On the central configurations of the N–body problem

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The N–body problem consists in study the motion of N pointlike masses, interacting among themselves through no other forces than their mutual gravitational attraction according to Newton's gravitational law.

Equations of motion

The equations of motion are

$$m_k \mathbf{r}''_k = \sum_{j=1, j\neq i}^{n} \frac{G m_j m_k}{r_{jk}^3} (\mathbf{r}_j - \mathbf{r}_k),$$

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r_{jk} is the euclidean distance between m_j and m_k.

Inertial barycentric system

The center of mass of the system satisfies

$$\frac{\sum_{k=1}^{N} m_k \mathbf{r}_k}{m_1 + \ldots + m_N} = \mathbf{a}t + \mathbf{b},$$

where **a** and **b** are constant vectors.

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We can consider that the center of mass is at the origin of the inertial system; i.e.

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$$\sum_{k=1}^{N} m_k \mathbf{r}_k = \mathbf{0}.$$

A such inertial system is called inertial barycentric system. In the major part of this talk we will work in an inertial barycentric system.

Homographic solutions

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Two configurations are similar if we can pass from one to the other doing a dilatation and/or a rotation.

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In 1772 Lagrange found two additional homographic solutions in the 3–body problem. Now, the configuration formed by the 3 bodies is an equilateral triangle.

Central configurations

At a given instant $t = t_0$ the configuration of the N–bodies is central if the gravitational acceleration $\mathbf{r}''_k$ acting on every mass point m_k is proportional with the same constant of proportionality to its position $\mathbf{r}_k$ (referred to an inertial barycentric system); i.e.

$$\mathbf{r}''_k = \sum_{j=1, j\neq k}^{n} \frac{Gm_j}{r_{jk}^3}(\mathbf{r}_j - \mathbf{r}_k) = \lambda \mathbf{r}_k,$$

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Laplace Theorem: The configuration of the N bodies in a homographic solution is central at any instant of time.

It is important to note that homographic solutions with rotation and eventually with a dilatation only exist for planar central configurations.

For spatial central configurations all the homographic solutions only have a dilation.

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(3) If the N bodies are going simultaneously at infinity in parabolic motion (i.e. the radial velocity of each particle tends to zero as the particle tends to infinity), then the particles tend to a central configuration.

Where appear the central configurations (II)

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Classes of central configurations

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In what follows we will talk about the classes of central configurations defined by this equivalent relation.

Collinear central configurations

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F.R. Moulton, The straight line solutions of n bodies, Ann. of Math. **12** (1910), 1–17.

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A. Albouy and V. Kaloshin, Finiteness of central configuration of five bodies in the plane, preprint, 2011.

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In the $(1+n)$–body problem the infinitesimal particles interact between them under the gravitational forces, but they do not perturb the largest mass.

Let $\mathbf{r}(\varepsilon) = (\mathbf{r}_0(\varepsilon), \mathbf{r}_1(\varepsilon), \ldots, \mathbf{r}_n(\varepsilon))$ be a planar central configuration of the N–body problem with $N = 1 + n$, associated to the masses $m_0 = 1$ and $m_1 = \ldots = m_n = \varepsilon$, which depend continuously on ε when $\varepsilon \to 0$.

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We say that $\mathbf{r} = (\mathbf{r}_0, \mathbf{r}_1, \ldots, \mathbf{r}_n)$ with $\mathbf{r}_i \neq \mathbf{r}_j$ if $i \neq j$ and $i, j \geq 1$, is a central configuration of the planar $(1 + n)$–body problem if there exists the $\lim_{\varepsilon \to 0} \mathbf{r}(\varepsilon)$ and this limit is equal to $\mathbf{r}$.

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PROPOSITION: All central configurations of the planar $(1+n)$–body problem lie on a circle $\mathbb{S}^1$ centered at $\mathbf{r}_0$.

This is the reason for which the central configurations of the $(1+n)$–body problem have applications to the dynamics of the coorbital satellite systems.

