Regular maps with nilpotent automorphism groups

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According to a folklore result, every regular map on an orientable surface with abelian automorphism group belongs to one of three infinite families of maps with one or two vertices. Here we deal with regular maps whose automorphism group is nilpotent. We show that each such map decomposes into a direct product of two maps $H \times K$, where $\mathrm{Aut}(H)$ is a 2-group and K is a map with a single vertex and an odd number of semiedges. Many important properties of nilpotent maps follow from this decomposition. We show, for example, that apart from two well-defined classes of maps on at most two vertices and their duals, every nilpotent regular map has both its valency and face-size divisible by 4. We give a complete classification of nilpotent regular maps of nilpotency class 2 and 3, and prove that for every integer $c \geq 1$ there are only finitely many simple regular maps whose automorphism group is nilpotent of class c. This is a joint work with Y. F. Ban, S. F. Du, Y. Liu, A. Malnič, and R. Nedela.