):

$$\langle P_N\rangle=\frac{\sigma^2}{N_p};\qquad E[B_N]=\sqrt{\frac{\pi E_s}{2 f_s N_p}}\sigma$$ - Noise Background

$$Var[B_N]=0.43\frac{E_s\sigma^2}{f_s N_p}$$ - Noise Variance

$$SNR=\frac{(E_s-E[B_N])^2 f_s N_p}{0.43\,\sigma^2 E_s}$$

2) Incoherent Averaging (Post processing):

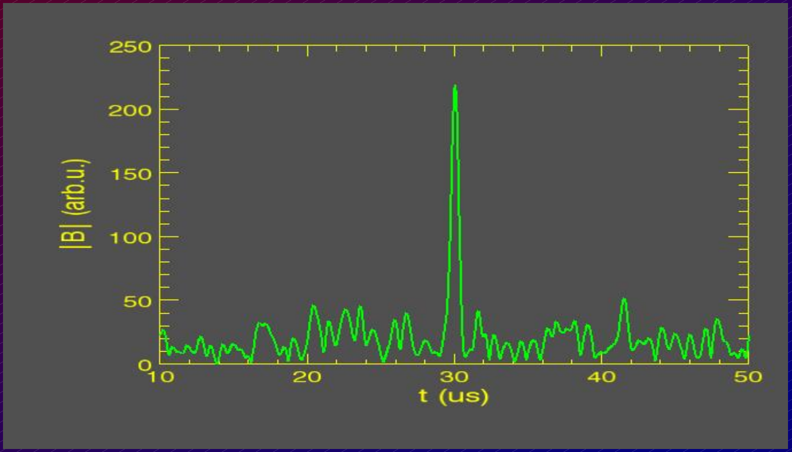
$$B_{av}(\tau)=B(\tau)+\frac{1}{N_p}\sum_{i=1}^{N_p}B_N(\tau)=B(\tau)+n_B$$

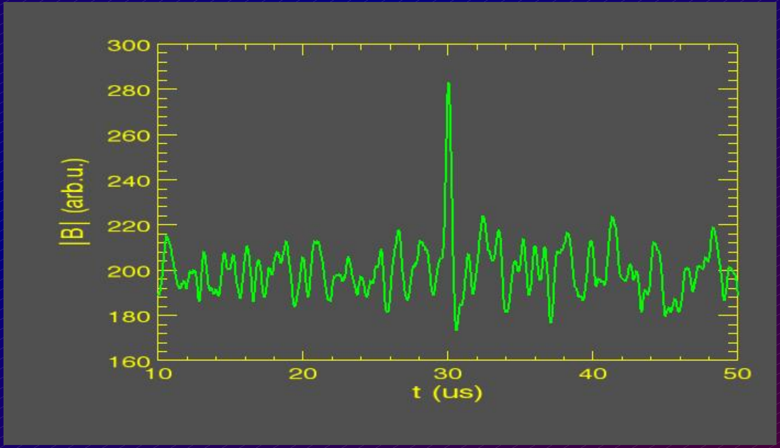
$$E[n_B]=\sqrt{\frac{\pi E_s}{2 f_s}}\sigma;$$ - Independent on N_P

$$SNR=\frac{\left(E_s-\sqrt{\frac{\pi E_s}{2 f_s}}\sigma\right)^2}{0.43\,\sigma^2 E_s}f_s N_p$$

