MATTHIAS ASCHENBRENNER
University of California, Los Angeles

Vapnik-Chervonenkis density in model theory

VC dimension, introduced in probability theory by Vapnik and Chervonenkis, is a numerical parameter which can be used to measure the combinatorial complexity of a family of sets. Its relation to Shelah's notion of theories without the independence property (also known as "NIP" or "dependent" theories) is familiar to model-theorists, due to work of Laskowski, who showed that the complete theory of a first-order structure is NIP if and only if each definable family of sets has finite VC dimension. Examples of NIP theories are all (weakly) o-minimal theories. Another parameter associated to a family of sets, of relevance in combinatorics and probability theory, is its VC density. I will describe how my work (jointly with Andreas Fischer) on a problem about Lipschitz maps undertaken during the special semester on O-minimal Structures and Real Analytic Geometry at the Fields Institute lead to more recent results (jointly with A. Dolich, D. Haskell, D. Macpherson, and S. Starchenko) on uniform bounds for the VC density of definable families of sets.

GAL BINYAMINI
Weizmann Institute of Science

Multiplicity and order of contact for regular and singular foliations

I will begin by considering the order of contact between trajectories of singular polynomial vector fields and algebraic hypersurfaces. The goal is to obtain upper bounds depending solely on the degrees of the field and the hypersurface, where possible. I will describe an algebraic approach to this problem. I will then consider a generalization of this algebraic approach to the study of contact between leafs of higher-dimensional foliations and algebraic surfaces of complementary dimension. If time permits, I will also discuss an application of this approach to the study of non-isolated intersection multiplicities.

ANDREI GABRIELOV
Purdue University

Semi-monotone sets and triangulation of tame monotone families

Let S(t), for t¿0, be a monotone (decreasing) family of compact sets in a compact subset K of R^n. Both S(t) and K are assumed to be definable in an o-minimal structure (for example, real semialgebraic). The following problem emerges from a conjecture formulated by Gabrielov and Vorobjov (2009) in connection with their work on approximation of a definable set by homotopy equivalent compact sets: Construct a triangulation of K so that restriction of S(t) to each open simplex is equivalent to one of the $1+2^n$ "standard" families. The list of standard families is based on lex-monotone Boolean functions in n Boolean variables. This can be done for $n < 4$. A weaker conjecture claims that K admits a regular cell decomposition such that restriction of S(t) to each k-cell is a family of regular k-cells, and its boundary is a family of regular (k-1)-cells. To prove this conjecture, Basu, Gabrielov and Vorobjov (2010) introduced semi-monotone sets, a generalization of convex sets. Definable semi-monotone sets are PL-regular cells. They are related to regular Boolean functions, for which the result of any quantifier elimination does not depend on the order of quantifiers. (Joint work with S. Basu and N. Vorobjov.)

PHILIPP HIERONYMI
University of Illinois at Urbana-Champaign

A dichotomy for expansions of the real field

If an expansion of the real field does not de fine the set of integers, then every nonempty, bounded, nowhere dense unary de finable set has Minkowski dimension zero. (Joint with A. Fornasiero and C. Miller. Preprint, arXiv 1105.2946.)

GARETH O. JONES
University of Manchester

Integer-valued definable functions

Suppose that f is an analytic function definable in the real exponential field taking integer values at natural number arguments. I'll talk about a version of Polya's theorem for such functions, that is, if f doesn't grow too quickly then f is in fact a polynomial. I'll also show how to extend this to functions of several variables. This is all joint work with Margaret Thomas and Alex Wilkie.

DAN MILLER
Emporia State University

Lebesgue classes and a preparation theorem for real constructible functions

TOBIAS KAISER
Universität Passau

Spherical blowing up

We present the concept of spherical blowing up. Compared to classical blowing up the projective space is replaced by the sphere. Spherical blowing up is suitable to real geometry since it allows the use of directions and enables us to study questions about orientation. We present many examples. In the semialgebraic setting we show an algebraic formulation by the spherical spectrum of a graded ring. (Joint work with Niels Schwartz)

JANA MARIKOVÁ
McMaster University and Western Illinois University

Definable sets in o-minimal fields with convex valuations

Let R denote an o-minimal field and let (R,V) be its expansion by a predicate for a convex valuation ring. Then the class of structures (R,V) such that the corresponding residue field with structure induced from R is o-minimal, is first-order axiomatisable. We investigate the question whether this setting yields a good generalization of the T-convex case. This is work in progress, joint with C. Ealy.

