# Comparison theorems for $p$-elliptic equations with laden levelsets as coefficients and some other topics.

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Part 1. Joint work with **Boris E Levitskij**.

We generalize Talenty's symmetrization methods to new comparison theorems for a wider class of equation.

Let $\Omega \subset \mathbb{R}^m$ be an open set with finite volume. Let $0 < q \leqslant p-1$. Let $\Phi(s) = s^{\alpha/(p-1)}$, where $\alpha < \frac{1}{m} + \frac{p-1-q}{q}$. For each measurable function $u : \Omega \to \mathbb{R}$ and each point $x \in \Omega$ we set

$$g_0(x,u) = \Phi^{p-1}(\mathrm{meas}\{y \in \Omega : |u(y)| > |u(x)|\})\,.$$

Let $u$ be a positive weak solution of the following equation

$$-\sum_{k=1}^{m} \frac{\partial}{\partial x_k}\big(g_0(x,u)|\nabla u|^{p-2}\frac{\partial u}{\partial x_k}\big) = f(x) + k|\nabla u|^q\,, \tag{1}$$

equipped with zero Dirichlet boundary conditions. Here $f \in L^1(\Omega)$ and $k \geqslant 0$. Let $\Omega^\star$ be the spherical symmetrization of the set $\Omega$. In other words $\Omega^\star$ is a ball in $\mathbb{R}^m$ of the same volume as the measure of $\Omega$. Let $u^\star : \Omega^\star \to \mathbb{R}$ and $f^\star : \Omega^\star \to \mathbb{R}$ denote the spherical symmetrization of the functions $u$ and $f$ respectively, and let $V : \Omega^\star \to \mathbb{R}$ be the maximal weak solution for

$$-\sum_{k=1}^{m} \frac{\partial}{\partial x_k}\big(g_0(x,V)|\nabla V|^{p-2}\frac{\partial V}{\partial x_k}\big) = f^\star(x) + k|\nabla V|^q \tag{2}$$

in $\Omega^\star$, which is the symmetrized version of equation (1). Equation (2) is also equipped with zero Dirichlet boundary conditions.

We prove that $V$ exist and unique and that $u^\star \leqslant V$. Also we prove that $|\nabla u|_{L^p(\Omega)} \leqslant |\nabla V|_{L^p(\Omega^\star)}$.

Generalizations of this result are also considered.

Part 2. Other topics.