Energy Production and Differential Games

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Energy Production: Cheap-Exhaustible-Dirty through Costly-Renewable-Clean

- Energy is produced from a variety of sources which are distinguished by differing production costs, exhaustibility and emissions.
- Oil & Coal: cheap, exhaustible, dirty.
- Solar, Wind, Hydro: expensive, inexhaustible, clean.
- Natural gas: cheaper, plentiful (fracking), cleaner.
- Focus here on competitive interaction and effects of heterogeneous costs and exhaustibility.

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Games with Asymmetric Costs

- In markets governed by a small number of competitive players (oligopolies), game theory provides a natural way to frame the outcome of competition.
- In most situations, firms have different costs of production perhaps due to size (larger firms are more efficient), or different technologies (energy : oil, gas, solar, wind).
- Games with asymmetric costs are relatively understudied (except in duopolies) because much less tractable than the symmetric case. But new issues arise:
  - Static game: some firms may be inactive in Nash equilibrium. They are blockaded by the lower costs of their competitors.
  - Dynamic game: higher cost firms enter the market at different times as prices rise.

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Energy Producers with Heterogeneous Costs

- N energy producers:
  - One from oil (or coal) with exhaustible reserves;
  - $N-1$ from alternative (renewable) technologies (solar, wind, ...)
- They are differentiated by per-unit costs of production:
  - Take oil extraction cost to be zero (for simplicity);
  - Renewables have costs $0 \leq s_1 \leq s_2 \leq \cdots \leq s_{N-1} < 1$.
- But oil has implicit scarcity value which increases as it runs out. When reserves are plentiful, player 0 has a monopoly. At what times (and reserve levels) do renewables enter?
- As oil runs out, energy price rises, but as others enter, we move from monopoly through duopoly to oligopoly: increased competition, so does the price fall with entry?
- Is the price smooth as market structure changes?

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Dynamic Cournot Model for Energy Production

- The oil producer (Player 0) has reserves $x(t)$ at time t, and chooses his production rate $\bar{q}_0(x(t))$, depleting reserves as

$$\frac{dx}{dt} = -\bar{q}_0(x(t))\mathbb{1}_{\{x(t)>0\}}.$$

  Others produce energy at rates $\bar{q}_i(x(t))$, $i = 1, \ldots, N-1$.
- Price given by linear inverse demand function:

$$P(t) = 1 - \bar{q}_0(x(t)) - \sum_{j=1}^{N-1} \bar{q}_j(x(t)).$$

  Note maximum (choke) price is 1.
- Players maximize discounted lifetime profit. Player 0's value function:

$$v_0(x) = \sup_{\bar{q}_0} \int_0^{\infty} e^{-rt} \bar{q}_0(x(t))P(t)\mathbb{1}_{\{x(t)>0\}}\,dt.$$

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Aside: Static Cournot Game

- In a *static* Cournot game between N players with ordered costs $(s_0, s_1, \cdots, s_{N-1})$, the number of active players in equilibrium depends on the distribution of the costs. Let

$$G_i(s_0, s) = \max_{q_i \geq 0} q_i\,(1 - Q - s_i), \qquad Q = \sum_{j=0}^{N-1} q_j.$$

- Let $S^{(n)} = \sum_{j=0}^{n-1} s_j$. If $n \leq N-1$ players participate, the equilibrium total supply is: $Q^{\star,n} = \frac{n - S^{(n)}}{(n+1)}$.

Proposition

Let $\bar{Q}^\star = \max\left\{Q^{\star,n} | 0 \leq n \leq N-1\right\}$. *Then the unique Nash equilibrium quantities are given by*

$$q_i^\star(s_0, s) = \max\left\{1 - \bar{Q}^\star - s_i, 0\right\}, \quad G_i = (q_i^\star)^2, \quad 0 \leq i \leq N-1.$$

The number of active players in the unique equilibrium is $m = \min\left\{n \mid Q^{\star,n} = \bar{Q}^\star\right\}$. *(The others are blockaded).*

Value Functions and Feedback Strategies

We look for a *Markov Perfect* Nash equilibrium. Player 0's value function:

$$v_0(x) = \sup_{\bar{q}_0} \int_0^\infty e^{-rt} \bar{q}_0(x(t)) P(t) \mathbb{1}_{\{x(t)>0\}} dt.$$