DANIEL PANAZZOLO
Université de Mulhouse, France

Center manifolds for holomorphic three-dimensional vector fields

According to the theorem of resolution of singularities for vector fields in three dimensions, any singularity can be reduced to a certain list of models called "canonical". We will discuss the problem of existence of invariant surfaces (called centers manifolds) defined in sectorial vicinity of such singularities. (Joint work with M. McQuillan).

SANJAY PATEL
McMaster

o-minimal expansions by quasi-analytic functions of one variable

We will report on a work in progress concerning expansions of the real field by functions belonging to a quasi-analytic algebra of functions of one variable. We propose some simple conditions on the algebra which should be sufficient to guarantee the expansion is o-minimal. The o-minimality will be established by constructing suitable algebras of functions of several variables so that a resolution of singularities method of van den Dries and Speissegger may be applied.

FERNANDO SANZ
Universidad de Valladolid

On restricted Analytic Gradients on Analytic Isolated Surface Singularities

Let $(X, 0)$ be a real analytic isolated surface singularity at the origin 0 of $\mathbb{R}^n$ equipped with a real analytic metric g. Given an analytic function $f : (\mathbb{R}^n, 0) \to \mathbb{R}$, we prove that the trajectories in $X \setminus 0$ for the restricted gradient $\nabla_{g|_X}(f|_X)$ do not oscillate. Such a trajectory is thus a sub-pfaffian set.

TAMARA SERVI
CMAF Lisbon

Quantifier elimination for generalised quasi-analytic algebras of real functions

We consider the expansion of the real field by certain algebras of functions such that to each germ at zero of a function in the algebra we can associate injectively a generalised (divergent) power series. We show that these structures are o-minimal and polynomially bounded (in fact, all the known examples of o-minimal polynomially bounded expansions of the real field by functions are generated by such kind of algebras). Our main result is a quantifier elimination property for these structures (in a reasonable language). (Joint work with Jean-Philippe Rolin.)

MASAHIRO SHIOTA
Nagoya University

How to avoid method of integration of vector fields in singularity theory

This is an answer to a question posed by A. Gabrielov regarding the existence of a local cone structure of semialgebraic sets satisfying a strong natural condition on Cartesian coordinate system. If the required homeomorphism is only of class C^0, then it is easily constructed by the integration of a vector field in the usual method of singularity theory. However, we require it to be of class semialgebraic and the flow obtained by the integration is not necessarily semialgebraic. Hence we use a method introduced in the papers with Coste other than that of integration of vector fields. By extending this method we obtain a semialgebraic homeomorphism.

MICHAEL TYCHONIEVICH
Ohio State University

The set of restricted complex exponents for expansions of the reals

We introduce the set of defi nable restricted complex powers for expansions of the real fi eld and calculate it explicitly for expansions of the real field itself by collections of restricted complex powers. We apply this computation to establish a classi fication theorem for expansions of the real field by families of locally closed trajectories of linear vector fields.

LOU VAN DEN DRIES
University of Illinois at Urbana-Champaign

A progress report on the model theory of H-fields

This is joint work with Matthias Aschenbrenner and Joris van der Hoeven. We consider the valued differential field of transseries as a worthy object for modeltheoretic analysis. Does it even have a viable modeltheory? I will discuss progress we made on this since the special semester at Fields. For example, we can now show that any H-field has an immediate maximally valued H-field extension. (At the time of the special semester at Fields, this was not even known for Hardy fields of rank 1.)

NICOLAI VOROBJOV
University of Bath

Approximation of definable sets by compact families, and upper bounds on homotopy and homology

We suggest a construction for approximating a large class of sets, denable in an ominimal structure over the reals, by compact sets. The class includes sets defined by arbitrary Boolean combinations of equations and inequalities, and images of such sets under denable maps, e.g., projections. Based on this construction, we prove k-equivalence of denable sets to compact denable sets. This leads to improvements of the known upper bounds on Betti numbers of semialgebraic sets defined by quantifier-free formulae and to a singly exponential bound on Betti numbers of sub-Pfaffian sets. (Joint work with Andrei Gabrielov.)