MagnetoHemoDynamics in MRI devices

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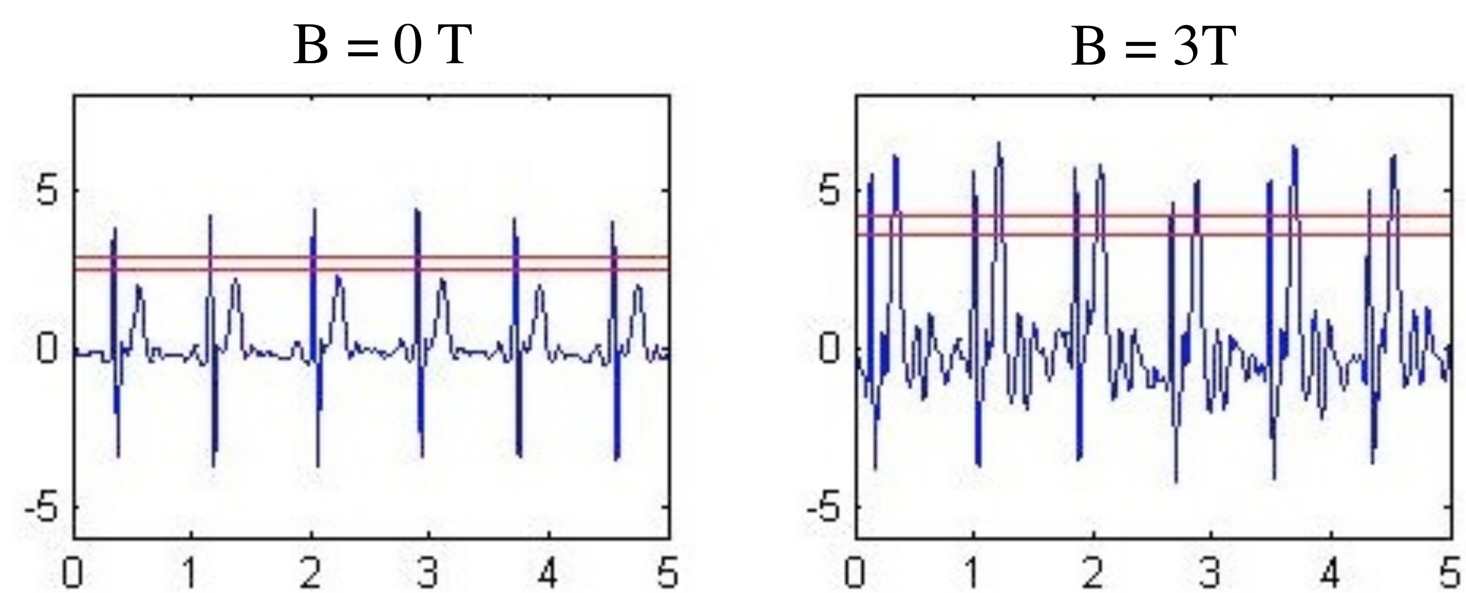
MHD Artifact in MRI

Philips MRI, 3 Teslas

- Permanent uniform magnetic field (typically 1.5 Teslas)
- Today: 3 Teslas (human), 10 Teslas (animals)
- Tomorrow : 10 Teslas (human), 17 Teslas (animals)

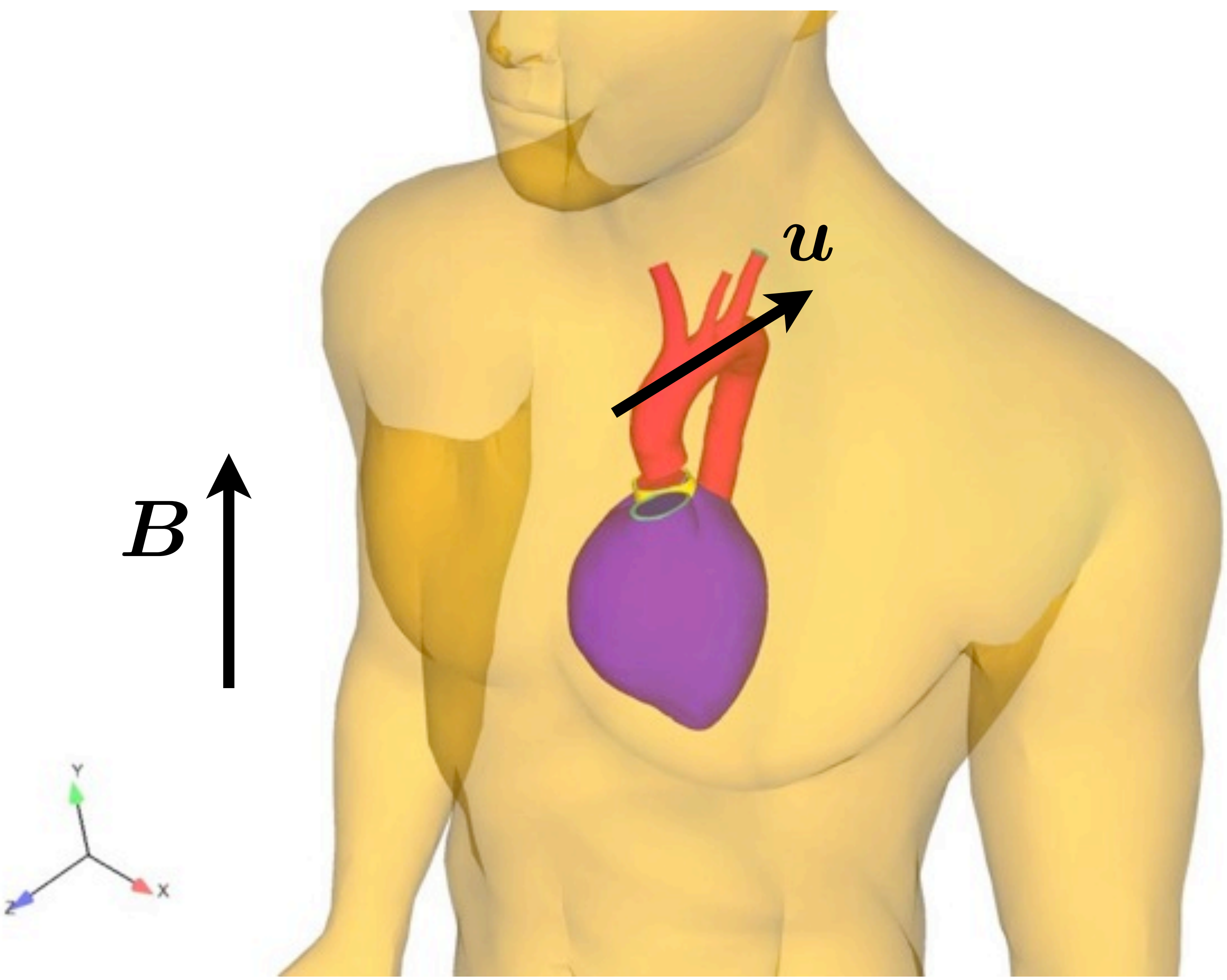
MHD Artifact in MRI

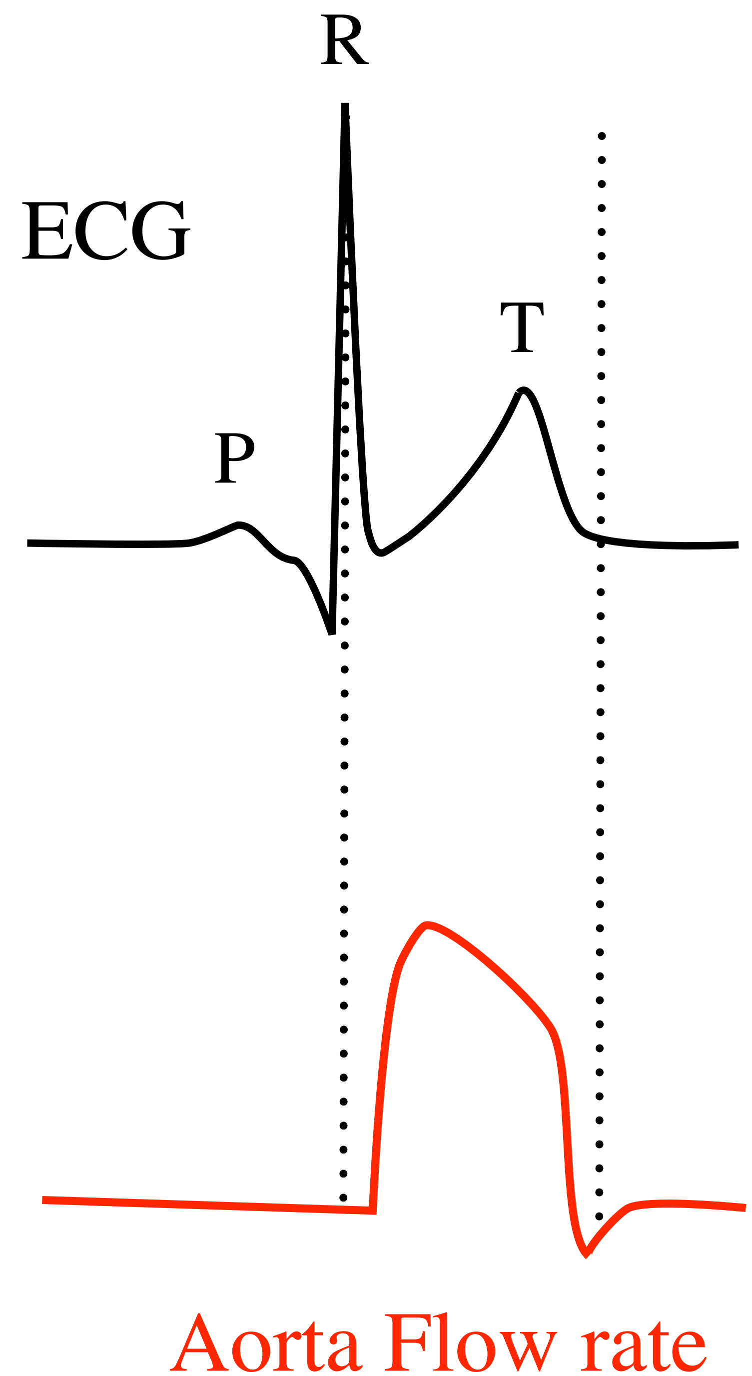
- Electrocardiograms (ECG): synchronize MRI sequences ("gating")
- Several known artifacts. Among them : MHD
- T wave may be as large as the R wave : may result in triggering problems for MR image acquisition



D. Abi Abdallah, A. Drochon, O. Fokapu(UTC)

MHD induced current



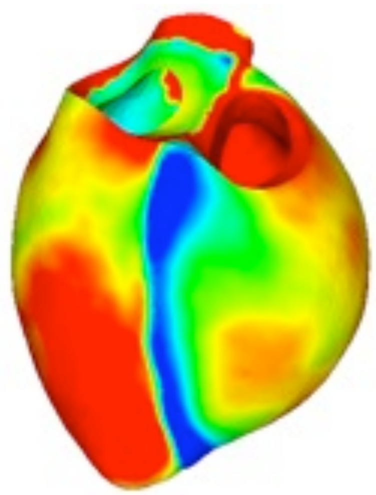


$$\boldsymbol{j} = \sigma(\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B})$$

