A simple equilibrium model for commodity markets

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Commodity market

- Commodity with a physical and a futures markets.
- On the physical market, one trades the commodity itself, for immediate delivery (spot market) : this is where producers meet industrial users.
- On the futures market, one trades financial contracts (paper) : this is where hedging against a rise in the price of the underlying asset meets hedging against a decrease. This brings in new agents, who are interested not in the commodity itself, but in the risk : speculators.

Some very old subjects and questions

- The relation between P_F and P_1 (Storage theory).
  Market is in contango if $P_F > P_1$, and in backwardation if $P_F < P_1$.
  If inventory > 0, arbitrage (cash-and-carry) implies contango (except in the case where there is a convenience yield).
- The relation between P_F and $E\left[\tilde{P}_2\right]$ (Normal backwardation theory)
  If $P_F \neq E\left[\tilde{P}_2\right]$, futures market is biased.
  Keynes: futures markets exhibit systematic downwards bias.
  $P_F < E\left[\tilde{P}_2\right]$ because there is an imbalance on the futures market: short hedging positions are higher than long positions. This is why speculators are needed (and remunerated).
- How does speculation impact the welfare of the agents in the economy? How does it influence prices?

Very recent and very frequent concerns: 2008 crisis, G20, CFTC decisions about the physical commodity trading (July 2013), etc.

Earlier works

- Anderson & Danthine (1983) : interplay between hedgers and speculators ; bias in the futures price
- Guesnerie & Rochet (1993) : analysis of coordination strategies ("eductive" equilibrium)
- Deaton & Laroque (1992): role of inventories and prices dynamics
- Routledge &al (2000) : term structures of futures prices
- Baker & Routledge (2012) : welfare ; Pareto optimal risk allocations

We study all aspects simultaneously. We propose a very simple model, perhaps the simplest possible, and we perform an equilibrium analysis.

The model

- 2 periods, $t = 1$ and $t = 2$. All decisions are taken at $t = 1$ and a source of uncertainty (Ω, P) operates between $t = 1$ and $t = 2$.
- 1 commodity. Produced in quantity ω_1 at $t = 1$ and $\tilde{\omega}_2$ at $t = 2$. ω_1 is observed, $\tilde{\omega}_2$ is not.
- 1 spot market, at $t = 1$ and $t = 2$. There is a non-negativity constraint on inventories.
- 1 futures market. Positions are initiated at $t = 1$ and settled at $t = 2$. The maturity of the contract is $t = 2$.
- There are 3 prices: 2 spot prices P_1 and $\tilde{P}_2$, and a futures price P_F. P_1 and P_F are observed, $\tilde{P}_2$ is not.
- Interest rate is set to 0.
- Aim of the paper: Determining P_1, P_F and $\tilde{P}_2$ by equilibrium conditions (all markets clear).

The agents

- **Spot traders**, who intervene only on the spot markets.
- **Processors**, or industrial users, who use the commodity to produce some goods which they sell to end users. Because of the inertia of the production process or because they sell their production forward, they have to decide at $t = 1$ how much to produce at $t = 2$.
  They cannot store the commodity, so they have to buy all of their input at $t = 2$.
- **Storers**, who have storage capacity, and who can use it to buy the commodity at $t = 1$ and release it at $t = 2$.
- **Speculators**, who only trade futures.

Rational behaviour

- All agents (firms), except the spot traders have mean-variance utilities: if they make a profit $\tilde{\pi}$ they derive utility:

$$E[\tilde{\pi}] - \frac{1}{2}\alpha \mathrm{Var}[\tilde{\pi}], \text{ with } \alpha = \alpha_I, \alpha_P, \alpha_S$$

- They make optimal decisions at $t = 1$, based on the conditional expectation of $\tilde{P}_2$, which will be determined in equilibrium.

