Strategic R&D in Cournot Markets

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Outline

1. Strategic R&D: Motivation
   - Cournot Games
2. Dynamic R&D Control
   - Game Model
   - Separable Controls
   - Coupled Controls
3. Extensions

Resource and Commodities Markets

- Long-term prices largely driven by production levels among several large producers
- Lots of evidence of strategic behavior by participants
- Noncooperative dynamic game
- Many sources of uncertainty
  - changing production costs
  - fluctuating demand
  - policy changes
  - technological advances
- Fertile application area for stochastic games

Role of R&D

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- Motivation:
  - Green (solar, biofuel) vs. fossil fuel energy production
  - R&D is key for switching to inexhaustible technologies
  - Game-theoretic effects can be significant

Game Model

- **Cournot** market: players control supply
- Production levels q_t^i; production costs c_t^i
- Price is given by inverse demand curve P based on aggregate supply $\vec{q} = \sum_i q^i$
- Profit from production is $q_t^i \cdot (P(\vec{q}_t) - c_t^i)$
- Each producer i looks at her total discounted revenue:

$$\mathbb{E}\left[\int_0^\infty e^{-rt}\left\{(P(\vec{q}_t) - c_t^i)q_t^i - \mathcal{C}(a_t^i)\right\} dt\right].$$

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- Look for **closed-loop** Markov Nash equilibrium
- **State**: c_t^i is stochastic, can be lowered through R&D
- Everything else is deterministic

Intuition

- Recall the static Cournot duopoly with production costs c^1 and c^2
- For simplicity, will focus on linear demand $P(\vec{q}) = 1 - \sum_i q^i$
- The respective revenue is $R_1 := q^1(1 - q^1 - q^2 - c^1)$ and $R_2 := q^2(1 - q^1 - q^2 - c^2)$
- Interior eqm solution is $q^{i,*} = \frac{1+c^i-2c^{\bar{i}}}{3}$ yielding revenue rate $(q^{i,*})^2$
- Game value $v_i = \frac{(1+c^i-2c^{\bar{i}})^2}{9r}$

Blockading

- Production rate must be non-negative
- If $c^i > \frac{1+c^{\bar{i}}}{2}$, producer i is blockaded and does not produce at all, $q_i^* = 0$. In that case have monopoly with $q_{\bar{i}}^* = (1 - c^{\bar{i}})/2$
- Blockading when c^i is large (close to 1) relative to $c^{\bar{i}}$. No blockading if $c^i < 0.5$

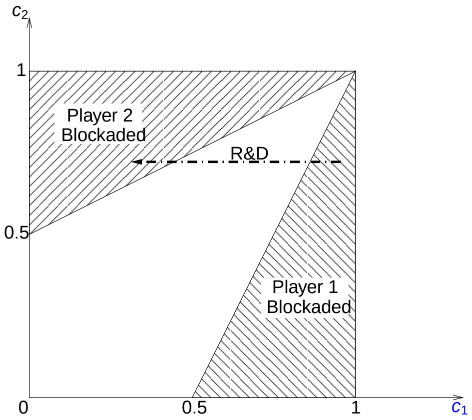

Figure : Fixed-cost Cournot game.

Existing Literature

- Industrial Organization: optimizing R&D investment by a monopolist facing uncertainty
- Within exhaustible resource context: Kamien and Schwartz (1978), Lafforgue (2008), numerous papers addressing climate change mitigation.
- But: Only single agent – no game effects
- Game Theory: impact of technological change on strategic competition
- Fudenberg & Tirole (1985), Weeds (2002)
- But: One-shot games – focus on coordination/preemption, no dynamic effects
- Cournot Games: Hotelling (1931), Sircar et al. (2010–)
- But: no R&D – production costs are fixed

R&D Control

- Model technology as a discrete ladder: $c(1) > c(2) > ... \geq 0$
- If currently at n-th stage, a breakthrough moves the producer to $n+1$-st stage of technology
- $c(n) = \exp(-bn)$: efficiency improves proportionally by $b\%$
- $c(n) = (1 - bn)_+$: absolute improvements in efficiency – eventually will reach “zero” costs