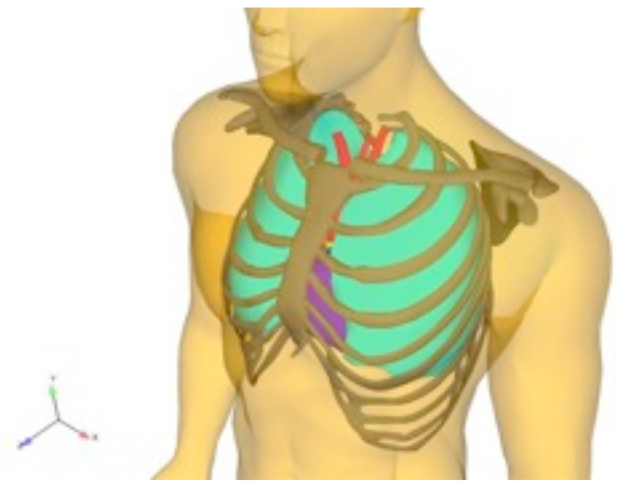
MHD and blood flows

- MHD and blood flows:
  - ★ *Kinouchi et al. (Bioelectromag., 1996), Tenforde (Prog. Biophys. Mol. Biol., 2005). Simplified 2D stationary computations : no fluid, no electrophysiology. Prediction 10 Teslas:*
    - 2200 mA/cm^2 in aorta, 115 mA/cm^2 in the heart
    - Normal cardiac current density 10-1000 mA/cm^2
    - Flow rate reduction : 5 %
- *In vivo* observation *Chakeres et al. (J. Mag. Res. Imag. 2003):* at 8 Teslas, no flow reduction, but consistent pressure increase
- MHD artifact
  - ★ *Gupta et al. (IEEE Tr. Biomed Eng., 2008) : analytical solution in a straight pipe + ECGSIM*
  - ★ *Nijm et al. (Comp. Card., 2008), Kainz et al. (Phys Med Biol, 2010)*

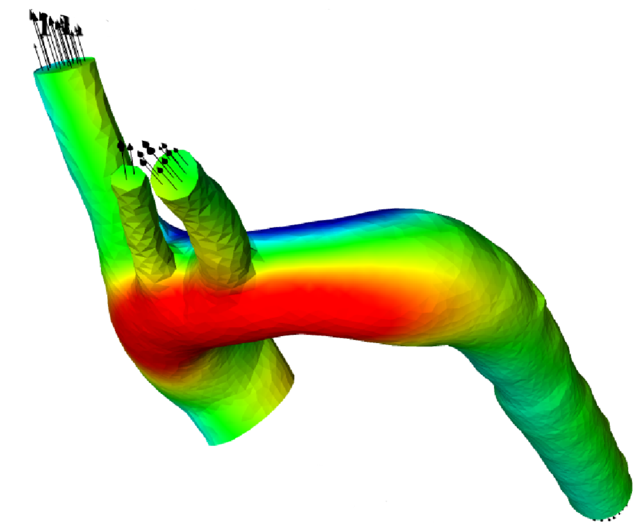
Roadmap



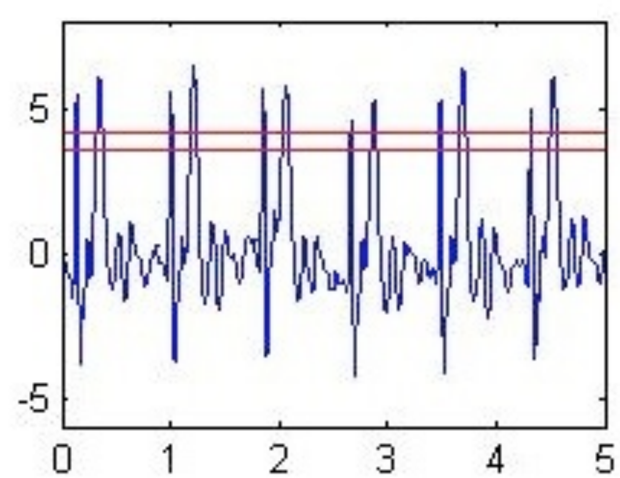
Electrophysiology in the heart



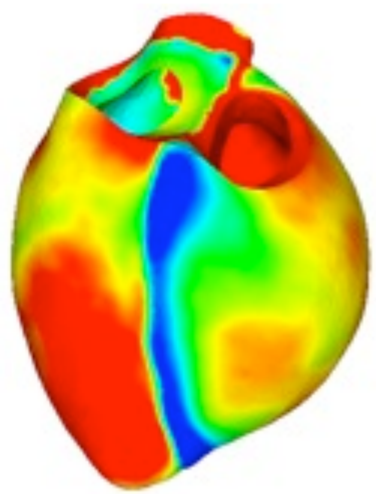
Electrostatic in the torso



MHD in the aorta



Roadmap



Electrophysiology in the heart

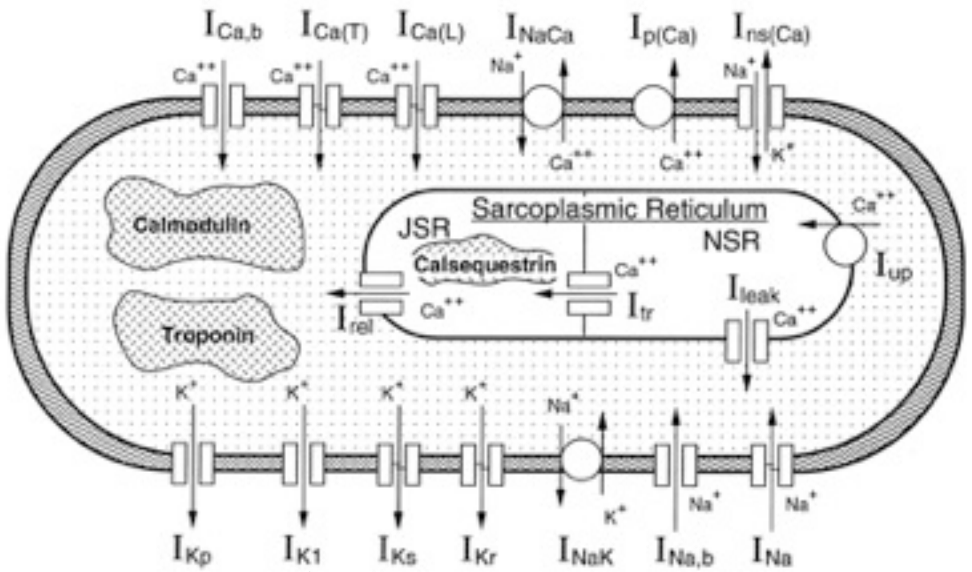
Electrostatic in the torso

MHD in the aorta

Cell scale

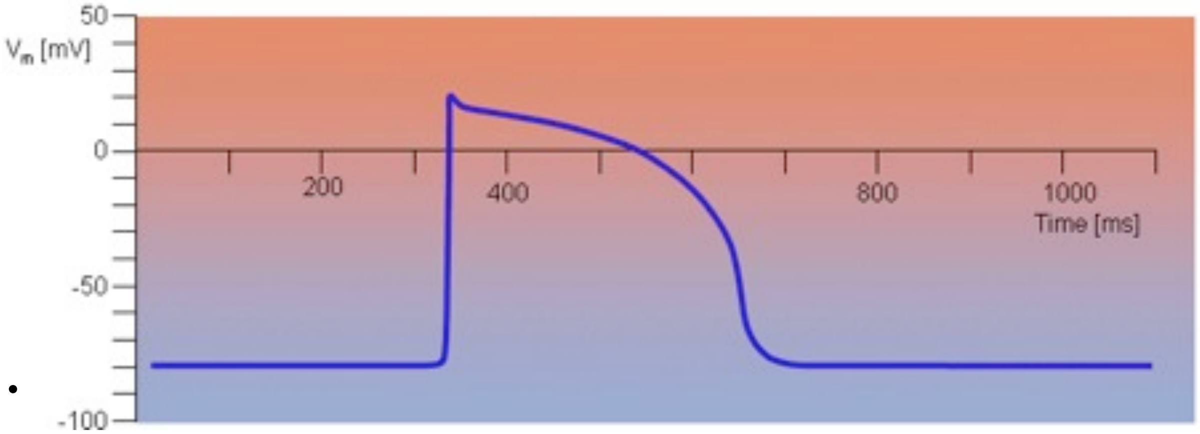
Physiological models

- In F. Sachse Springer 2004 :
  28 models of cardiac cells !
- Noble 60, Luo Rudy 91 & 94, ...
- Up to sixty state variables : very difficult to parametrize



Phenomelogical models

- The purpose is to reproduce the shape of the action potential:
- Typically 2 or 4 state variables
- FitzHugh 61, Nagumo et al. 62,
- Fenton-Carma 98, Mitchell-Schaeffer 03...



Tissue scale

- Bidomain equations :

$$\begin{cases} A_{\mathrm{m}}\left(C_{\mathrm{m}}\dfrac{\partial V_{\mathrm{m}}}{\partial t}+I_{\mathrm{ion}}(V_{\mathrm{m}},\boldsymbol{g})\right)-\mathrm{div}(\boldsymbol{\sigma}_{\mathrm{i}}\boldsymbol{\nabla}u_{\mathrm{i}})=A_{\mathrm{m}}I_{\mathrm{app}}, & \text{in} \quad \Omega_{\mathrm{H}} \\ \mathrm{div}(\boldsymbol{\sigma}_{\mathrm{e}}\boldsymbol{\nabla}u_{\mathrm{e}})=-\,\mathrm{div}(\boldsymbol{\sigma}_{\mathrm{i}}\boldsymbol{\nabla}u_{\mathrm{i}}), & \text{in} \quad \Omega_{\mathrm{H}} \\ \dfrac{\partial\boldsymbol{g}}{\partial t}+G(V_{\mathrm{m}},\boldsymbol{g})=0, & \text{in} \quad \Omega_{\mathrm{H}} \\ \boldsymbol{\sigma}_{\mathrm{i}}\boldsymbol{\nabla}u_{\mathrm{i}}\cdot\boldsymbol{n}=0, & \text{on} \quad \Gamma_{\mathrm{epi}} \\ \boldsymbol{\sigma}_{\mathrm{e}}\boldsymbol{\nabla}u_{\mathrm{e}}\cdot\boldsymbol{n}=0, & \text{on} \quad \Gamma_{\mathrm{epi}} \end{cases}$$

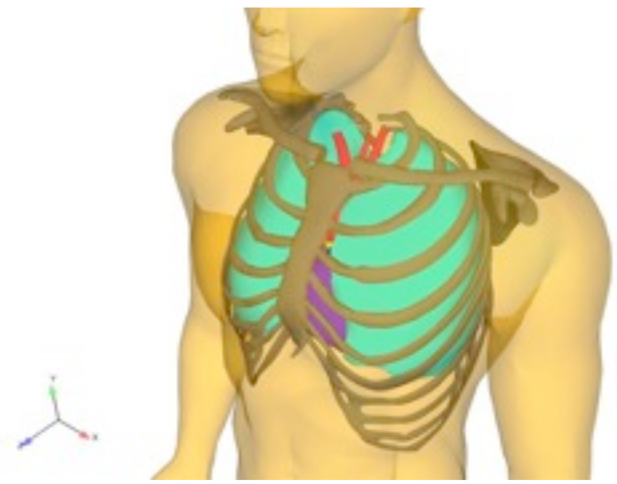
- Anisotropic conductivity

$$\boldsymbol{\sigma}_{\mathrm{i},e}(x)=\sigma^{\mathrm{t}}_{\mathrm{i},e}I+(\sigma^{\mathrm{l}}_{\mathrm{i},e}-\sigma^{\mathrm{t}}_{\mathrm{i},e})\boldsymbol{a}(x)\otimes\boldsymbol{a}(x)$$