- **Spot traders**.
  If price at time $t = 1, 2$ is P_t, the demands from spot traders are

$$\mu_1 - mP_1 \quad \text{and} \quad \tilde{\mu}_2 - mP_2$$

these demands can be positive or negative

Speculators

- **Speculators**. The profit resulting from a futures position f_S is:

$$\pi_S(f_S) = f_S(\tilde{P}_2 - P_F)$$

Perfect convergence of the basis
Portfolio effects are ignored
Transaction horizon is $t = 2$

$$f_S^* = \frac{\mathrm{E}[\tilde{P}_2] - P_F}{\alpha_S \mathrm{Var}[\tilde{P}_2]}$$

If $E\left[\tilde{P}_2\right] > P_F$, $f_S > 0$.
Long position; consistent with normal backwardation

The inventory holders

Storage is costly: holding a quantity x costs $\frac{1}{2}Cx^2$. If they buy $x \geq 0$ on the spot market at $t = 1$, resell it on the spot market at $t = 2$, and take a position f_I on the futures market, the resulting profit is:

$$\pi_I(x, f_I) = x(\tilde{P}_2 - P_1) + f_I(\tilde{P}_2 - P_F) - \frac{1}{2}Cx^2$$

The optimal positions are:

$$x^* = \frac{\max\{P_F - P_1, 0\}}{C}; \qquad f_I^* = \frac{\mathrm{E}[\tilde{P}_2] - P_F}{\alpha_I \mathrm{Var}[\tilde{P}_2]} - x^*$$

The storer creates the link between the spot and the futures prices.
Consistent with the storage theory.
The futures position is made of a speculative component, plus a short position.

Processors

Processors decide at time $t = 1$ how much input y to buy at $t = 2$, and which position f_P to take on the futures market. The input y results in an output $(y - \frac{\beta}{2}y^2)$ which is sold at a price P (decreasing returns to scale). It is assumed that P is known at time $t = 1$. The resulting profit is:

$$\pi_P(y, f_P) = P\left(y - \frac{\beta}{2}y^2\right) - y\tilde{P}_2 + f_P(\tilde{P}_2 - P_F)$$

The optimal positions are:

$$y^* = \frac{\max\{P - P_F, 0\}}{\beta P}; \qquad f_P^* = \frac{\mathrm{E}[\tilde{P}_2] - P_F}{\alpha_P \mathrm{Var}[\tilde{P}_2]} + y^*$$

Separation between the physical and the futures decision.

Clearing the markets

- **Futures market**

$$N_S f_S^* + N_P f_P^* + N_I f_I^* = 0$$

$$\mathrm{E}[\tilde{P}_2] - P_F = \frac{\mathrm{Var}[\tilde{P}_2]}{\frac{N_P}{\alpha_P} + \frac{N_I}{\alpha_I} + \frac{N_S}{\alpha_S}} (N_P y^* - N_I x^*)$$

Hedging pressure theory.
Formal expression for the bias in the futures price.
The sign of the bias depends only on the sign of $(N_P y^* - N_I x^*)$
No bias when $(N_P y^* = N_I x^*)$.

Clearing the markets

- **Spot market at** $t = 1$. On the supply side: the harvest ω_1. On the demand side: the inventories $N_I x^*$ bought by the storers and the demand of the spot traders.

$$\omega_1 = N_I x^* + \mu_1 - mP_1$$
$$P_1 = \frac{1}{m}\left(\mu_1 - \omega_1 + N_I x^*\right)$$

- **Spot market at** $t = 2$. On the supply side, the harvest $\tilde{\omega}_2$, and the inventories $N_I x^*$ sold by the storers. On the other side, the input $N_P y^*$ bought by the processors and the demand of the spot traders.

$$\tilde{\omega}_2 + N_I x^* = N_P y^* + \tilde{\mu}_2 - m\tilde{P}_2$$
$$\tilde{P}_2 = \frac{1}{m}\left(\tilde{\mu}_2 - \tilde{\omega}_2 - N_I x^* + N_P y^*\right)$$