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- Expend effort $a_t \Rightarrow$ breakthroughs occur at rate λa_t
- N^i_t: point process for the technology advances of player i. Given (a^i_t), (N^i_t) is a Poisson process with controlled intensity $\lambda^i a^i_t$
- Dynamic production costs are $c^i_t = c(N^i_t)$
- R&D incurs costs $\mathcal{C}(a_t)$ per unit time; convex + increasing
- For convenience assume finite number of stages N_i for player i

Dynamic R&D

- Only uncertainty is from (N^i_t). Between R&D advances the game is deterministic. Can be viewed as a sequence of coupled static Cournot games
- There is dynamic interaction between R&D and production. Players may be blockaded, and may also choose to expend no effort (making some states (c^1, c^2) absorbing)
- Continuous-time strategies: q^i_t, a^i_t
- Strategies assumed to be in feedback form for (N^1_t, N^2_t)

Finding Nash Equilibrium

- Given initial technology stages (n_1, n_2), game values are denoted by $v_i(n_1, n_2)$
- τ^i is the time of first R&D success by player i – controlled by effort (a^i_t)
- Given (q^i, a^i), v_i's satisfy the recursions

$$v_1(n_1, n_2) = \mathbb{E}\Big[\int_0^{\tau^1 \wedge \tau^2} e^{-rs}\{q^1_s(P(\vec{q}_s) - c^1(n_1)) - \mathcal{C}(a^1_s)\}\, ds \\ + e^{-r\tau^1 \wedge \tau^2}[1_{\{\tau^1 < \tau^2\}} \cdot v_1(n_1 + 1, n_2) + 1_{\{\tau^1 > \tau^2\}} \cdot v_1(n_1, n_2 + 1)]\Big]$$

- By the **piecewise deterministic** property, under every Markov Nash equilibrium, $q^i_t \equiv q^i, a^i_t \equiv a^i$ are constant for $t \in [0, \tau^1 \wedge \tau^2]$
- So $\tau^1 \wedge \tau^2 \sim Exp(\lambda^1 a^1 + \lambda^2 a^2)$

Duopoly Game Values

- Using properties of Poisson arrival times, Nash equilibria are characterized by

$$v_1(n_1,n_2) = \sup_{q,a} \frac{q(1-q-q^{2,*}-c^1(n_1)) - \mathcal{C}(a) + \lambda^1 a v_1(n_1+1,n_2) + \lambda^2 a^{2,*} v_1(n_1,n_2+1}{\lambda^1 a + \lambda^2 a^{2,*} + r}$$

- Similar equation for $v_2(n_1,n_2)$
- $q^{1,*}$ is obtained directly as $\frac{1+c^1(n_1)-2c^2(n_2)}{3}$
- Differentiating wrt a's, obtain a system of two nonlinear equations in $a^{1,*}, a^{2,*}$ characterizing the Nash equilibrium

Recursive Static Games

$$
\begin{array}{ccc}
v_1(n_1, n_2+1) & & \\
\big\downarrow \lambda^2 a^2 & & \\
v_1(n_1, n_2) & \xleftarrow{\lambda^1 a^1} & v_1(n_1+1, n_2)
\end{array}
$$

- Can solve recursively on a lattice
- Boundary condition is $v_i(N_1, N_2) = \frac{(1+c^1(N_1)-2c^2(N_2))^2}{9r}$; also when $n_1 = N_1$, no further R&D is possible for P1 (1-dim optim by P2)
- $a^{1,*}$ depends on $v_1(n_1+1, n_2) - v_1(n_1, n_2) > 0$ and $v_1(n_1, n_2+1) - v_1(n_1, n_2) < 0$
- $\mathcal{C}(a) = a^2/2 + \kappa a$:
  - Have a system of two coupled quadratic equations for $a^{i,*}$
  - if $\kappa > 0$ then R&D may be unprofitable, so $a^* = 0$ is possible
  - Analytic expressions to determine whether R&D is zero