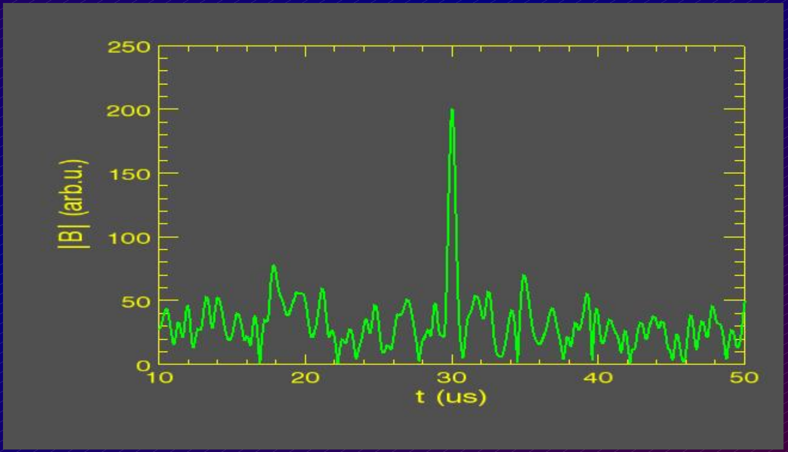
SNR of Coherent vs Incoherent Averaging

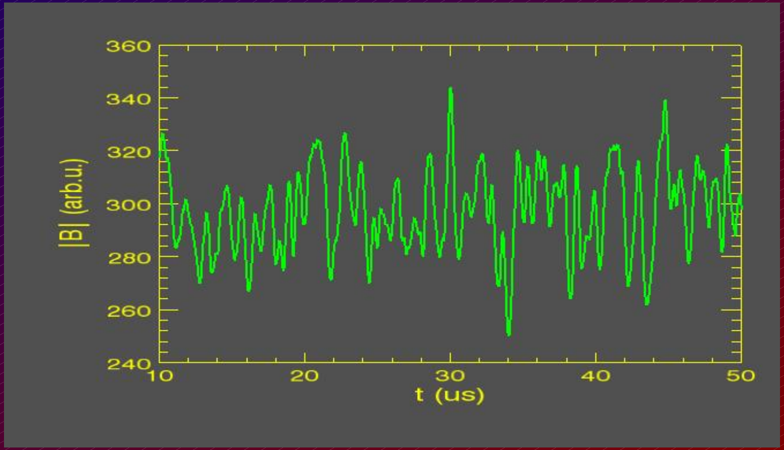
Two methods of signal detection with different level of the input noise (zero-mean Gaussian noise with std deviation σ)


Coherent Averaging of 100 chirps


Incoherent Averaging of 100 chirps

σ = 100
SNR = -23 dB


Coherent Averaging of 100 chirps


Incoherent Averaging of 100 chirps

σ = 150
SNR = -25 dB

SNR and Laser Safety Limit

Maximum Permissible Exposure (1064nm, 10-7 – 10s):

$$E_{MPE} = 5.5 \cdot T^{1/4} [J/cm^2]$$

SNR of Matched Filter Processing:

$$SNR_{MF} \sim E_s \sim A_s^2 T_{ch}$$

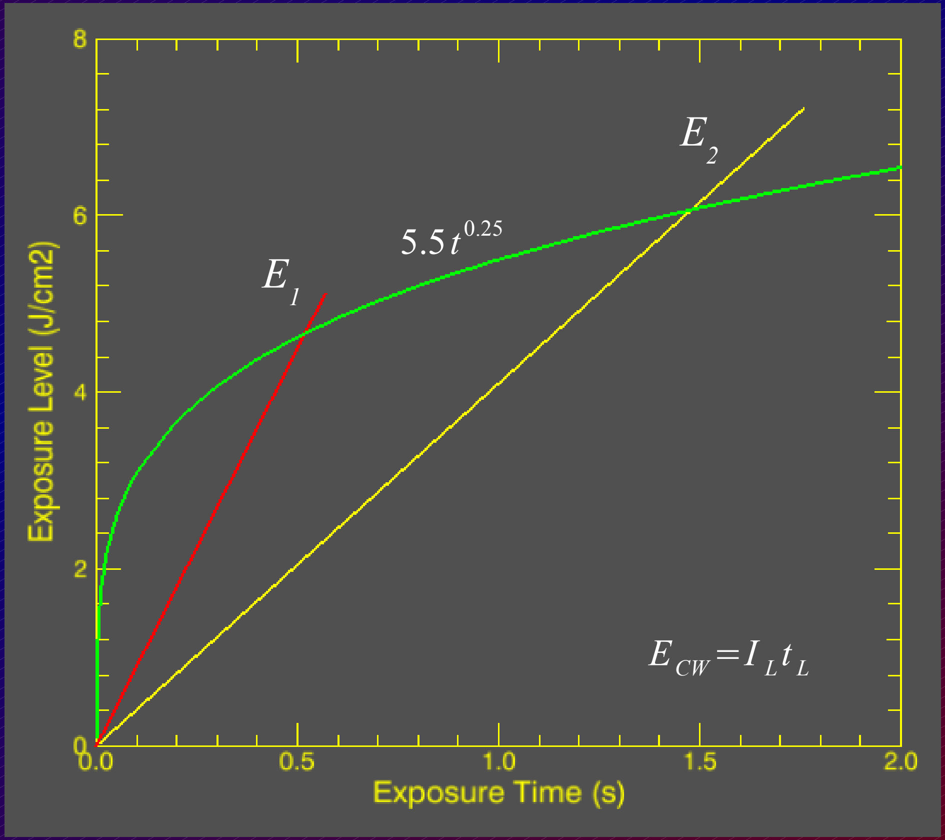
Signal Amplitude:

