Indices in a number field.

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Let K be a number field, A be its ring of integers and $\hat{A}$ be the set of elements of A which are primitive over $\mathbb{Q}$. For any $\theta \in \hat{A}$ denote by $F_\theta(x)$ the characteristic polynomial of θ sur $\mathbb{Q}$. Set $i(\theta) = gcd_{x\in\mathbb{Z}} F_\theta(x)$ and $i(K) = lcm_{\theta\in\hat{A}} i(\theta)$. C. R. Mac Cluer has characterized the fields K for which $i(K)$ is nontrivial. In this common work with O. Kihel, we show that this integer $i(K)$ is linked to the so called common factor of indices in a number field or innessential discriminant divisor. For any $\theta \in \hat{A}$, write its disciminant in the form $D(\theta) = I(\theta)^2 D_K$, where D_K denotes the absolute discriminant and $I(\theta)$ the index of θ. p is said to be a common factor of indices in K if $p \mid I(\theta)$ for any $\theta \in \hat{A}$. It is shown in particular that if the exists some prime number which is a commmon factor of indices in K then $i(K)$ is nontrivial. Moreover fix a prime p and define the integers

$$\rho(p) = |\{\bar{\theta} \in A/pA, \quad p \mid i(\theta)\}|$$

and

$$\mu(p) = |\{\bar{\theta} \in A/pA, \quad p \mid I(\theta)\}|.$$

Explicit formulas for these integers are given and we show that they may be usefull for determining, in some cases, the spliting type of p in A.