By solving the Berger–Nirenberg problem for a special case we identify the critical sets of bounded analytic functions with the zero sets of the weighted Bergman space $\mathcal{A}_1^2$. We also show that for any nonconstant bounded analytic function in the unit disk there is always an indestructible Blaschke product with the same critical points. This leads to a sharpening of Nehari's extension of the Ahlfors–Schwarz lemma, which in a sense is best possible.