$A_s \sim I_L$ - Laser Irradiance [W/cm^2]

Assuming: $I_L = I_{MPE} = 5.5 \cdot T_{ch}^{-3/4}$

Then: $SNR \sim I_{MPE}^2 \cdot T_{ch} \sim T_{ch}^{-1/2}$

For $I = I_{MPE}$ shorter chirp duration is expected to give higher SNR



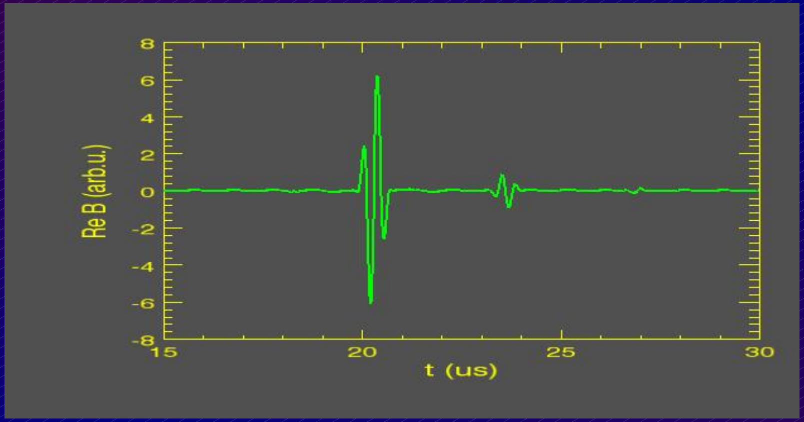
🟥 P = 1.76 W; 🟨 P = 0.8 W (diam = 5 mm)

🟩 Laser safety curve

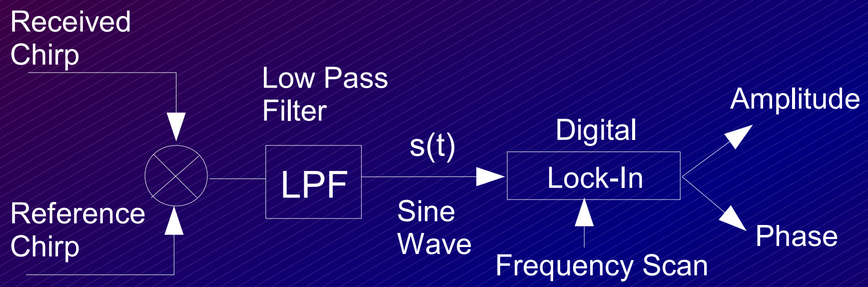
Amplitude and Phase of Correlation Processing