- If the anisotropy is the same in both media : mono-domain equations

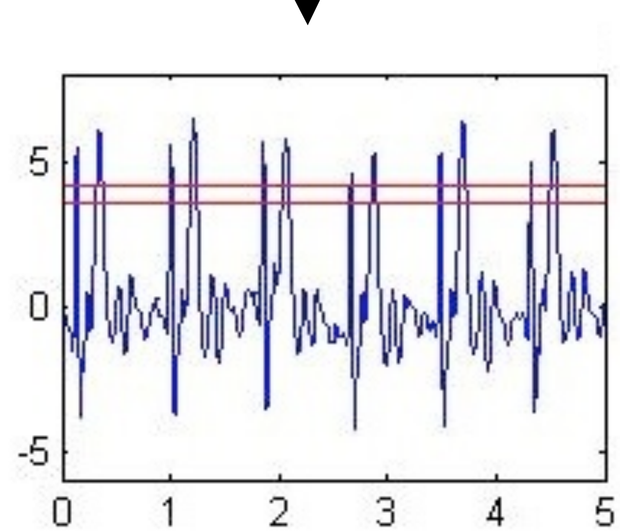
Roadmap

Electrophysiology in the heart

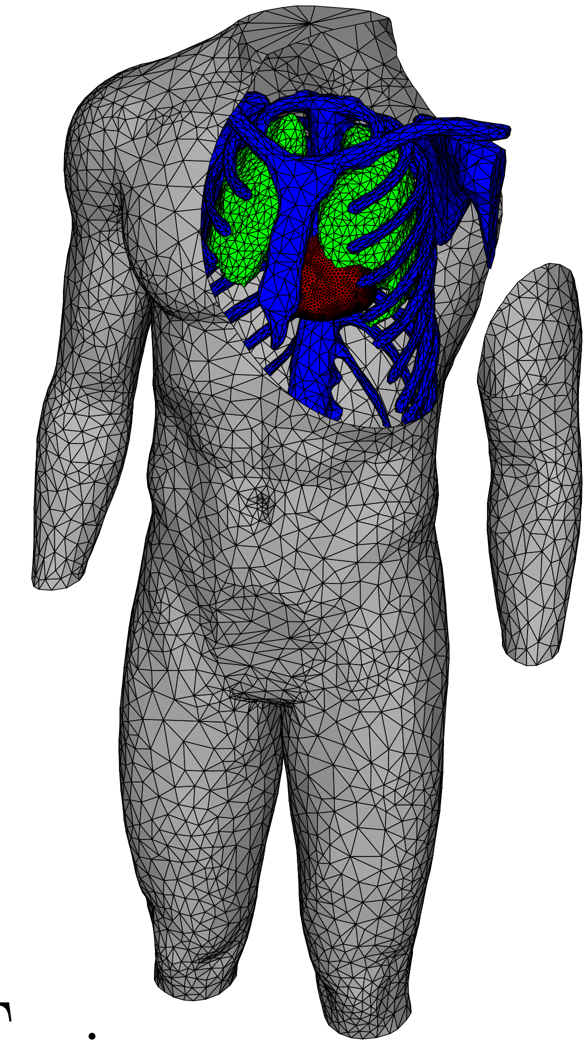


Electrostatic in the torso

MHD in the aorta



Heart-torso coupling



- Torso: passive conductor

$$\begin{cases} \mathrm{div}(\boldsymbol{\sigma}_\mathrm{T} \boldsymbol{\nabla} u_\mathrm{T}) = 0, & \text{in} \quad \Omega_\mathrm{T} \\ \boldsymbol{\sigma}_\mathrm{T} \boldsymbol{\nabla} u_\mathrm{T} \cdot \boldsymbol{n}_\mathrm{T} = 0, & \text{on} \quad \Gamma_\mathrm{ext} \end{cases}$$

- Strong coupling conditions:

$$\begin{cases} u_\mathrm{e} = u_\mathrm{T}, & \text{on} \quad \Gamma_\mathrm{epi} \\ \boldsymbol{\sigma}_\mathrm{e} \boldsymbol{\nabla} u_\mathrm{e} \cdot \boldsymbol{n} = \boldsymbol{\sigma}_\mathrm{T} \boldsymbol{\nabla} u_T \cdot \boldsymbol{n}, & \text{on} \quad \Gamma_\mathrm{epi} \end{cases}$$

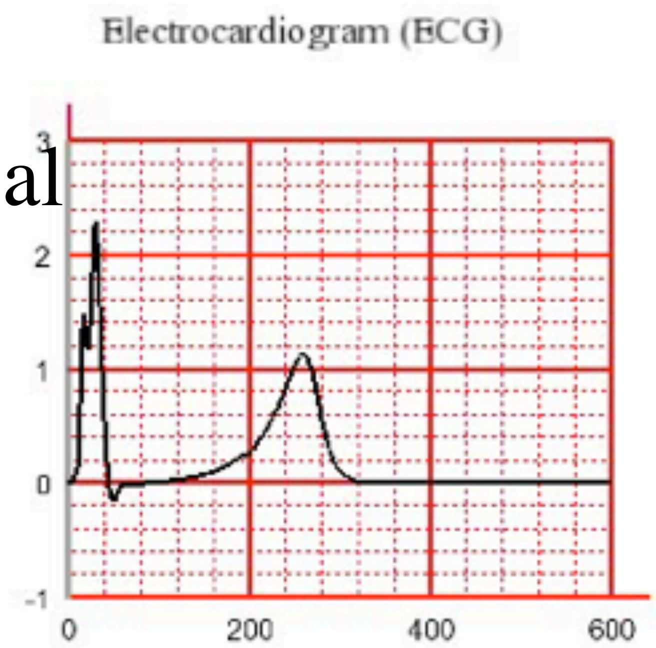
- Weak coupling conditions:

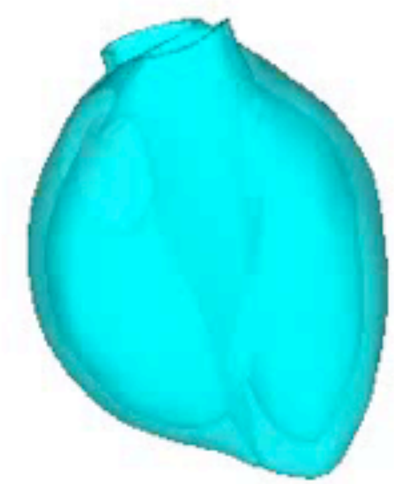
$$\begin{cases} u_\mathrm{e} = u_\mathrm{T}, & \text{on} \quad \Gamma_\mathrm{epi} \\ \boldsymbol{\sigma}_\mathrm{e} \boldsymbol{\nabla} u_\mathrm{e} \cdot \boldsymbol{n} = 0, & \text{on} \quad \Gamma_\mathrm{epi} \end{cases}$$

(Krassowsca-Neu 94, Clements et al. 04, Pierre 05, Lines et. al 06,...)

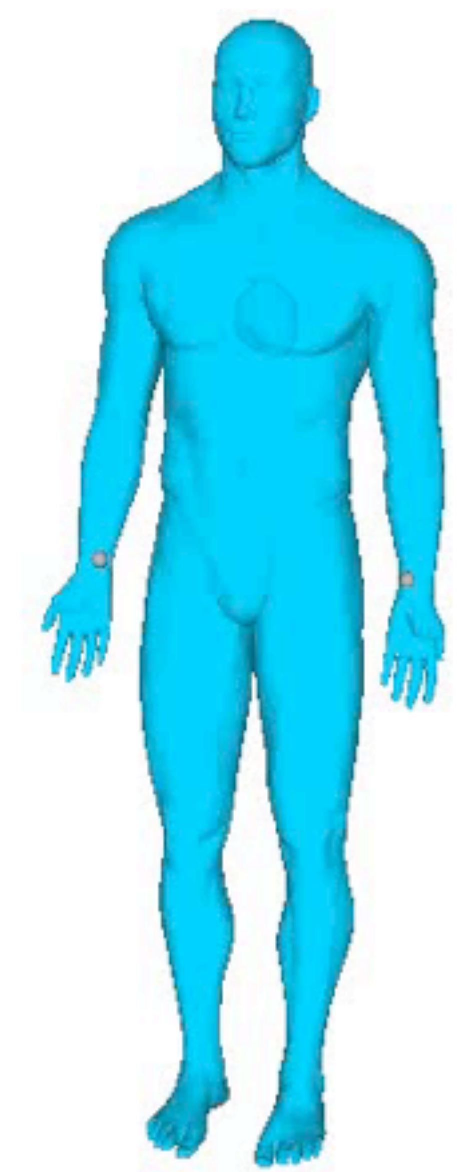
Body surface potential

- Strong / Weak coupling with the torso
- Monodomain / Bidomain equations & fibers
- Mitchell-Schaeffer phenomenological model
- 3 different cells
- Careful initialization of the simulation





extra-cellular potential

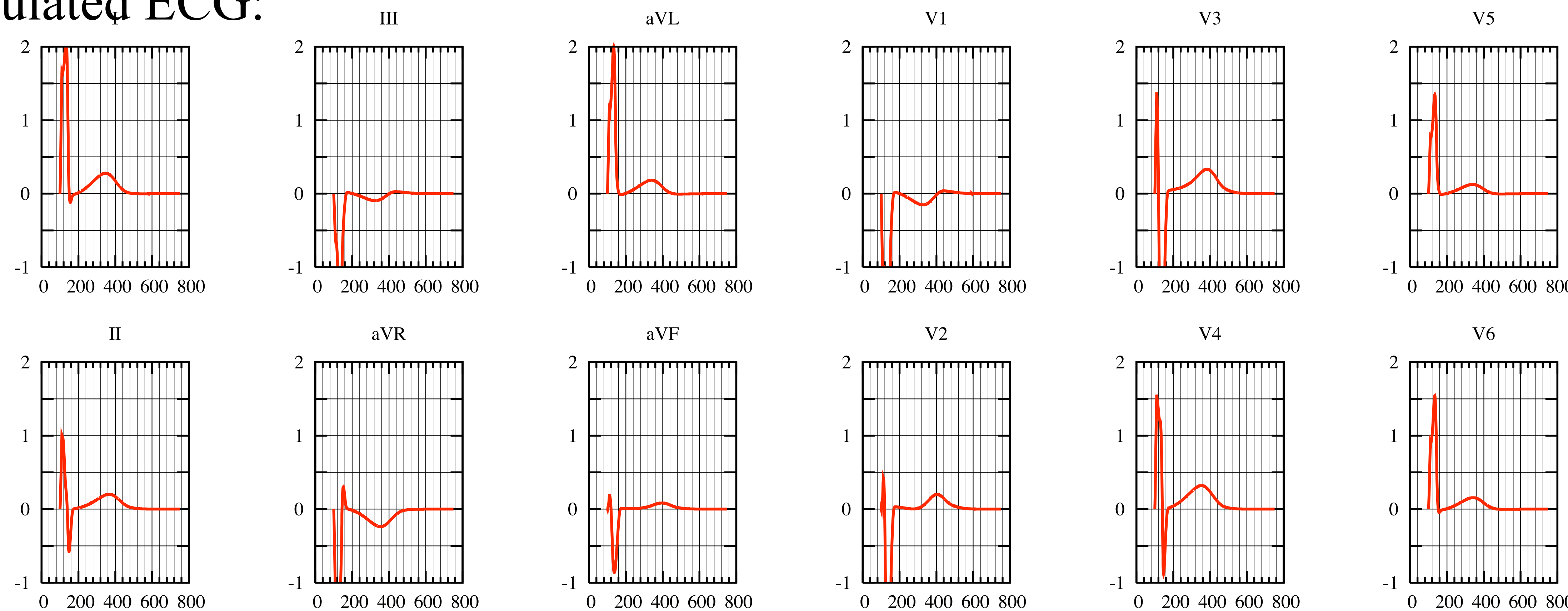


body surface potential

Boulakia, Fernández, Cazeau, JFG, Zemzemi, *Annals Biomed Engng. 2010*

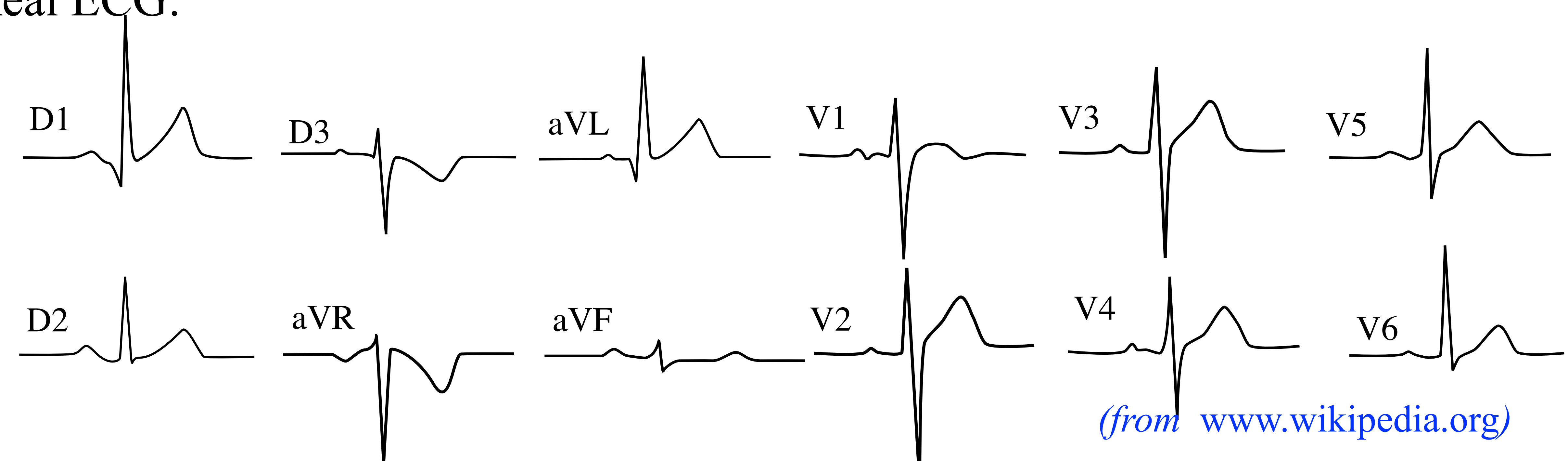
12-lead ECG

- Simulated ECG:



Fernández, Boulakia, Cazeau, JFG, Zemzemi, *Annals Biomed Engng*. *2010*

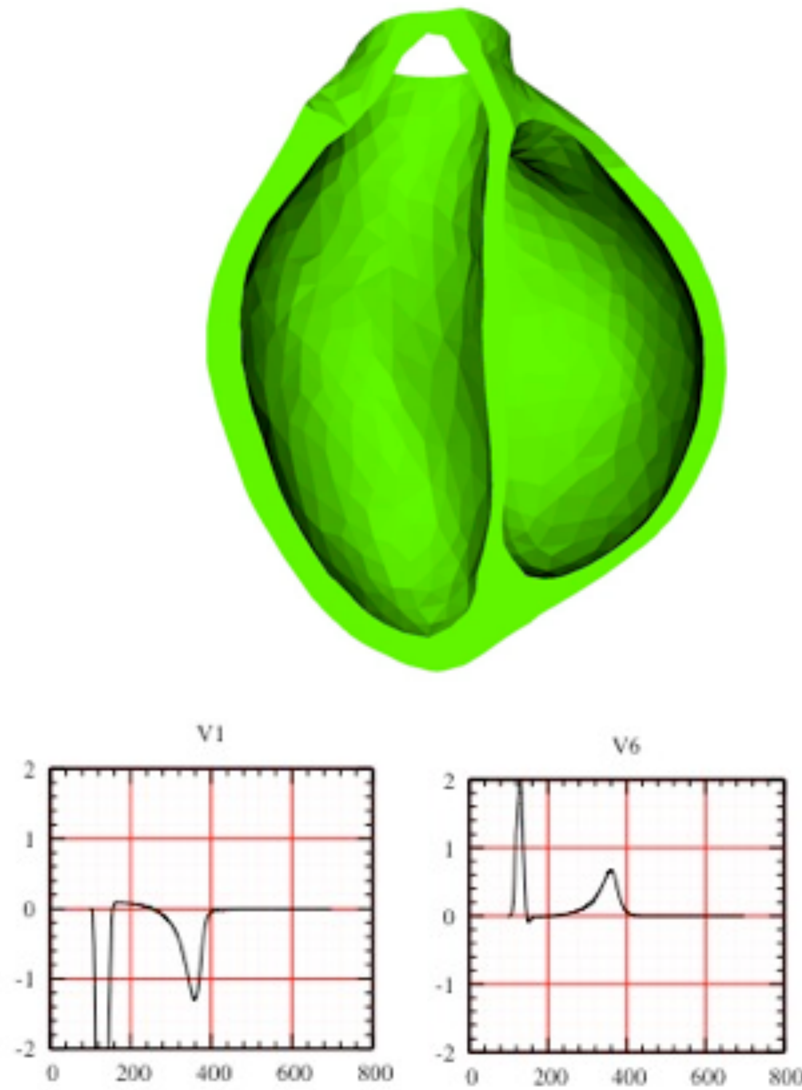
- Real ECG:



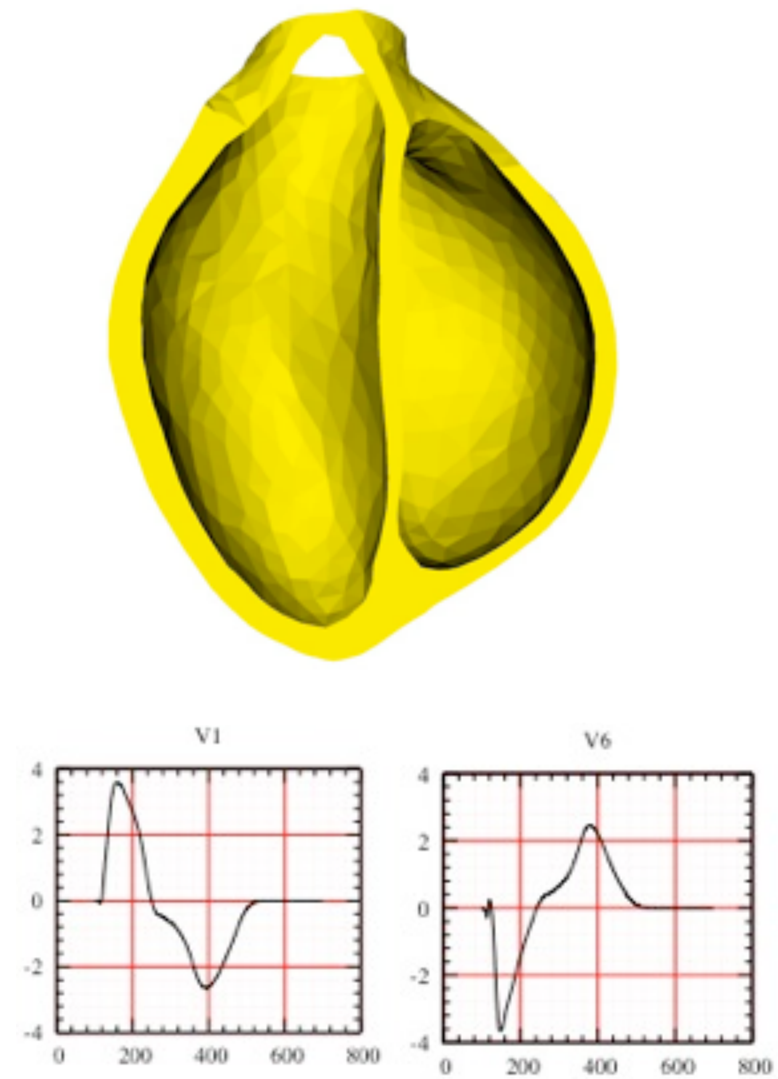
(from www.wikipedia.org*)*

Example 1: Electro-mechanical coupling

Healthy case

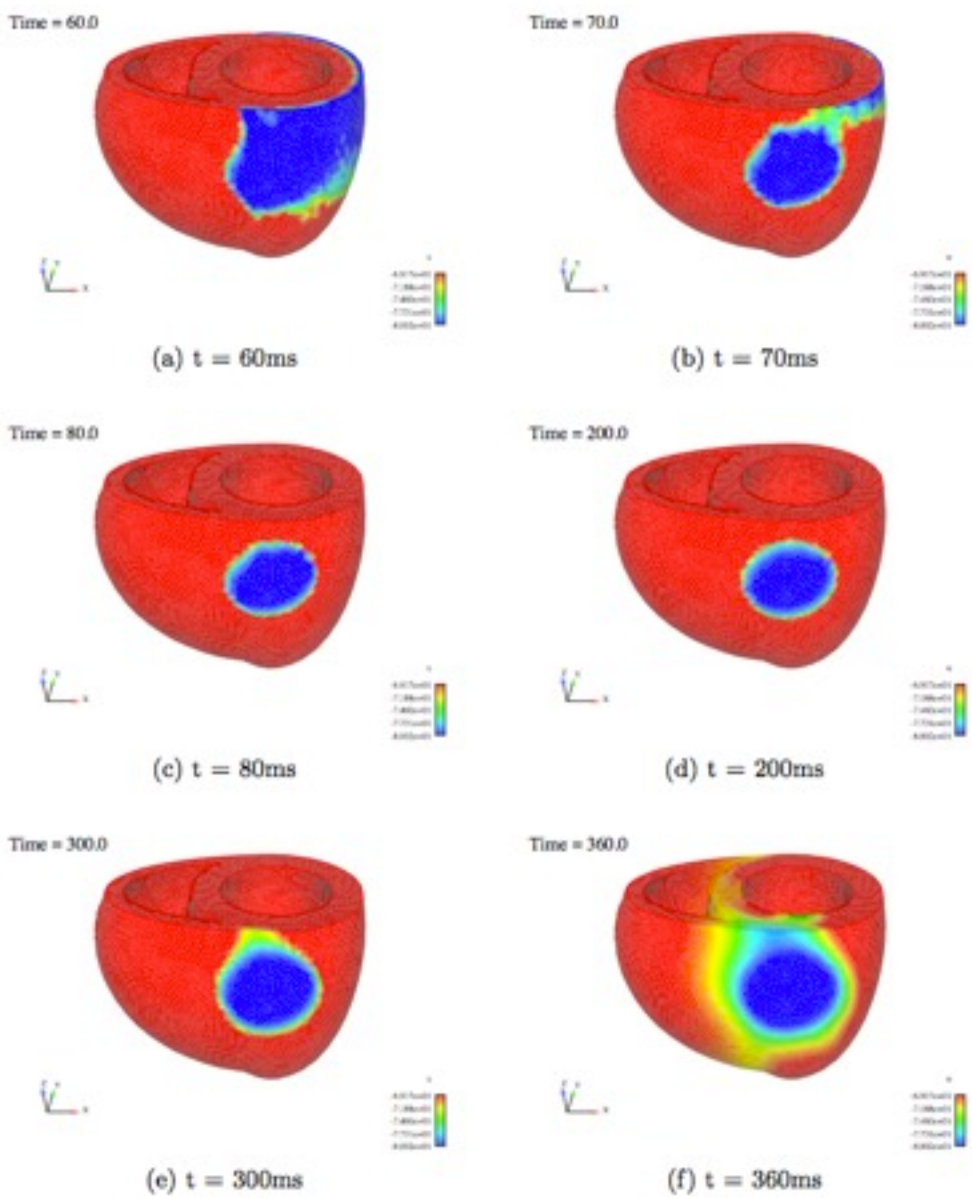


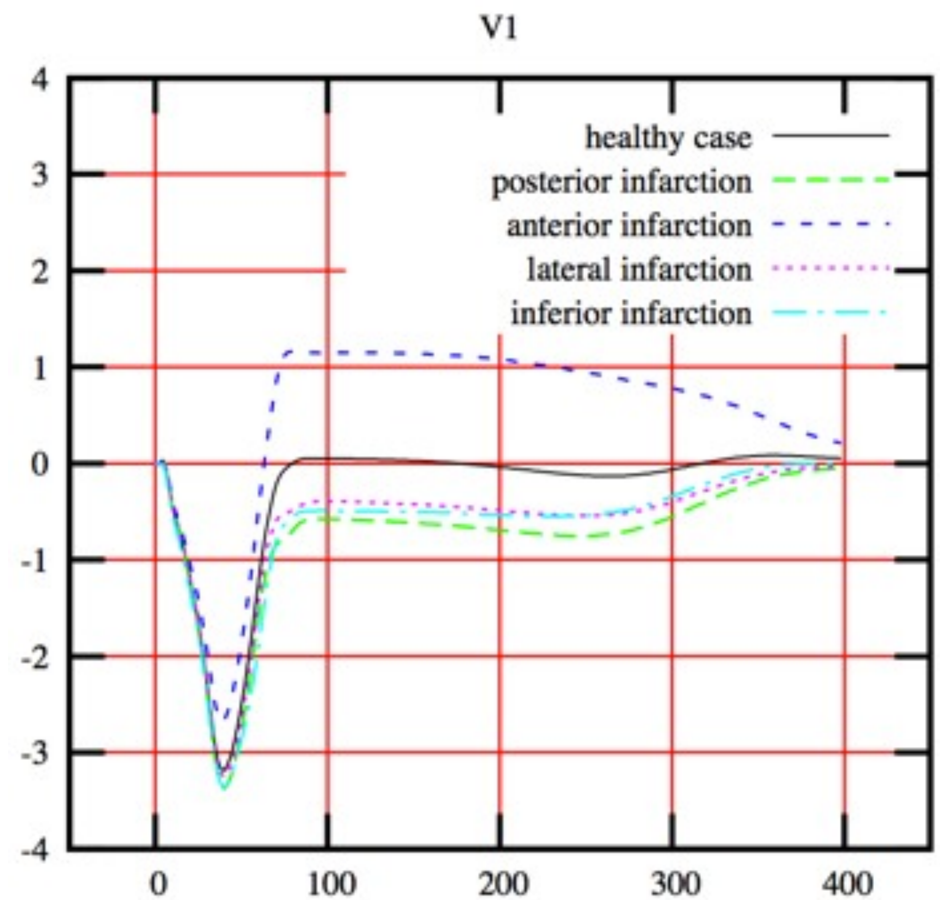
Right bundle branch block



Chapelle, Fernández, JFG, Moireau, Sainte-Marie, Zemzemi, *FIMH 2009*

Example 2: infarct





- **Anterior** infarct : ST elevation
- **Posterior** infarct: ST depression

Example:Anterior infarct

Simulation : E. Schenone & M. Boulakia

Statistical classification

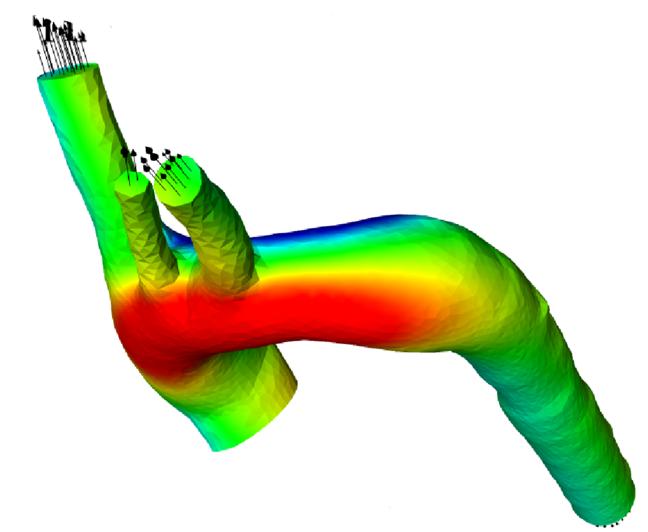
- **Prometeo** project *(F. Ieva & AM Paganoni, Politecnino di Milano)*
- Pilot analysis: database of
  - 25 normal ECG
  - 10 LBBB
  - 13 RBBB
- Statistical clustering...

- **Our normal, LBBB and RBBB ecg are correctly classified !**

Roadmap

Electrophysiology in the heart

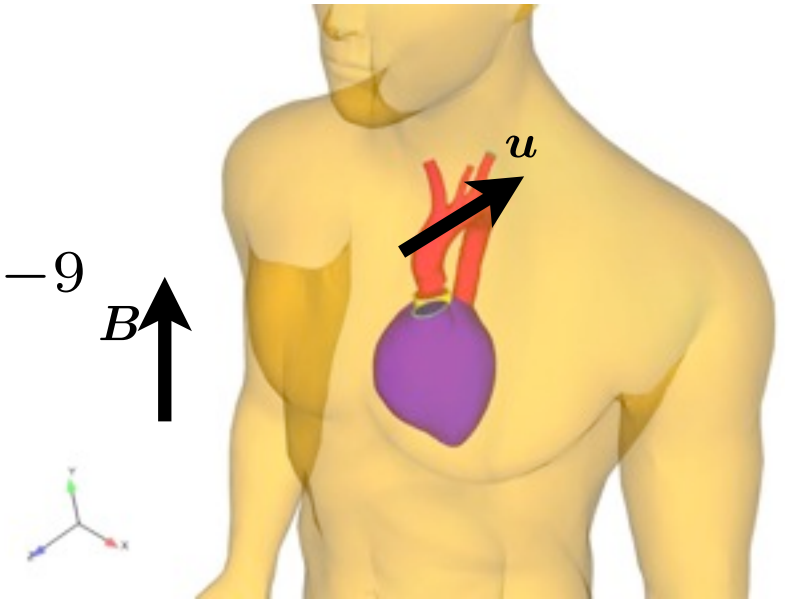
Electrostatic in the torso



MHD in the aorta

MHD in blood flows

- Nondimensional parameters:
  - Magnetic Reynolds: $Rm = \mu_0 \sigma_0 U_0 L_0 \approx 10^{-9}$
  - Hartman number: $Ha = B_0 L_0 \sqrt{\dfrac{\sigma_0}{\eta}} \approx 0.1$

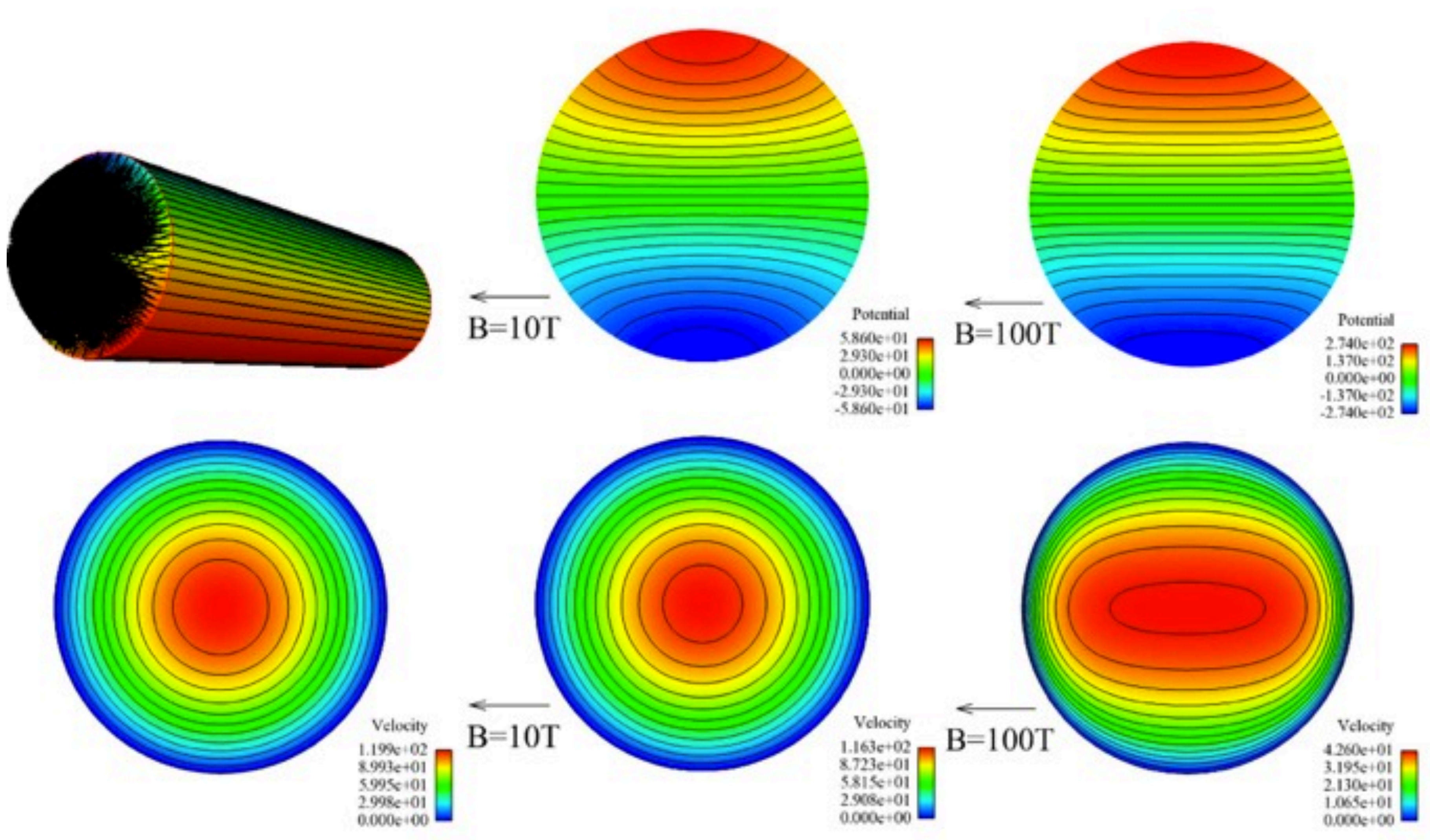


- Quasi-static approximation ($\partial_t \boldsymbol{B} \approx 0$): $\boldsymbol{E} = -\boldsymbol{\nabla} \phi_a$
- Ohm law: $\boldsymbol{j} = \sigma(\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B}) = \sigma(-\boldsymbol{\nabla} \phi_a + \boldsymbol{u} \times \boldsymbol{B})$

$$
\left\{
\begin{array}{rcl}
\dfrac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} - \dfrac{1}{Re} \Delta \boldsymbol{u} + \boldsymbol{\nabla} p & = & -\dfrac{Ha^2}{Re} \boldsymbol{\nabla} \Phi_a \times \boldsymbol{B} + \dfrac{Ha^2}{Re} (\boldsymbol{u} \times \boldsymbol{B}) \times \boldsymbol{B}, \\
\mathrm{div}\, \boldsymbol{u} & = & 0, \\
\mathrm{div} \left(\dfrac{\sigma}{\sigma_0} \boldsymbol{\nabla} \Phi_a \right) & = & \mathrm{div} \left(\dfrac{\sigma}{\sigma_0} \boldsymbol{u} \times \boldsymbol{B} \right)
\end{array}
\right.
$$