The equilibrium equations (1)

$$\begin{cases} P_1 &= \frac{1}{m}\left(\mu_1 - \omega_1 + n_I X^\star\right) \\ \tilde{P}_2 &= \frac{1}{m}\left(\tilde{\mu}_2 - \tilde{\omega}_2 - n_I X^\star + n_P Y^\star\right) \\ P_F &= \mathrm{E}[\tilde{P}_2] + \frac{\mathrm{Var}[\tilde{P}_2]}{\frac{N_P}{\alpha_P}+\frac{N_I}{\alpha_I}+\frac{N_S}{\alpha_S}}\left(n_P Y^\star - n_I X^\star\right) \end{cases}$$

where $n_I = N_I / C$, $n_P = \frac{N_P}{\beta P}$,

$$X^\star = \max\{P_F - P_1, 0\},$$

and

$$Y^\star = \max\{P - P_F, 0\}.$$

The equilibrium equations (2)

Substituting the values for $X^\star$, $Y^\star$ and $\tilde{P}_2$ into the equations for P_1 and P_F we get the system:

$$mP_1 - n_I \max\{P_F - P_1, 0\} = \mu_1 - \omega_1$$

$$mP_F + \rho\left(n_I \max\{P_F - P_1, 0\} - n_P \max\{P - P_F, 0\}\right) = E[\tilde{\mu}_2 - \tilde{\omega}_2]$$

$$\text{where: } \rho = 1 + \frac{1}{m}\frac{\mathrm{Var}[\tilde{\mu}_2 - \tilde{\omega}_2]}{\frac{N_P}{\alpha_P} + \frac{N_I}{\alpha_I} + \frac{N_S}{\alpha_S}}$$

which is a system of two nonlinear equations for two unknowns P_1 and P_F.

Solving the equilibrium equations

We solve by investigating the piecewise linear map:

$$F\left(P_1, P_F\right) = \begin{pmatrix} mP_1 - n_I \max\left\{P_F - P_1, 0\right\} \\ mP_F + \rho n_I \max\left\{P_F - P_1, 0\right\} - \rho n_P \max\left\{P - P_F, 0\right\} \end{pmatrix}$$

We show that the equilibrium exists and that it is unique. Note that:

$$F\left(P_1, P_F\right) = \begin{pmatrix} \mu_1 - \omega_1 \\ E\left[\tilde{\mu}_2 - \tilde{\omega}_2\right] \end{pmatrix}$$

are precisely the equilibrium conditions.

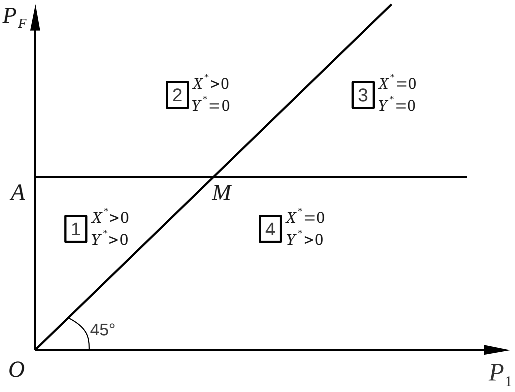


Figure: Phase diagram of physical and financial decisions.

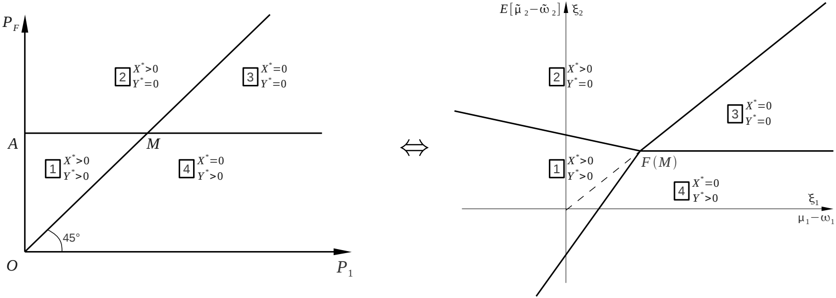


Figure: One-to-one relationship between diagrams.

