Let f be an analytic function in $\mathbb{D} = \{z : |z| < 1\}$, $r_M[f]$ and $r_T[f]$ be the orders of f defined by the maximum modulus and the Nevanlinna characteristic, respectively. Connection between these orders and zero distribution of the function is well-known, when f is a canonical product of genus greater than 1. But it is not the case, when f is bounded in the unit disc, in particular, when f is a Blaschke product.

For an analytic function f in $\mathbb{D}$, and $p \geq 1$, we define

$$m_p(r, f) = \left(\frac{1}{2\pi} \int_0^{2\pi} \left| \log |f(re^{it})| \right|^p dt \right)^{1/p} \qquad (0 < r < 1),$$

We write (see [2])

$$r_p[f] = \limsup_{r \uparrow 1} \frac{\log m_p(r, f)}{-\log(1-r)}.$$

Note that $r_1[f] = r_T[f]$, and $r_M[f] \leq r_p[f] + [1/p]$, $p > 0$.

We define r_∞-order of f as

$$r_\infty[f] = \lim_{p \to \infty} r_p[f].$$

It is clear that, $r_M[f] \leq r_\infty[f]$. Moreover, Linden [1] proved that $r_M[f] = r_\infty[f]$ provided that $r_M[f] \geq 1$.

In the first part of the talk, we characterize zero distribution of the canonical products with given r_∞-order. As applications we obtain new factorization theorem and logarithmic derivative estimate for analytic functions of order $r_M[f] \leq 1$.

It is well-known that $r_T[f] \leq r_M[f] \leq r_T[f] + 1$. The same inequality holds for $r_\infty[f]$ instead of $r_M[f]$. So, it is natural to consider the following problem.

Problem : Given $0 \leq s \leq r \leq s + 1 < +\infty$, describe the class of analytic functions in $\mathbb{D}$ such that $r_T[f] = s$ and $r_M[f] = r$.

In the second part of the talk we deal with this problem and the similar one for the pair of orders $r_T[f]$ and $r_\infty[f]$.

References :

1. C.N.Linden, *Integral logarithmic means for regular functions*, Pacific J. of Math. 138 (1989), no.1, 119-127.

2. C.N.Linden, *The characterization of orders for regular functions*, Math. Proc. Cambridge Phil. Soc. 111 (1992), no.2, 299-307.