Unilateral R&D

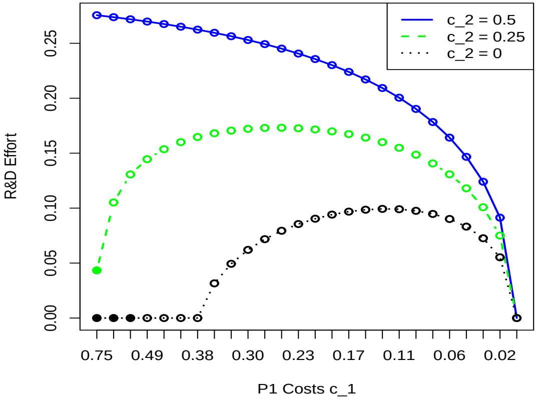
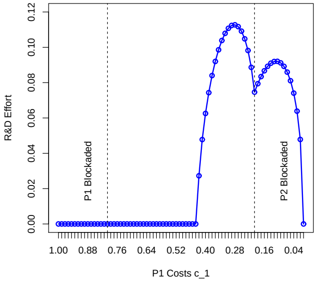


Figure : Effort Curves for Unilateral R&D in a Cournot Duopoly. Quadratic effort cost $\mathcal{C}(a) = a^2/2 + 0.2a$ with $\lambda = 5, r = 0.1$. Here $c^1(n) = 0.75 - 1.5\sqrt{n}$ ($q^1(n)$ is linear)

Unilateral R&D



- Levels of R&D over time may have different shapes
- Affected by: **expectation** of future profits (less future gains as get close to $c^1 = 0$), **shape** of the cost curve $n \mapsto c^1(n)$ (marginal efficiency of R&D), **current** revenue levels
- If initial c^1 is too high relative to c^2, become too discouraged and do nothing (never enter the market)
- Monopoly is more conducive to R&D than duopoly

Bilateral R&D

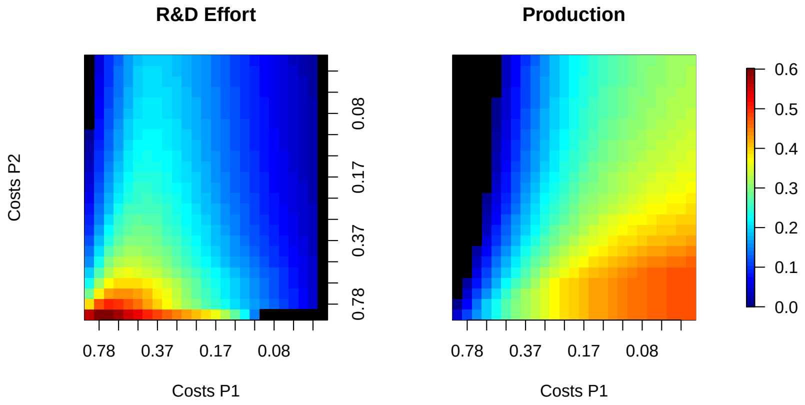


Figure : Right panel shows the effort $a^1(n_1, n_2)$ and left panel the production rate $q^1(n_1, n_2)$. Quadratic costs $\mathcal{C}(a) = a^2/2 + 0.2a$ with $\lambda = 5, r = 0.1$. $c^i(n) = e^{-n/8}$

Bilateral R&D

- R&D effort levels are asymmetric: put most effort when slightly ahead of competitor
- Therefore, player with lower costs tends to extend her advantage ("mean-aversion")
- Expectations of future profits can spur R&D even if currently blockaded out of the market
- Zero R&D can happen even without blockading
- Conversely, if c^i is very low compared to competitor, may become complacent and stop R&D

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- Conversely, if c^i is very low compared to competitor, may become complacent and stop R&D
- Outside input (subsidies) can spur endogenous advances both for very inefficient technologies and for efficient monopolies
- Competition is dynamically unstable (tends to collapse into a monopoly)

Sample Path of (N_t^1, N_t^2)