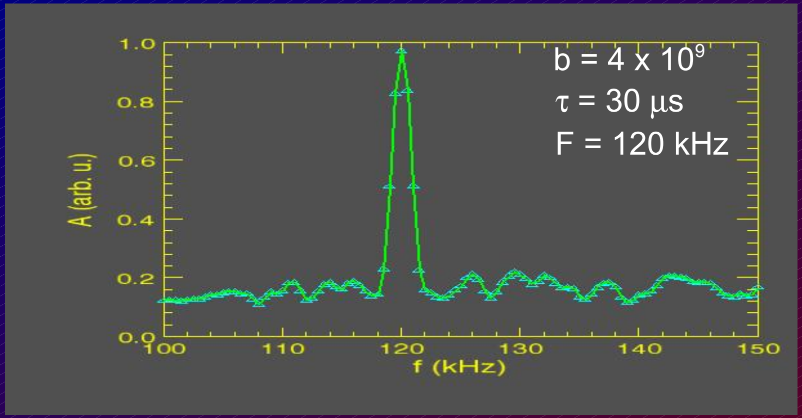
Correlation function of a linear frequency modulated chirp

$$B(\tau)=\frac{A^2 T_{ch}}{2}\,\frac{\sin\left[\frac{\pi m \tau}{T_{ch}}\left(1-\frac{\tau}{T_{ch}}\right)\right]}{\pi m \tau / T_{ch}}\cos(\omega_0 \tau)$$



Heterodyne mixing (Stretch Processor):





Reference: $r(t)=r_0 \exp[i(2\pi f_1 t+\pi b t^2)]$

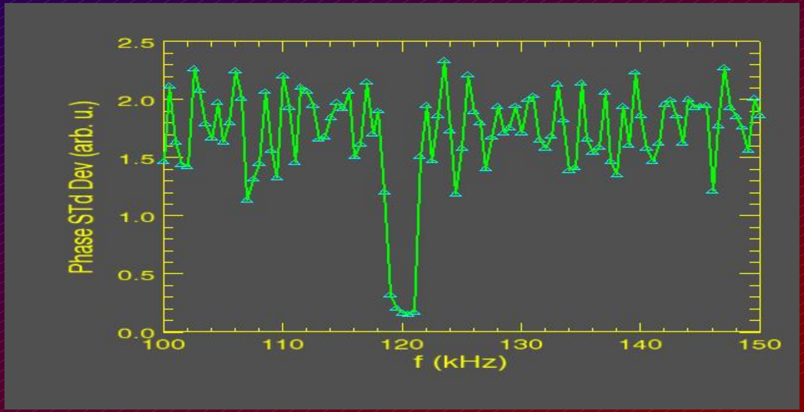
Delayed chirp: $s(t)=s_0 \exp[i(2\pi f_1 (t+\tau)+\pi b (t+\tau)^2)]$

Downshifted signal:

$$V(t)=s(t)*r(t)\sim r_0 s_0 \exp[i(2\pi b \tau t+2\pi(f_1 \tau+\frac{b}{2}\tau^2))]$$

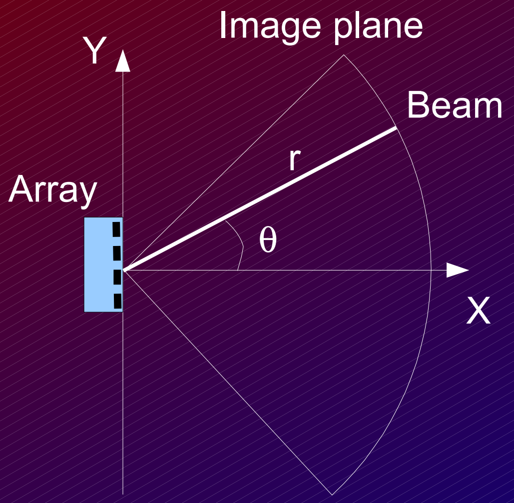
Sine wave frequency: $F = b\tau$

Phase: $\Theta=2\pi(f_1\tau+\frac{b}{2}\tau^2)$



Phased Array Correlation Imaging

Correlation Phased Array – multichannel matched filter processing and beamforming in frequency domain