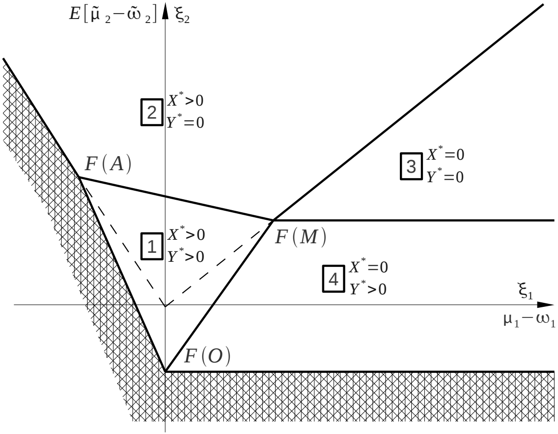
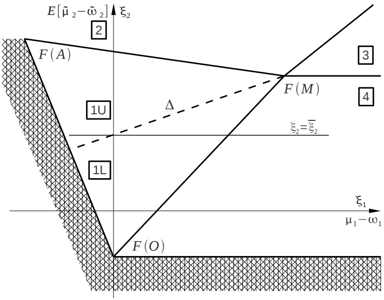


Figure: Phase diagram of physical and financial decisions (final).



1U	$P_1 < P_F$ $X^\star > 0$	$P_F < E[\tilde{P}_2]$ $f_S > 0$	$P_F < P$ $Y^\star > 0$
Δ	$P_1 < P_F$ $X^\star > 0$	$P_F = E[\tilde{P}_2]$ $f_S = 0$	$P_F < P$ $Y^\star > 0$
1L	$P_1 < P_F$ $X^\star > 0$	$P_F > E[\tilde{P}_2]$ $f_S < 0$	$P_F < P$ $Y^\star > 0$
2	$P_1 < P_F$ $X^\star > 0$	$P_F < E[\tilde{P}_2]$ $f_S > 0$	$P_F > P$ $Y^\star = 0$
3	$P_1 > P_F$ $X^\star = 0$	$P_F = E[\tilde{P}_2]$ $f_S = 0$	$P_F > P$ $Y^\star = 0$
4	$P_1 > P_F$ $X^\star = 0$	$P_F > E[\tilde{P}_2]$ $f_S < 0$	$P_F < P$ $Y^\star > 0$

Figure: Relationships between prices, physical and financial positions.

Political economy and policy evaluation

Our model allows for:

- Various kinds of comparative statics:
  - with or without futures market
  - with different levels of risk aversion
  - ...
- The computation of indirect (equilibrium) utilities.

Typical example: impact of an increase in the number of speculators

- on prices (level and variances) and quantities hold in the physical market
- on welfare

Changing the number of speculators

What's that?

- Could be an exogenous, unexpected fact.
  - Metaphor for variations in liquidity,
  - or risk aversion of the rest of the world.
- Could be a decision.
  - Access to derivative markets made easier or stricter.
  - Limits on positions could be changed, ...

Impact of speculators on prices and quantities

Let us see what happens in Regime 2, where $\tilde{Y}^\star = 0$

- $E[\tilde{P}_2] > P_F \longrightarrow$ speculators are long $\longrightarrow P_F$ increases with N_S;
- $\tilde{X}^\star$ increases because $(E[\tilde{P}_2] - P_F)$ diminishes and $(P_F - P_1)$ raises.

	$\tilde{P}_F$	$\tilde{X}^\star$	$\tilde{Y}^\star$	$\tilde{P}_1$	$\tilde{P}_2$	$\mathrm{Var}[\tilde{P}_F]$	$\mathrm{Var}[\tilde{P}_1]$	$\mathrm{Var}[\tilde{P}_2]$
2	$\nearrow$	$\nearrow$		$\nearrow$	$\searrow$	$\searrow$	$\searrow$	$\nearrow$
4	$\searrow$		$\nearrow$	$\longleftrightarrow$	$\nearrow$	$\longleftrightarrow$	$\longleftrightarrow$	$\longleftrightarrow$
1U	$\nearrow$	$\nearrow$	$\searrow$	$\nearrow$	$\searrow$	$\searrow$	$\searrow$	$\nearrow$
1L	$\searrow$	$\searrow$	$\nearrow$	$\searrow$	$\nearrow$	$\searrow$	$\searrow$	$\nearrow$

- Inventories are the transmission channel for shocks in space (paper / physical market) and in time (1 /2).
- Consequences for regulatory authorities.