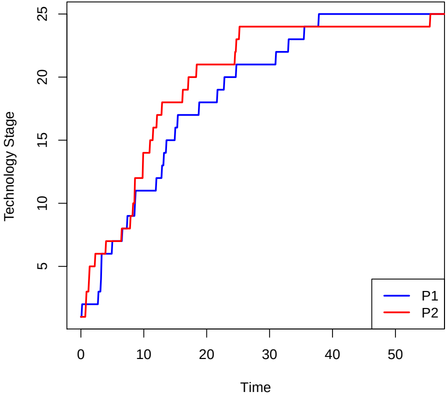
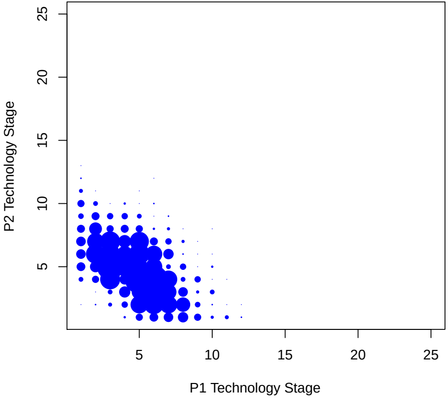


Figure : Sample path of (N_t^1, N_t^2). Costs are $c^i(n) = e^{-n/8}$. Quadratic effort curve $\mathcal{C}(a) = a^2/2 + 0.2a$ with $\lambda = 5, r = 0.1$.

Distribution of (N_t^1, N_t^2)

$t = 2$



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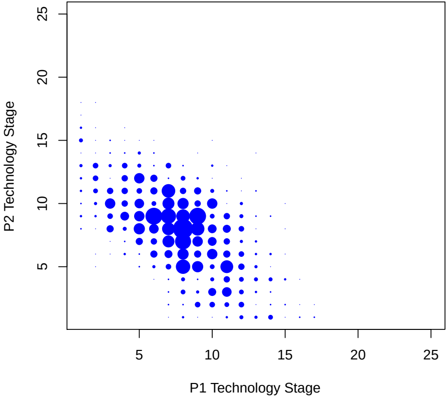
$$t = 4$$



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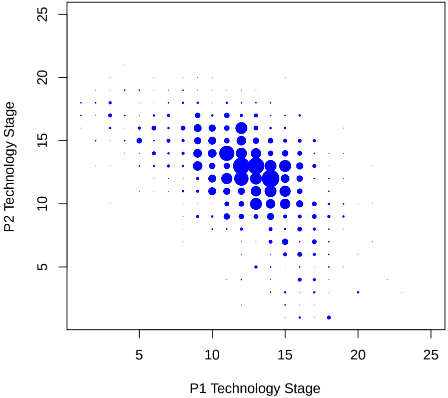
$$t = 8$$



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Distribution of (N_t^1, N_t^2)

$$t = 15$$

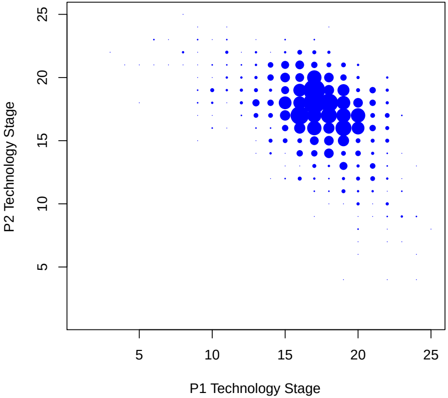
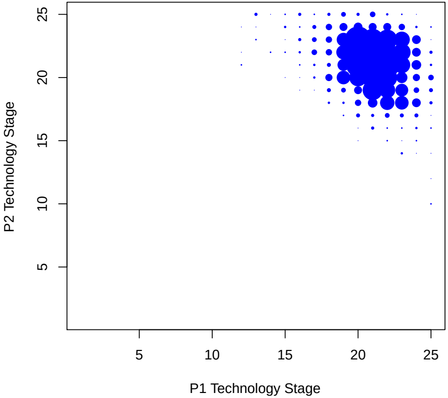


Figure : Distribution of (N_t^1, N_t^2). Costs are $c^i(n) = e^{-n/8}$. Quadratic effort curve $\mathcal{C}(a) = a^2/2 + 0.2a$ with $\lambda = 5, r = 0.1$.