PROPOSITION: Let $\mathbf{r} = (\mathbf{r}_0, \mathbf{r}_1, \dots, \mathbf{r}_n)$ be a central configuration of the planar $(1+n)$–body problem. Denoting by θ_i the angle defined by the position of $\mathbf{r}_i$ on the circle $\mathbb{S}^1$, we have

$$\sum_{j=1, j\neq k}^{n} \sin(\theta_j - \theta_k) \left[1 - \frac{1}{8|\sin^3(\theta_j - \theta_k)/2|} \right] = 0$$

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G.R. Hall, Central configurations in the planar $1+n$ body problem, preprint, 1988 (unpublished).

J. Casasayas, J. Llibre and A. Nunes, Central configurations of the $1+n$–body problem, Celestial Mechanics and Dynamical Astronomy **60** (1994), 273–288.

Numerical results due to

H. Salo and C.F. Yoder, The dynamics of coorbital satellite systems, Astron. Astrophys. **205** (1988), 309–327.

n	Number of central configurations
2	2
3	3
4	3
5	3
6	3
7	5
8	3
9	1

For the n's of the table they also study the linear stability of the central configurations.

R. MOECKEL, Linear stability of relative equilibria with a dominant mass, J. of Dynamics and Differential Equations **6** (1994), 37–51.

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J.M. Cors, J. Llibre and M Ollé, Central configurations of the planar coorbital satellite problem, Celestial Mechanics and Dynamical Astronomy **89** (2004), 319–342.

Analytical results

(1) The numerical results of the table for $n = 2, 3, 4$, have been proved analytically by

Euler and Lagrange for $n = 2$,
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A. Albouy and Yanning Fu, Relative equilibria of four identical satellites, Proc. Royal Soc. A **465** (2009), 2633–2645.

(2) For $n \geq 2$ it is known that the regular n–gon, having the infinitesimal particles in its vertices and with the large mass in its center, is a central configuration.

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CONJECTURES. For the central configurations of the $(1+n)$–body problem:

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If conjecture (1) is proved, then we only need to prove conjecture (2) for $5 \leq n \leq 8$.

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Double nested central configurations for the planar $2n$–body problem

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S. Zhang and Q. Zhou, Periodic solutions for the $2n$–body problems, Proc. Amer. Math. Soc. **131** (2002), 2161–2170.

$p = 2$ and $n \geq 2$.

Triple and Quadruple nested central configurations for the planar $3n$– or $4n$–body problem

J. Llibre and L.F. Mello, Triple and Quadruple nested central configurations for the planar n–body problem, Physica D **238** (2009) 563–571.

$p = 3, 4$ and $n \geq 2, 3, 4$.

THEOREM For all $p \geqslant 2$ and $n \geqslant 2$, we prove the existence of central configurations of the pn–body problem where the masses are at the vertices of p nested regular n–gons with a common center. In such configurations all the masses on the same n–gon are equal, but masses on different n–gons could be different.

Central configurations of p nested n–gons

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M. Corbera, J. Delgado and J. Llibre, On the existence of central configurations of p nested n–gons, Qualitative Theory and Dynamical Systems **8** (2010), 255-265.

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Central configurations of nested regular polyhedra for the spatial $2n$–body problem

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THEOREM We consider $2n$ masses located at the vertices of two nested regular polyhedra with the same number of vertices. Assuming that the masses in each polyhedron are equal, we prove that for each ratio of the masses of the inner and the outer polyhedron there exists a unique ratio of the length of the edges of the inner and the outer polyhedron such that the configuration is central.

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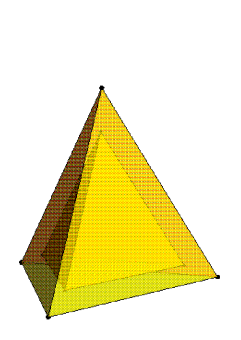
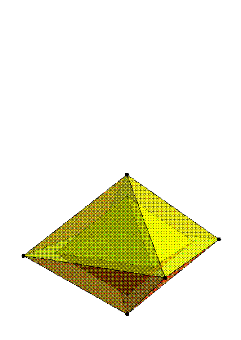
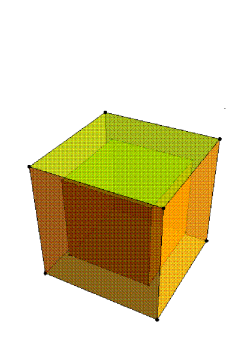


