The main object of the talk is a class of analytic functions f in the unit disk with the property

$$(1) \qquad v(z) := \log|f(z)| \leq C_f \mathrm{dist}(z,E)^{-q},$$

where $q > 0$ and E is a closed subset of the unit circle. We prove that zeros $\{z_n\}$ of such function satisfy the condition

$$(2) \qquad \sum (1-|z_n|)\ \mathrm{dist}\,(z_n,E)^{(q+\kappa(E)-1+\varepsilon)_+} < \infty,$$

for all $\varepsilon > 0$. Here $\kappa(E)$ is the upper Minkowski dimension of E. Note that $1-\kappa(E) = \beta(E)$, $\beta(E)$ being a type of the set E, introduced by P. Ahern and D. Clark in 1976.

We also study the general class of subharmonic functions v in the disk subject to (1) and obtain Blaschke-type conditions for their Riesz measures (generalized Laplacians) $\mu = (1/2\pi)\Delta v$, which is similar to (2).

The result is optimal in various senses, and we consider the corresponding examples. We also discuss the inverse problem : given a discrete set Z (measure μ) in the disk subject to (2) (or its subharmonic analog), whether there exists an analytic (subharmonic) function with bound (1).

The results are applied to the study of critical points of Blaschke products and to the behavior of the discrete spectrum of contraction operators close to unitary ones.