$\sigma_{\mathrm{blood}} \approx 0.5 S/m$

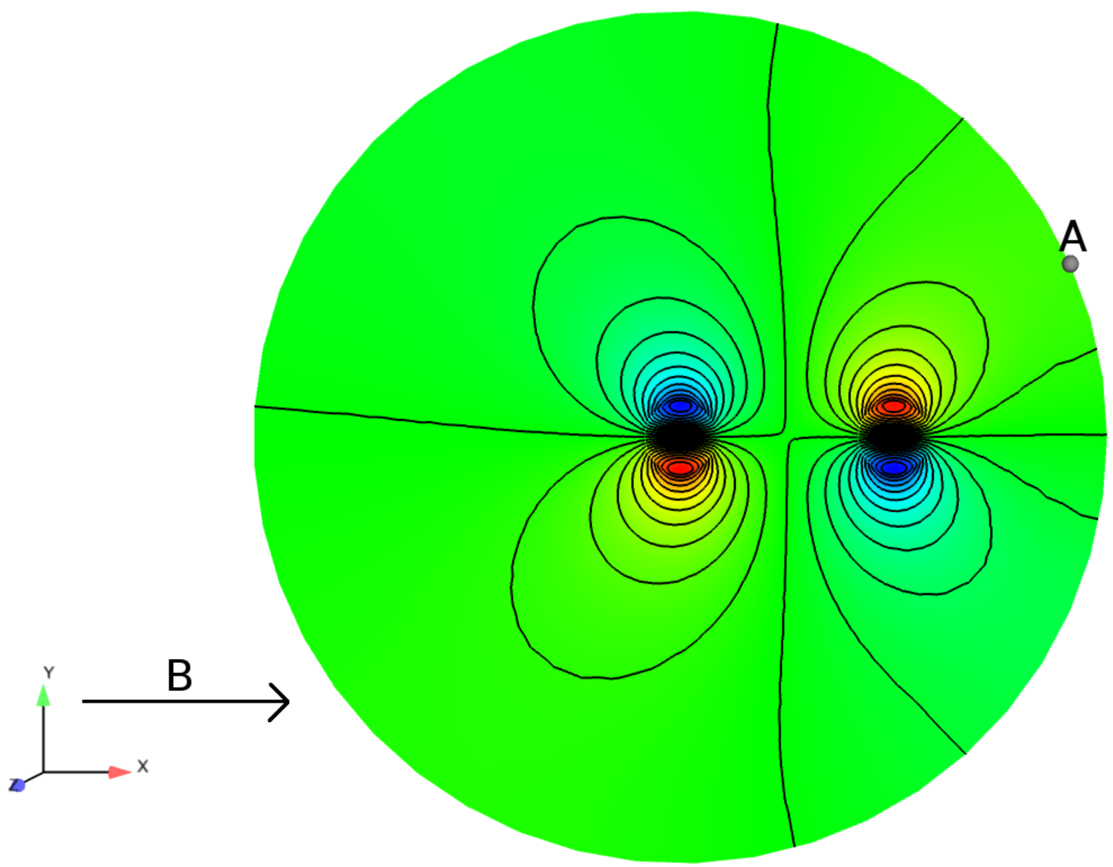
Code verification

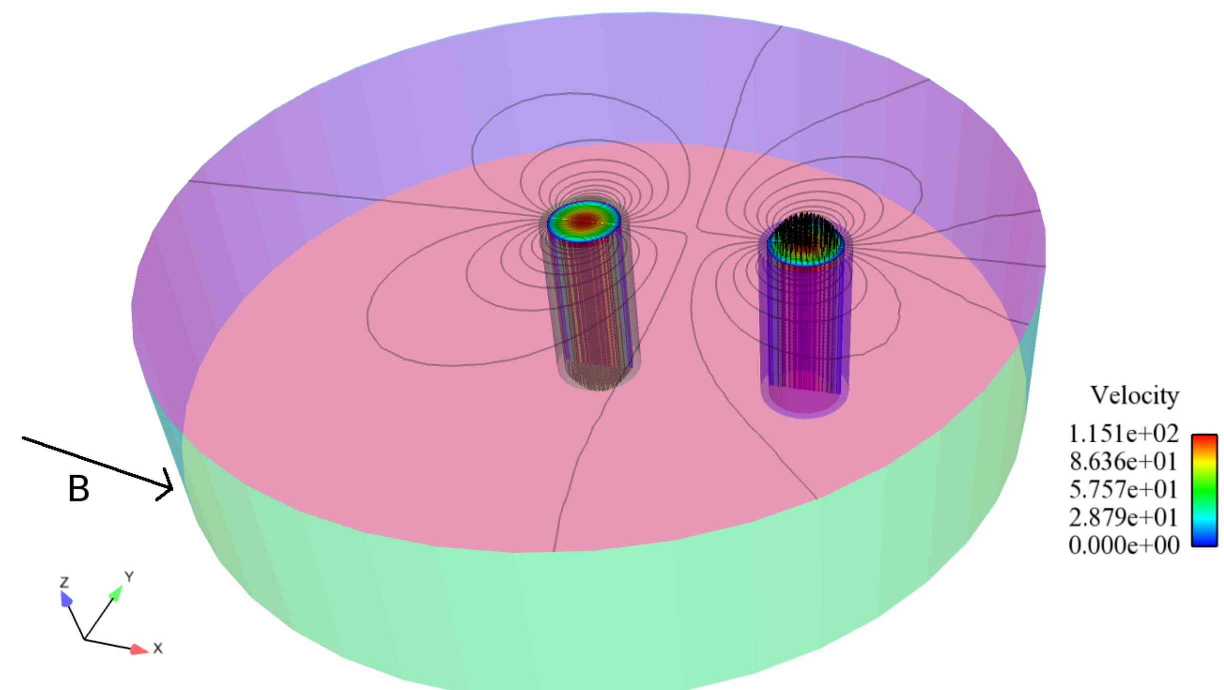


- Analytical solution of the full MHD equation (Bessel functions…)
- Gold (1962), Abi-Abdallah *et al*. (2009)

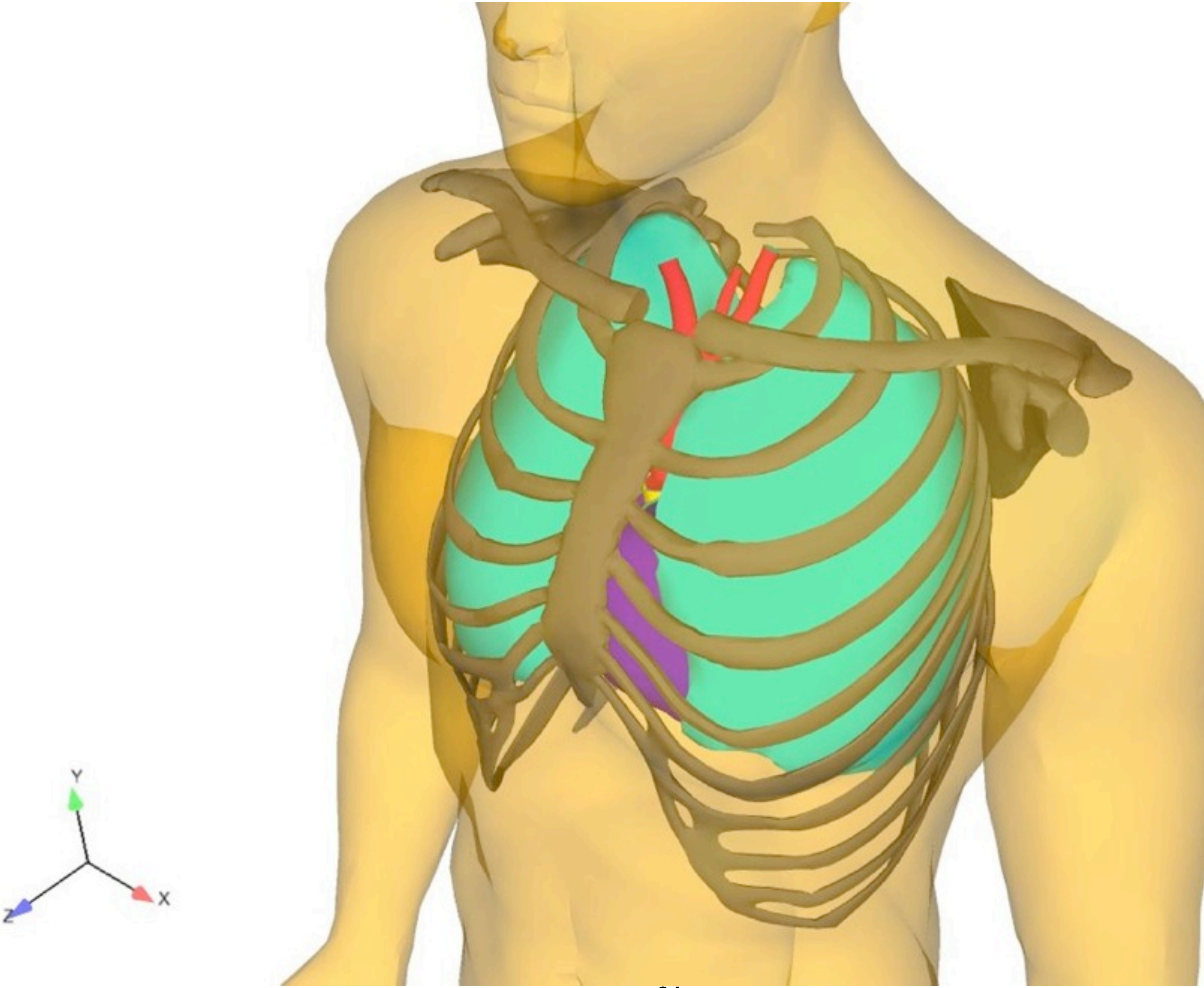
Code verification

- 3D test from a 2D benchmark proposed by *Tenforde et al. 1996*
- Excellent agreement with their results

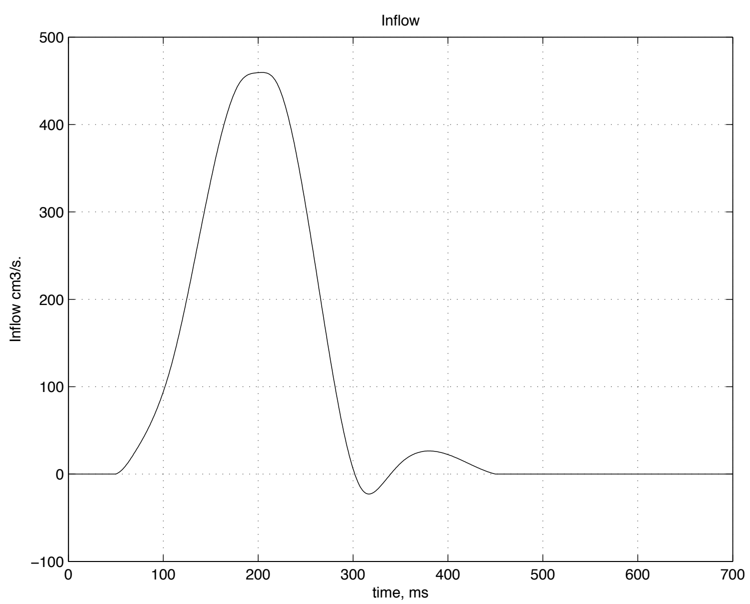




Computational domain



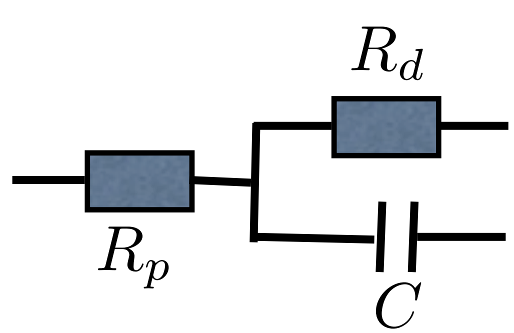
Inlet BC:

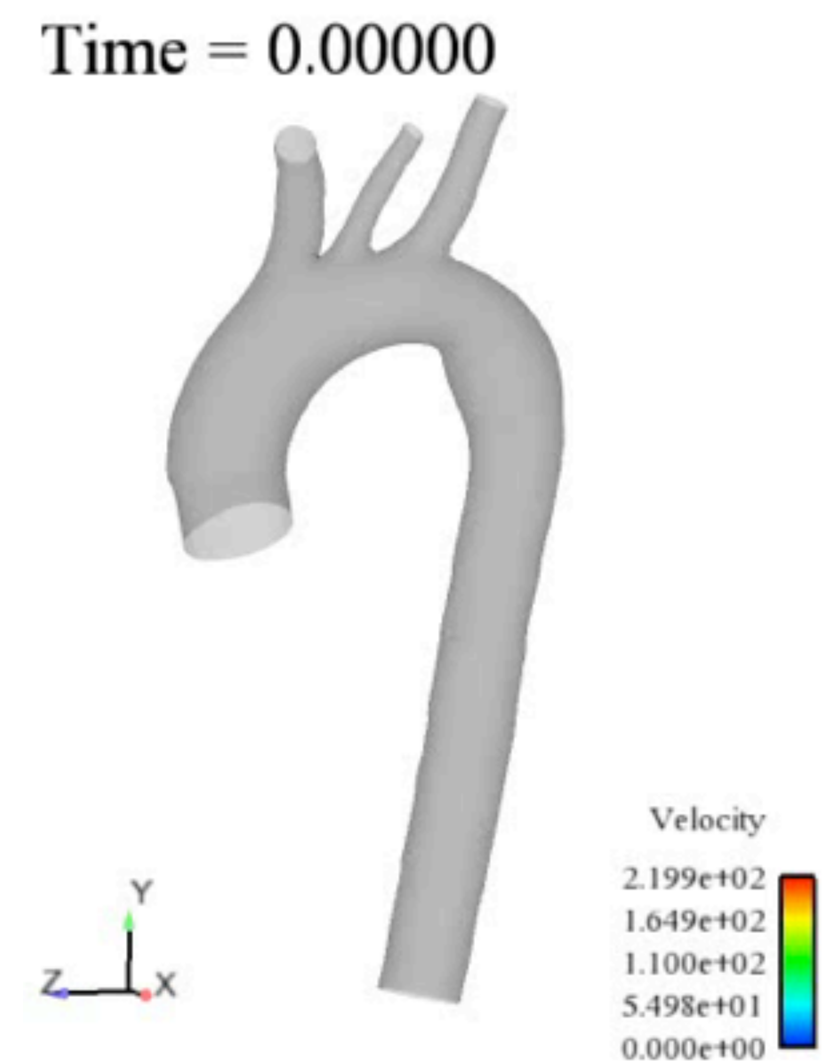


Flow rate (about 5L/min):

