Map Gaps

Thomas Tucker
Colgate University, Hamilton, New York
ttucker@colgate.edu

Call the non-negative integer g a *gap* for a certain type of symmetric map if there is no surface of genus g containing a map of that type. This talk will discuss gap phenomena in a wide variety of contexts. For example, chiral regular maps and nondegenerate regular maps (no primal or dual multiple edges) both have gaps at $g = p + 1$, for $p > 12$ prime and $p \not\equiv 1 \bmod 6, 8$ or 10, by recent work of Conder, Širàň, and Tucker. Computer lists for some of the Graver-Watkins types of edge-transitive maps have gaps for low genus, although it is not known whether any type other than chiral regular maps have infinitely many gaps. There are also gap questions for groups acting on surfaces. For the strong symmetric genus $\sigma^o(G)$, there are no gaps, but the question is open for the symmetric genus $\sigma(G)$. One can fix a group G and ask for all g such that G acts (faithfully) on the surface of genus g. By Kulkarni's Theorem one can determine all but finitely many such g, but even for cyclic groups those finitely many can be impossible to determine (for number theoretic reasons). Similar questions for non-orientable surfaces will also be discussed.