Generalized Multi-Factor Commodity Spot Price Modeling through Dynamic Cournot Resource Extraction Models

Bilkan Erkmen
(joint work with Michael Coulon)

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    1. Reduced-form models that characterize the spot price as the solution to an SDE: Schwartz 1997, Schwartz & Smith 2000, Hilliard & Reis 1998
    2. Structural models that explicitly capture the supply - demand intraction, market mode (monopoly, oligopoly or competitive) and other economic dynamics: Sundaresan 1984, Reinganum & Stokey 1985, Dockner et al. 2001, Sircar et al. 2009

Research Motivation

While reduced-form and structural models are somehow connected (economic intuition underlying both types of models is similar), there does not appear to be an attempt to formally unite them.

The research motivation for this work is this absence of investigation into the precise relationship between reduced form and structural models of commodity prices.

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$$\begin{aligned} dS_t &= \left(\mu - \delta_t\right) S_t dt + \sigma_1 S_t dZ_t^1 \\ d\delta_t &= \kappa \left(\alpha - \delta_t\right) dt + \sigma_2 dZ_t^2 \end{aligned}$$

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- Three-factor model w/ stochastic convenience yield and interest rates:

$$\begin{aligned} dS_t &= \left(r_t - \delta_t\right) S_t dt + \sigma_1 dZ_t^1 \\ d\delta_t &= \kappa\left(\hat{\alpha} - \delta_t\right) dt + \sigma_2 dZ_t^2 \\ dr_t &= \alpha\left(m^* - r\right) dt + \sigma_3 dZ_t^3 \end{aligned}$$

  where $dZ_t^1 dZ_t^2 = \rho_1 dt$, $dZ_t^2 dZ_t^3 = \rho_2 dt$ and $dZ_t^1 dZ_t^3 = \rho_3 dt$.

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$$d\chi_t = -\kappa\chi_t dt + \sigma_\chi dZ_t^\chi$$
$$d\gamma_t = \mu_\gamma dt + \sigma_\gamma dZ_t^\gamma$$

where $dZ_t^\chi dZ_t^\gamma = \rho_{\chi\gamma} dt$.

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- Hilliard & Reis 1998 extends Schwartz 1997 three-factor model to include jump-diffusion in the commodity spot price in addition to stochastic convenience yield and interest rates.

$$
\frac{dS_t}{S_t} = \left(\mu - \delta_t\right) dt + \sigma_S dW_t^S + \kappa dq_t
$$

  where $\log\left(1+\kappa\right) \sim \mathcal{N}\left(\log\left(1+\mathbb{E}\left[\kappa\right]\right) - \frac{\omega^2}{2}, \omega^2\right)$ and $\left(q_t\right)_{t\geq 0}$ is a Poisson counter with intensity λ.

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1. Help establish a connection between reduced-form and structural models by endogenously deriving generalized forms of the Schwartz 1997 one-factor and Schwartz & Smith 2000 two-factor models from a simple stochastic dynamic Cournot resource extraction model.
2. Generalize the Cournot model to an arbitrary number of players, N, allowing to derive monopoly, oligopoly and competitive market modes.

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- Homogenous resource and hence no product differentiation

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- Let q_t^i be the resource extracted by player i at time t, then a strategy U_i for player i defined as $U_i : (X_t, \epsilon_t) \mapsto q_t^i$ where $q_t^i \leq X_t \quad \forall t \in [0, \infty)$ is a Markov strategy as it only depends on the current value of the state vector and let $\mathcal{U}_i$ be the set of such Markov strategies for player i

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- Resource stock is not perfectly measurable, i.e. there is continuous uncertainty regarding the actual stock level, and there are randomly occurring randomly sized jumps in the resource supply, characterizing the evolution of the resource stock as:

$$dX_t = \left(-\sum_{i=1}^{N} U_i\left(X_t, \epsilon_t\right) \right) dt + \sigma_X X_t dW_t + \left(e^{\theta_t} - 1 \right) X_t dN_t$$

  where $(N_t)_{t \geq 0}$ is a Poisson process with rate γ that is independent of $(W_t)_{t \geq 0}$. Letting $T_1, T_2, \ldots$ be the arrival times of the Poisson process, the sequence of $\theta_{T_1}, \theta_{T_2}, \ldots$ is i.i.d. and $\theta_{T_i} \sim \mathcal{N}\left(\mu_\theta, \sigma_\theta\right) \ \forall i$

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- As long term demand for the resource is linked to stable consumption levels, demand shocks are temporary in nature and should be mean-reverting, characterizing the demand shock process $(\epsilon_t)_{t \geq 0}$ as a mean-zero OU process:

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- The profit function is given by $\pi\left(q_t^i, p\right) = pq_t^i$ with $\partial\pi/\partial q_t^i > 0$

Problem Formulation

- Player i's value function J_t^i at time t is defined as:

$$J_t^i(x,\epsilon) = \mathbb{E}\left[\int_t^\infty e^{-rs}\log\left(\frac{U_i(X_s,\epsilon_s)\exp(\epsilon_s)}{\sum_{j=1}^N U_j(X_s,\epsilon_s)}\right) ds \mid X_t = x, \epsilon_t = \epsilon\right]$$

$$\text{s.t.}\quad dX_t = \left(-\sum_{i=1}^N U_i(X_t,\epsilon_t)\right) dt + \sigma_X X_t dW_t + \left(e^{\theta_t} - 1\right) X_t dN_t$$
$$d\epsilon_t = -\alpha\epsilon_t dt + \sigma_\epsilon dZ_t, \quad dW_t dZ_t = \rho dt$$

Since $(W_t)_{t\geq 0}$ and $(Z_t)_{t\geq 0}$ are correlated, we can define a new Brownian motion $\left(\tilde{W}_t\right)_{t\geq 0}$ that is independent of both $(W_t)_{t\geq 0}$ and $(Z_t)_{t\geq 0}$ and represent $dZ_t = \rho dW_t + \left(1-\rho^2\right)^{1/2} d\tilde{W}_t$.

- The objective of player i is to find optimal strategy U_i^* such that:

$$J_t^i\left(x,\epsilon | U_i^*, U_{-i}^*\right) \geq J_t^i\left(x,\epsilon | U_i, U_{-i}^*\right) \;\; \forall U_i \in \mathcal{U}_i \text{ and } \forall t$$

where $U_{-i} = (U_1, \ldots, U_{i-1}, U_{i+1}, \ldots, U_N)$.

We define $V_t^i(x,\epsilon) := J_t^i\left(x,\epsilon | U_i^*, U_{-i}^*\right)$ as the optimal value function for player i.

Solution Overview

We take the stochastic dynamic programming approach and write the HJB equation for player i's optimal value function $V_t^i(x,\epsilon)$ at time t:

$$\sup_{U_i \in \mathcal{U}_i} \left[\mathcal{G} * V_t^i(x,\epsilon) - rV_t^i(x,\epsilon) + \log\left(\frac{U_i(x,\epsilon)\exp(\epsilon)}{U_i(x,\epsilon) + \sum_{j=1,j\neq i}^{N} U_j^*(x,\epsilon)} \right) \right] = 0$$

where $\mathcal{G}$ is the infinitesimal generator.

For brevity, drop the function parameters x, ϵ, then $\mathcal{G} * V_t^i$ is given by PIDE:

$$\begin{aligned} \mathcal{G} * V_t^i &= \left(-U_i - \sum_{j=1,j\neq i}^{N} U_j^* \right) \frac{\partial V_t^i}{\partial x} - \alpha\epsilon \frac{\partial V_t^i}{\partial \epsilon} + \frac{1}{2}\frac{\partial^2 V_t^i}{\partial x^2}\sigma_X^2 x^2 \\ &+ \frac{1}{2}\frac{\partial^2 V_t^i}{\partial \epsilon^2}\sigma_\epsilon^2 + \frac{\partial^2 V_t^i}{\partial x \partial \epsilon}\sigma_X \sigma_\epsilon \rho x + \gamma \mathbb{E}\left[V_t^{i+} - V_t^i \right] \end{aligned}$$

where $V_t^{i+} = V_t\left(xe^{\theta}, \epsilon \mid \theta_t = \theta\right)$ accounting for the jump.

We proceed with the solution by fixing and differentiating the HJB equation w.r.t. U_i and then looking for a symmetric solution of the type $U_i^* = U_j^*\ \forall i,j \in [1,\ldots,N]$.

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- The optimal value function of player i is:

$$V_t^i(x,\epsilon) = \frac{1}{r}\left(\frac{1}{2} - N\right) - \frac{\sigma_X^2}{4r^2} + \gamma\frac{\mu_\theta}{2r^2} + \frac{1}{2r}\log\left(x\frac{r}{N^2}(2N-1)\right) + \frac{\epsilon}{\alpha + r}$$

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- The law of motion of the resource stock under optimal extraction behavior by players is:

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- The law of motion of the spot price of the resource is:

$$\begin{aligned}\frac{dp_t}{p_t} &= \left(-\alpha\epsilon_t + \frac{1}{2}r(2N-1) + \frac{1}{2}\sigma_\epsilon^2 - \frac{1}{2}\sigma_\epsilon\sigma_X\rho + \frac{3}{8}\sigma_X^2\right)dt \\ &\quad \left(\sigma_\epsilon\rho - \frac{1}{2}\sigma_X\right)dW_t + \sigma_\epsilon\left(1-\rho^2\right)^{1/2}d\tilde{W}_t + \left(e^{-\frac{1}{2}\theta_t} - 1\right)dN_t\end{aligned}$$

where $\left(\tilde{W}_t\right)_{t\geq 0}$ and $(W_t)_{t\geq 0}$ are independent Brownian motions.