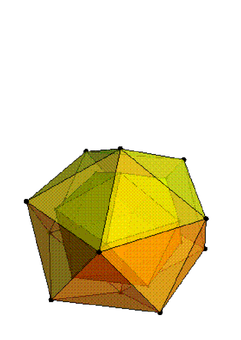
Nested regular tetrahedra



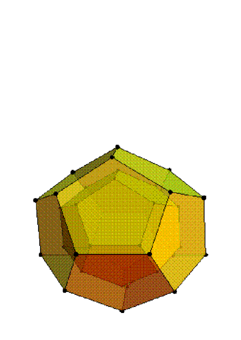
Nested regular octahedra



Nested regular cube



Nested regular icosahedra



Nested regular dodecahedra

Central configurations of 3 nested regular polyhedra for the spatial 3n-body problem

Idem with 3 nested regular polyhedra.

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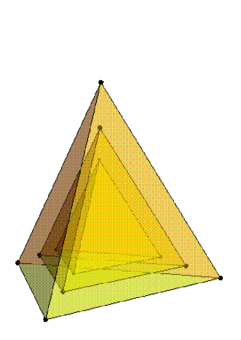
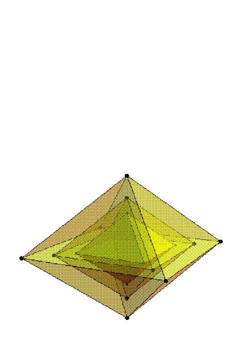
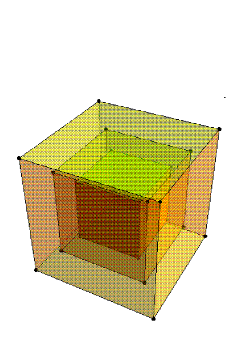


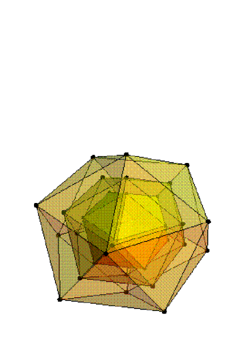
Nested regular tetrahedra



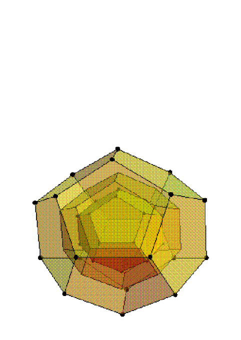
Nested regular octahedra



Nested regular cube



Nested regular icosahedra



Nested regular dodecahedra

On the existence of central configurations of p nested regular polyhedra

THEOREM For all $p \geqslant 2$, we prove the existence of central configurations of the pn–body problem where the masses are located at the vertices of p nested regular polyhedra having the same number of vertices n and a common center. In such configurations all the masses on the same polyhedron are equal, but masses on different polyhedra could be different.

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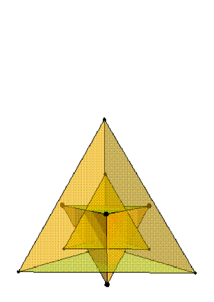
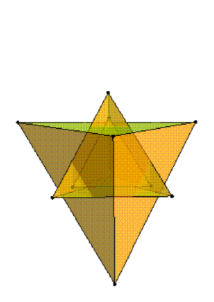
M. Corbera and J. Llibre, On the existence of central configurations of p nested regular polyhedra, Celestial Mechanics and Dynamical Astronomy **106** (2010), 197–207.

Idem but rotated with 3 nested regular polyhedra.

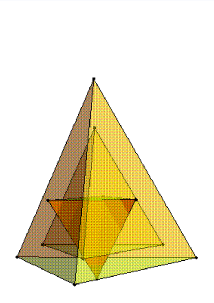
M. Corbera and J. Llibre, Central configurations of nested rotated regular tetrahedra, J. of Geometry and Physics **59** (2009), 137–1394.



$$1 \leqslant \rho < R$$



$$1 < R \leqslant \rho$$



$$0 < \rho < 1 < R$$

The contens