We will consider the oscillation of solutions $f \not\equiv 0$ of

$$f'' + A(z)f = 0, \tag{1}$$

where $A(z)$ is analytic in the unit disc $\mathbb{D}$. If f is a solution of (1), then f is analytic in $\mathbb{D}$, all (possible) zeros of f are simple, and all (possible) limit points of the zeros are on $\partial\mathbb{D}$. The equation (1) is called disconjugate (resp. non-oscillatory) if all solutions have at most one zero (resp. at most finitely many zeros) in $\mathbb{D}$. If (1) possesses a solution with infinitely many zeros in $\mathbb{D}$, then (1) is called oscillatory. These three concepts on oscillation were introduced over five decades ago. In 2005 the author called (1) as Blaschke-oscillatory (BO for short) if the zero sequence $\{z_n\}$ of any solution of (1) satisfies the Blaschke condition $\sum_n(1-|z_n|) < \infty$. A 1982 result due to Pommerenke implies that if

$$\int_{\mathbb{D}} |A(z)|^\alpha \, dm(z) < \infty \tag{2}$$

holds for $\alpha = 1/2$, where $dm(z)$ is the standard Euclidean area measure on $\mathbb{D}$, then (1) is BO. Conversely, in 2006 the author showed that if (1) is BO, then (2) holds for any $\alpha \in (0, 1/2)$. We note that if (1) is BO, then (2) does not necessarily hold for $\alpha = 1/2$. The coefficient $A(z) = -1/(1-z)^4$ gives a counterexample of such a case.

Since the zeros are always simple, we observe that separated and uniformly separated sequences are potential candidates for zero sequences of solutions of (1). This makes several results from interpolation theory available. The proofs also rely on sharp estimates (both pointwise and integrated) for logarithmic derivatives of Blaschke products.

This is the background to which the talk will be based on. Some open problems will be presented, unless they are solved before the conference.