Key Results (1/2)

Derivation of generalized Schwartz 1997 one-factor model

If you assume in the Dynamic Cournot model that there is no supply-side uncertainty and the only uncertainty is that of demand uncertainty, then we pick the parameters $\sigma_X = 0, \theta_t = 0 \ \forall t$ and $\rho = 0$. Then, based on the solutions on the prior page,

$$\begin{aligned} X_t &= x_0 \exp\left(-rt\left(2N-1\right)\right), \text{ with } X_0 = x_0 \\ p_t &= \left(rx_0\left(2N-1\right)\right)^{-1/2} \exp\left(rt\left(N-\frac{1}{2}\right)+\epsilon_t\right) \\ \frac{dp_t}{p_t} &= \left(-\alpha\epsilon_t + \frac{1}{2}r\left(2N-1\right)+\frac{1}{2}\sigma_\epsilon^2\right)dt + \sigma_\epsilon d\tilde{W}_t \end{aligned}$$

Rewriting ϵ_t in terms of p_t and then plugging into expression for dp_t/p_t

$$\begin{aligned} \text{Dynamic Cournot model} &: \quad dp_t = \alpha\left(\mu\left(t\right)-\log\left(p_t\right)\right)p_t dt + \sigma_\epsilon p_t d\tilde{W}_t \\ \text{Schwartz one-factor model} &: \quad dS_t = \kappa\left(\mu - \log\left(S_t\right)\right)S_t dt + \sigma S_t dZ_t \end{aligned}$$

where $\mu\left(t\right) = r\left(N-\frac{1}{2}\right)\left(\frac{1}{\alpha}+t\right)+\frac{\sigma_\epsilon^2}{2\alpha}-\frac{1}{2}\log\left(rx_0\left(2N-1\right)\right)$.

Key Results (2/2)

Derivation of generalized Schwartz & Smith 2000 two-factor model
Taking a detour and looking at the Schwartz & Smith 2000 model,

$$\begin{aligned} S_t &= \exp\left(\chi_t + \gamma_t\right) \\ d\chi_t &= -\kappa \chi_t dt + \sigma_\chi dZ_t^\chi \\ d\gamma_t &= \mu_\gamma dt + \sigma_\gamma dZ_t^\gamma, \quad dZ_t^\chi dZ_t^\gamma = \rho dt \end{aligned}$$

Comparing law of motion of S_t to p_t from the dynamic Cournot model,

$$\frac{dS_t}{S_t} = d\chi_t + d\gamma_t + \frac{1}{2}\left(d\chi_t\right)^2 + d\chi_t d\gamma_t + \frac{1}{2}\left(d\gamma_t\right)^2$$

$$\begin{aligned} \frac{dp_t}{p_t} = & \underbrace{\left[-\alpha \epsilon_t dt + \sigma_\epsilon \left(\rho dW_t + \left(1-\rho^2\right)^{1/2} d\tilde{W}_t\right)\right]}_{d\chi_t = -\kappa \chi_t dt + \sigma_\chi dZ_t^\chi} + \underbrace{\left[\frac{1}{2}\sigma_\epsilon^2 dt\right]}_{\frac{1}{2}(d\chi_t)^2} \\ & + \underbrace{\left[\left(\frac{1}{2}\mu_X + \frac{1}{4}\sigma_X^2\right) dt - \frac{1}{2}\sigma_X dW_t\right]}_{d\gamma_t = \mu_\gamma dt + \sigma_\gamma dZ_t^\gamma} + \underbrace{\left[\frac{1}{8}\sigma_X^2 dt\right]}_{\frac{1}{2}(d\gamma_t)^2} + \underbrace{\left[-\frac{1}{2}\sigma_X \sigma_\epsilon \rho dt\right]}_{d\chi_t d\gamma_t} \\ & + \underbrace{\left[\left(e^{-\frac{1}{2}\theta_t} - 1\right) dN_t\right]}_{\text{additional jump term}} \end{aligned}$$

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- Fitting and empirical analysis of the structural model