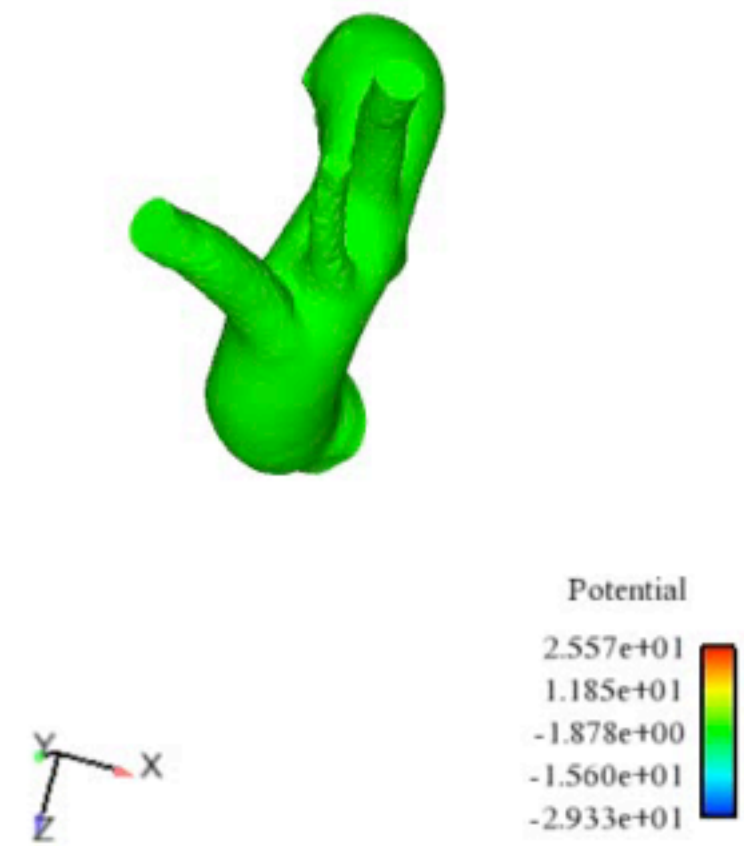
At the 4 Outlets:

3-element Windkessel





Velocity field

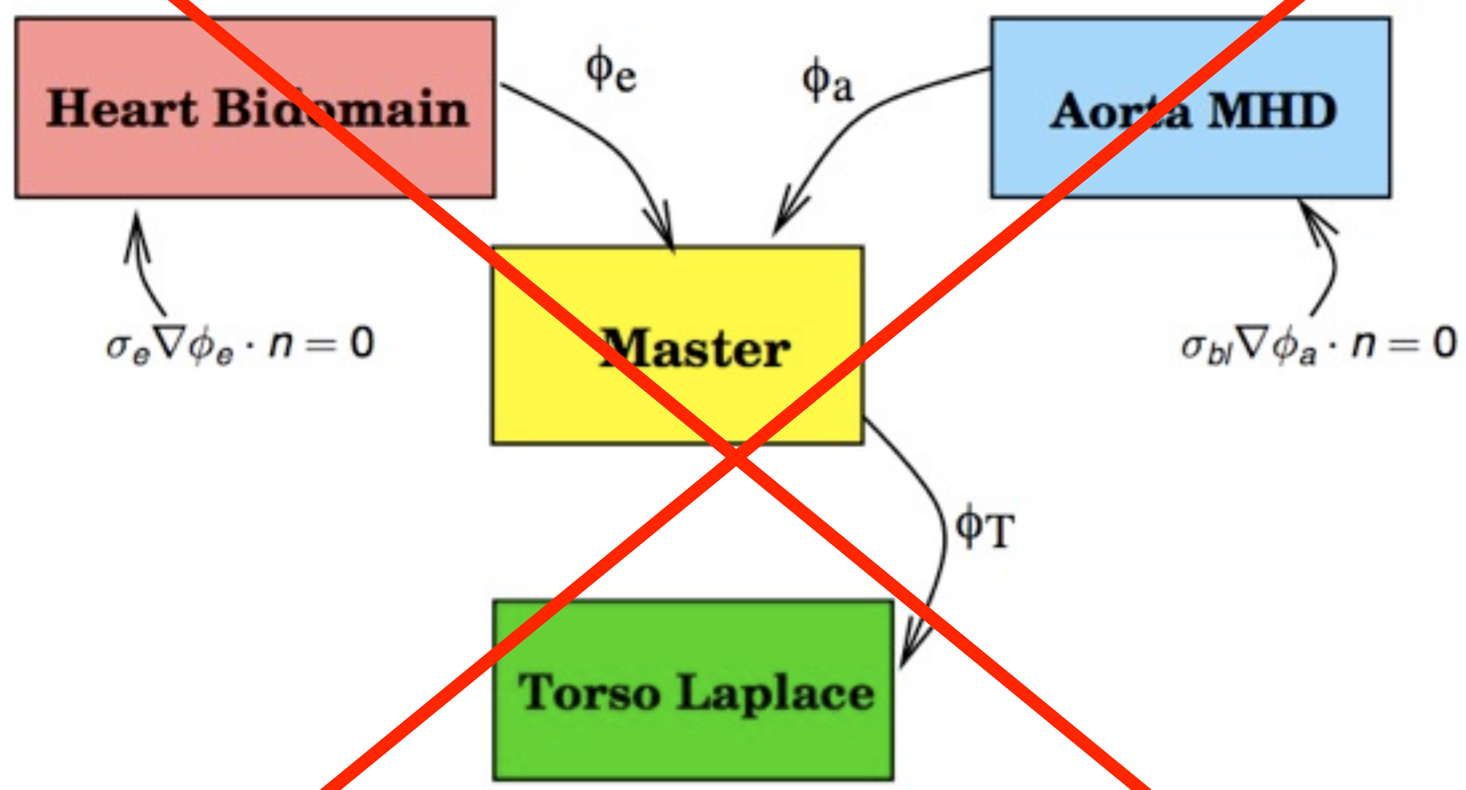


Potential

Aorta geometry: courtesy of C.A. Taylor

Coupling algorithm

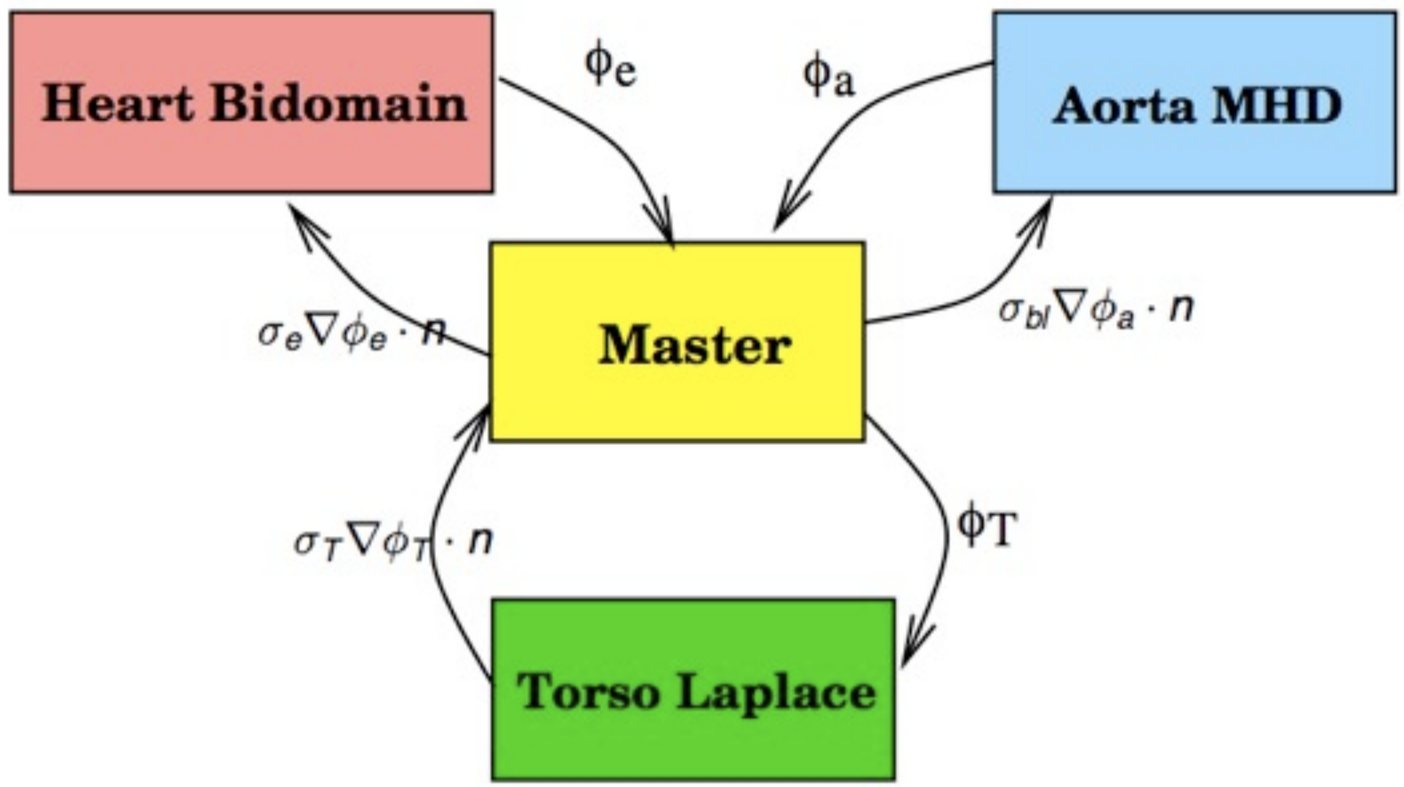
Weak coupling



$$\begin{cases} \boldsymbol{\sigma}_{\rm e} \boldsymbol{\nabla} \phi_{\rm e} \cdot \boldsymbol{n} = 0, & \text{on} \quad \partial\Omega_{\rm H}, \\ \boldsymbol{\sigma}_{\rm a} \boldsymbol{\nabla} \phi_{\rm a} \cdot \boldsymbol{n} = 0, & \text{on} \quad \partial\Omega_{\rm a}, \\ \phi_{\rm T} = \phi_{\rm e}, & \text{on} \quad \partial\Omega_{\rm H}, \\ \phi_{\rm T} = \phi_{\rm a}, & \text{on} \quad \partial\Omega_{\rm a}. \end{cases}$$

Coupling algorithm

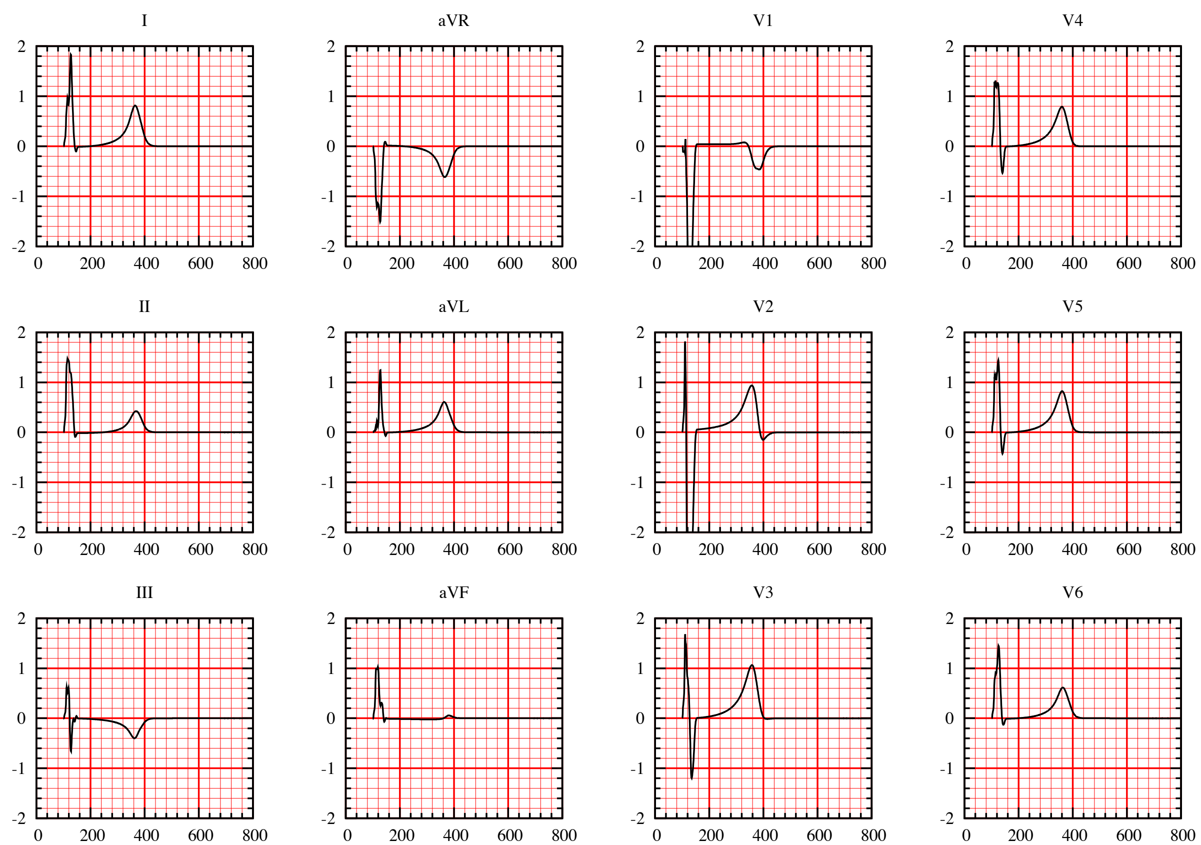
Strong coupling (relaxed Dirichlet-Neumann)