Distribution of (N_t^1, N_t^2)

$$t = 25$$



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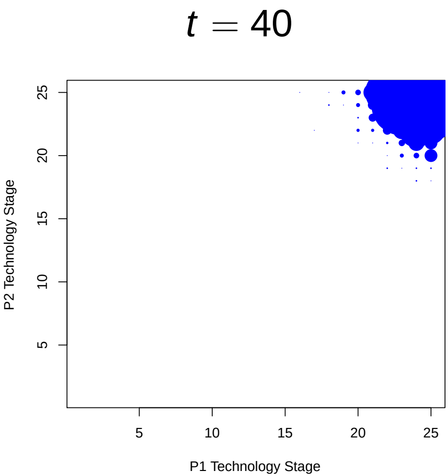

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Impact of Uncertainty

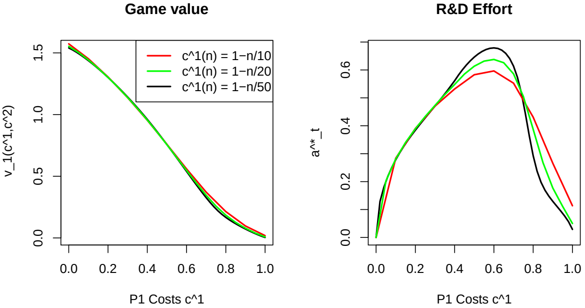

Figure : Comparison of game values $v_1(\cdot, c_2)$ and effort levels $a_1(\cdot, c_2)$. Linear technology progress $c^1(n) = 1 - n/M$, $\lambda = 0.4M$ with $c_2 = 0.7$ and $\kappa = 0.2$.

- Can study **effect of uncertainty** by linearly scaling the R&D ladder $c(n)$ and rate of progress λ
- Take $c(n) = f(n/M), \lambda = \lambda M$ where $c \mapsto f(c)$ is cont R&D curve on $[0, 1]$
- As $M \to \infty$, R&D success becomes deterministic: $dc_t = \lambda a_t \, dt$
- Impact is ambiguous: more uncertainty can spur/deter R&D investment!

R&D Complementary to Production

- Firm has **fixed labor supply** L. Allocate L between production and R&D: $a^i_t + q^i_t = L^i$
- Sharpens the trade-off between immediate revenue and future higher profits
- Cost of R&D is now implicit (quadratic if assume linear demand $P(\vec{q})$)
- Will tend to decrease R&D over time
- May be optimal to voluntarily lower/suspend production to advance technology (e.g. to lock-in monopoly)
- May allow a high-cost competitor to operate by temporarily focusing on R&D (i.e. strategic non-blockading)

Extensions: Exhaustible Resources

- When considering competition between old and new energy (fossil fuels vs. renewables), exhaustible reserves play a crucial role
- X_t – level of reserves at date t; $dX_t = -q_t dt$ lowered through production
- Oil industry (P1): low production costs c^1, but also marginal cost of exhaustibility
- Renewables industry (P2): high current production costs $c^2(0)$; potential for R&D
- P1 chooses (q_t^1); P2 chooses (q_t^2) and (a_t^2). State is (x, n)
- Leads to a **system of nonlinear ODEs** in x, coupled through n
- Can allow P1 to also explore for new reserves (L. & Sircar 2012)

Extensions: Switching Technologies

- Consider two integrated producers who can each use **either** cheap fossil fuels, or expensive backstops (oil sands)
- Resources allocated between production and R&D (advancing backstop technology)
- Uncertainty in advances will spur earlier R&D investments as marginal value of cheap reserves rises
- Related to the model of Harris, Howison and Sircar (2010)

Conclusion

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- Effect of exhaustibility
- Potential of R&D
- Strategic interactions (blockading, uncertainty, policy impact) lead to numerous non-trivial phenomena

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THANK YOU!

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