Indirect utilities

The indirect utility of the speculators is given by:

$$U_S = f_S^*(\mathrm{E}[\tilde{P}_2] - P_F) - \frac{1}{2}\alpha_S f_S^{*2}\mathrm{Var}[\tilde{P}_2],$$

where we substitute the value of f_S^*, which leads to the utility of speculation:

$$U_S = \frac{\left(\mathrm{E}[\tilde{P}_2] - P_F\right)^2}{2\alpha_S \mathrm{Var}[\tilde{P}_2]}. \quad (1)$$

Let us now turn to the storers:

$$U_I = \frac{\left(\mathrm{E}[\tilde{P}_2] - P_F\right)^2}{2\alpha_I \mathrm{Var}[\tilde{P}_2]} + \frac{(P_F - P_1)^2}{2C}. \quad (2)$$

For the processors we have, in a similar fashion:

$$U_P = \frac{\left(\mathrm{E}[\tilde{P}_2] - P_F\right)^2}{2\alpha_P \mathrm{Var}[\tilde{P}_2]} + \frac{(P_F - P)^2}{2\beta P}. \quad (3)$$

Impact of speculators on welfare

Increasing the number of speculators:

- Increases competition in the provision of hedging
- Benefits those who need most to hedge
- Hurts the other hedgers
- Who is who?
- Depends on the regime
  - If storers have big positions compared to processors' commitments, storers benefit, processors are hurt.
  - ...and the other way around.
- Nobody (neither storers nor processors) benefits or is hurt by nature.
- Easy to see consequences in terms of political economy.

What's next ?

- Portfolio effects
- Convenience yield
- Term structure
- Oligopolistic market for the commodity

Prices (Region1)

Note that $\xi_1 := \mu_1 - \omega_1$, $\tilde{\xi}_2 := \tilde{\mu}_2 - \tilde{\omega}_2$, $\xi_2 := \mathrm{E}[\tilde{\mu}_2 - \tilde{\omega}_2]$, $n_I := N_I / C$ and $n_P := \frac{N_P}{\beta P}$. (ξ_1, ξ_2) determine the regime.

$$P_1 = \frac{(m + (n_I + n_P)\rho)\frac{\xi_1}{m} + n_I \frac{\xi_2}{m} + n_I n_P \rho m^{-1} P}{m + (n_I + n_P)\rho + n_I + n_I n_P \rho m^{-1}}, \tag{4}$$

$$P_F = \frac{n_I \rho \frac{\xi_1}{m} + (m + n_I)\frac{\xi_2}{m} + (m + n_I) n_P \rho m^{-1} P}{n_I \rho + (m + n_I) + (m + n_I) n_P \rho m^{-1}}, \tag{5}$$

$$\tilde{P}_2 = \frac{\tilde{\xi}_2}{m} + \frac{n_I \frac{\xi_1}{m} - ((m + n_I) n_P m^{-1} + n_I)\frac{\xi_2}{m} + (m + n_I) n_P m^{-1} P}{n_I \rho + (m + n_I) + (m + n_I) n_P \rho m^{-1}}, \tag{6}$$

$$X^\star = \frac{-(m + n_P \rho)\frac{\xi_1}{m} + m\frac{\xi_2}{m} + n_P \rho P}{n_I \rho + (m + n_I) + (m + n_I) n_P \rho m^{-1}}, \tag{7}$$

$$Y^\star = \frac{-n_I \rho \frac{\xi_1}{m} - (m + n_I)\frac{\xi_2}{m} + (m + (1 + \rho) n_I) P}{n_I \rho + (m + n_I) + (m + n_I) n_P \rho m^{-1}}. \tag{8}$$