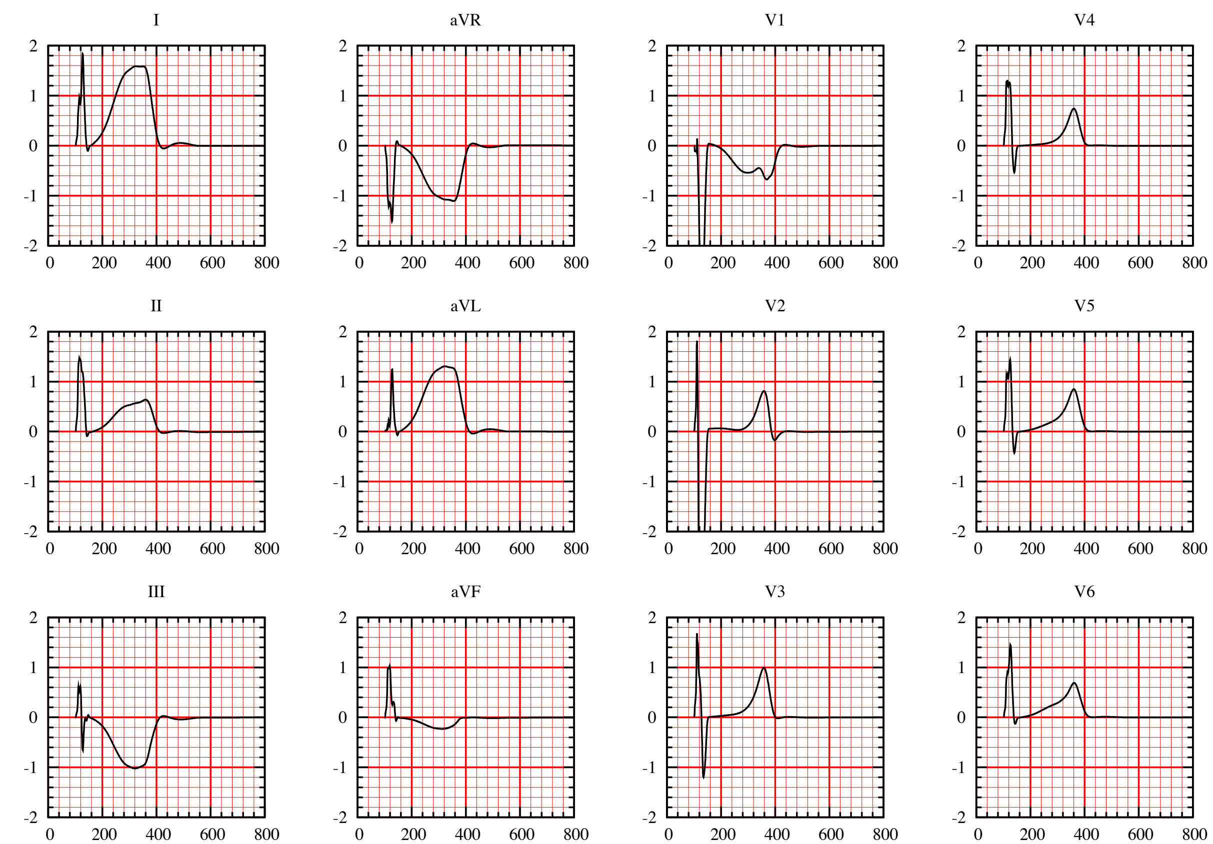
$$\begin{cases} \phi_e = \phi_T, & \text{on} \quad \partial\Omega_H, \\ \boldsymbol{\sigma}_e \boldsymbol{\nabla}\phi_e \cdot \boldsymbol{n} = \boldsymbol{\sigma}_T \boldsymbol{\nabla}\phi_T \cdot \boldsymbol{n}, & \text{on} \quad \partial\Omega_H, \\ \phi_a = \phi_T, & \text{on} \quad \partial\Omega_a, \\ \boldsymbol{\sigma}_a \boldsymbol{\nabla}\phi_a \cdot \boldsymbol{n} = \boldsymbol{\sigma}_T \boldsymbol{\nabla}\phi_T \cdot \boldsymbol{n}, & \text{on} \quad \partial\Omega_a. \end{cases}$$

MHD effect on the ECG

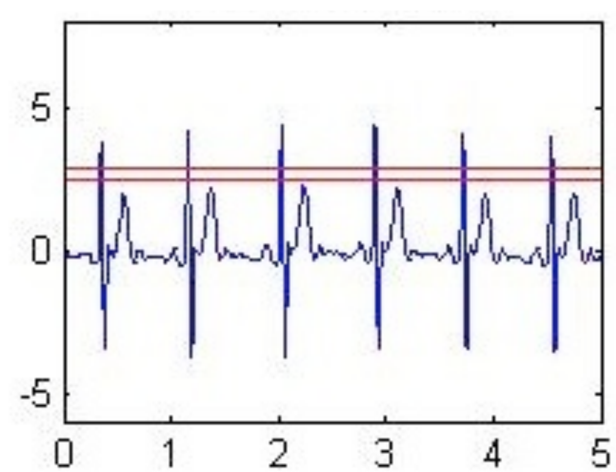
Without Magnetic Field



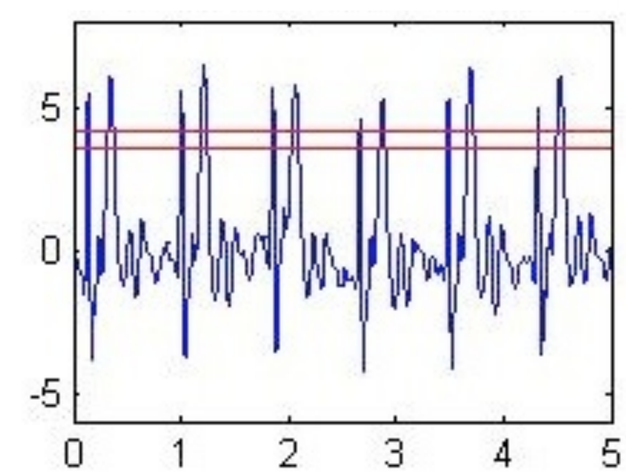
With Magnetic Field (*B = 3T*)

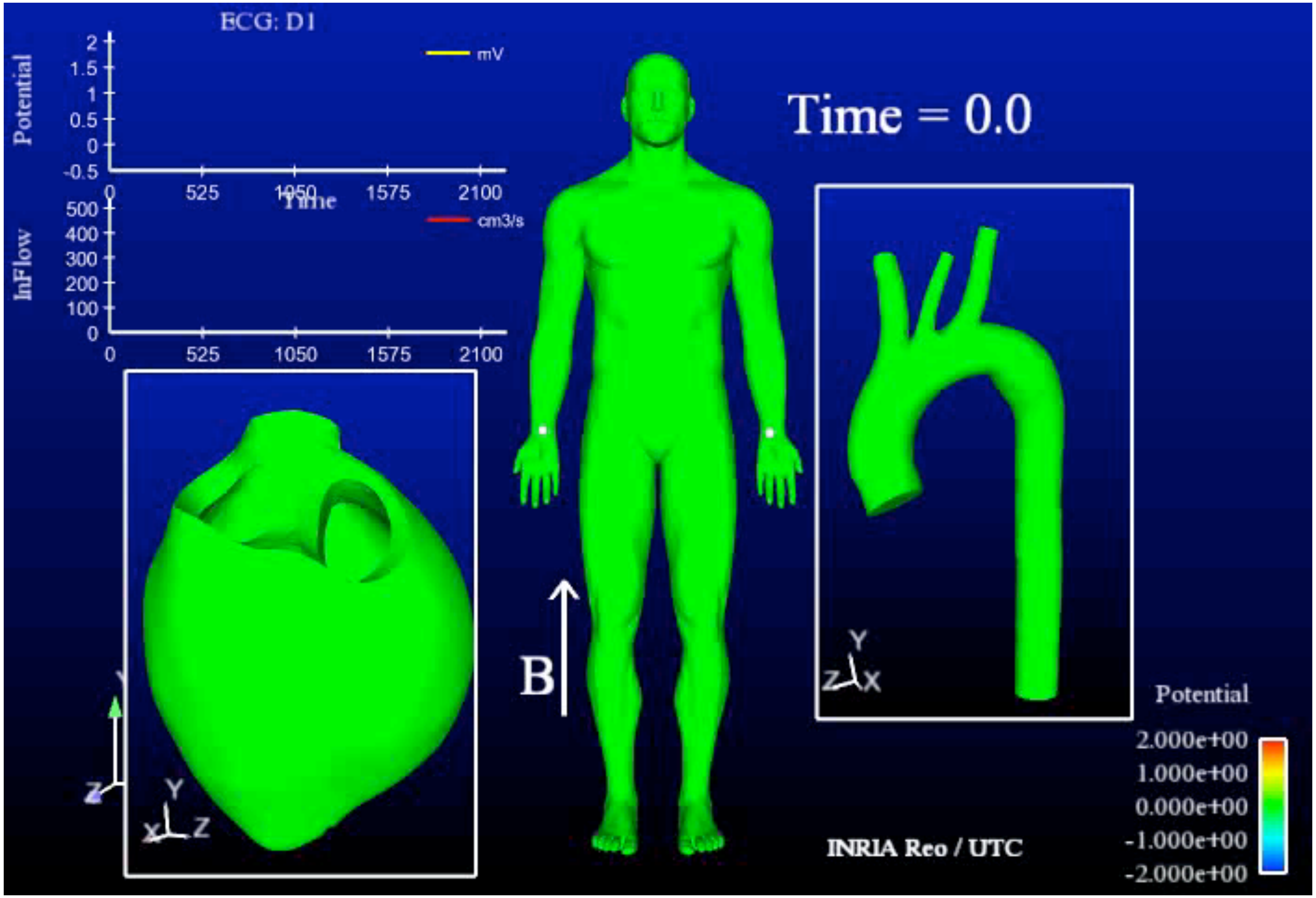


B = 0 T



B = 3T




ECG: D1
Potential
mV
Time
InFlow
cm3/s
Time = 0.0
B
Potential
2.000e+00
1.000e+00
0.000e+00
-1.000e+00
-2.000e+00
INRIA Reo / UTC

Conclusion

- Results:
  - ⋆ We do obtain a T-wave perturbation
  - ⋆ No significant flow perturbation *(to be confirmed)*
  - ⋆ No significant perturbation on the myocardium *(to be confirmed)*
- Possible future works:
  - ⋆ Improve the model: other vessels ? FSI ?
  - ⋆ Optimize the ECG lead locations to reduce the artifact
  - ⋆ Extract information from the perturbed signal