1) Array acquisition and FFT of signal matrix

$\tilde{B}_i(\omega)=\tilde{W}(\omega)\cdot\tilde{R}^*(\omega)\cdot\tilde{S}_i(\omega)$ - matrix N_e x N_t, N_t = 100k

2) Digital beamforming, i.e. spatial filtering by creating directional beams and beam steering

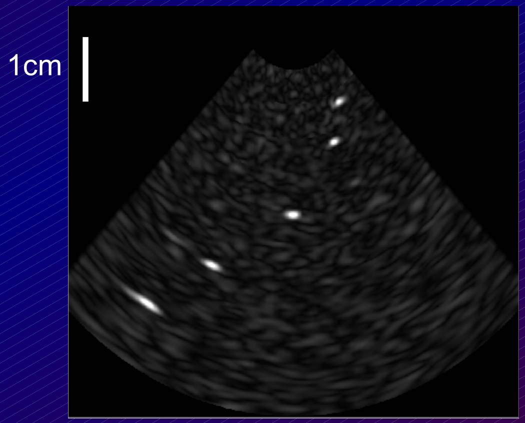
$$\tilde{U}(\omega,\theta)=\sum_{n=1}^{N_e} w_n\cdot\tilde{B}_n(\omega)\cdot\exp(-i\,\omega\,t_n(\theta)) \qquad t_n(\theta)=\frac{y_n}{c_a}\sin(\theta)+t_f$$

$k=\frac{\omega}{c_a}=k_x^2+k_y^2$ $\Rightarrow$ $\tilde{U}(k,\theta)$ - Spatial spectrum

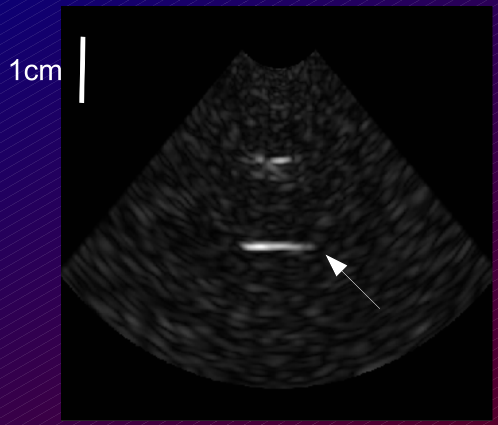
3) Backprojection:

$u(x,y)=FFT^{-1}[\tilde{U}]$ + Bilinear interpolation

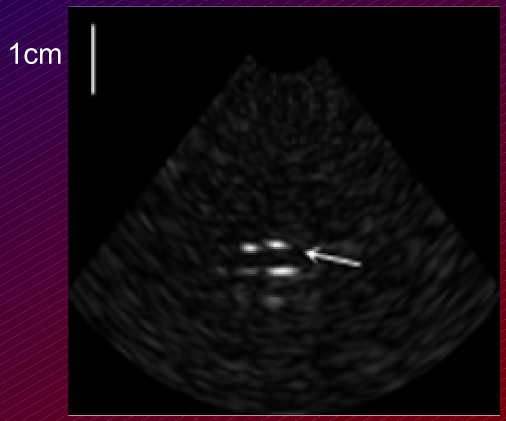
System PSF, SNR = -34dB



Optical inclusion in scattering phantom



Discrete chromophores in Intralipid



Conclusions

Depth-resolved PA imaging with CW laser sources is feasible using chirped optical excitation and correlation signal processing.

Correlation processing of coded PA response can significantly increase SNR (> 50 dB) and provide axial resolution < 1 mm.

High repetition rates (> 1 kHz) can easily implemented using inexpensive laser diodes.

To achieve maximum SNR performance multiple chirps must be averaged coherently in pre-processing and chirp duration should be set according to MPE.

Phase information can be potentially utilized for PA imaging using heterodyne mixing technique.

Phased array PA correlation imaging was demonstrated using conventional ultrasound array and frequency domain reconstruction algorithm.

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