When oil runs out, the remaining firms ($i = 1, \ldots, N-1$) with their inexhaustible resources repeatedly play a static game with profit flow $G_i(1, s)$:

$$w_i(x) = \sup_{\bar{q}_i} \int_0^\infty e^{-rt} \bar{q}_i(x(t)) \left(P(t) - s_i\right) \mathbb{1}_{\{x(t)>0\}} dt + \frac{1}{r} G_i(1, s).$$

The HJB equation is $rv_0 = G_0(v_0', s)$ with $v_0(0) = 0$, and the equilibrium production rates are:

$$\bar{q}_i^\star(x(t)) = q_i^\star(v_0'(x(t)), s), \qquad i = 0, \ldots, N-1.$$

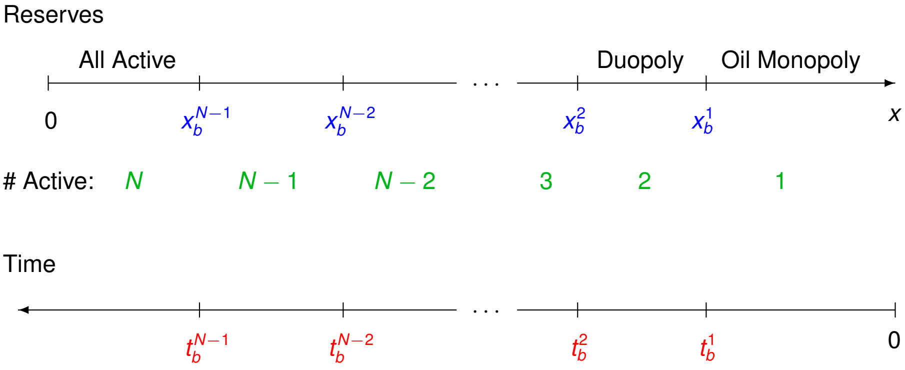
Oil producer's scarcity value (shadow cost) is encoded in $v_0'(x)$.

Blockading Points

For $n = 0, \dots, N-1$, let

$$x_b^n = \inf\{x \geq 0 : \bar{q}_n^\star(x) = 0\}, \qquad t_b^n = \inf\{t \geq 0 : \bar{q}_n^\star(x(t)) > 0\}.$$

Let $S^{(k)} = \sum_{j=1}^{k} s_j$ and assume s is s.t. $s_{N-1} < \frac{1+S^{(N-2)}}{N-1}$: guarantees everyone else participates when oils runs out.



Low Oil Reserves: Value Function

Proposition

For $x \in (0, x_b^{N-1})$, *Player* 0*'s value function is given by*

$$v^{(N)}(x) = \frac{1}{r}\left(\frac{1+S^{(N-1)}}{N+1}\right)^2 \left(1+\mathbf{W}\left(\theta(x)\right)\right)^2,$$

with $\theta(x) = -e^{-\mu_N x - 1}$, *and,* $\mu_N = \frac{r(N+1)^2}{2N(1+S^{(N-1)})}$, *and where* $\mathbf{W}(\cdot)$ *is the Lambert-*$\mathbf{W}$ *function.*

$$\bar{q}_0^\star(x(t)) = \frac{1}{(N+1)}\left(1 - N v^{(N)'}(x(t)) + S^{(N-1)}\right),$$

$$\bar{q}_i^\star(x(t)) = \frac{1}{(N+1)}\left(1 - (N+1)s_i + v^{(N)'}(x(t)) + S^{(N-1)}\right),$$

where $v^{(N)'}(x) = -(1+S^{(N-1)})\mathbf{W}\left(\theta(x)\right)/N$*.*

Blockading Point

Let $\alpha_n = (n+1)s_n - (1 + S^{(n-1)})$.

Proposition

The last blockading point is given by:

$$x_b^{N-1} = \frac{1}{\mu_N}\left[-1 + \frac{N\alpha_{N-1}}{1+S^{(N-1)}} - \log\left(\frac{N\alpha_{N-1}}{1+S^{(N-1)}}\right)\right],$$

provided $\alpha_{N-1} > 0$*, otherwise* $x_b^{N-1} = \infty$*. Suppose that for* $n \in \{2, \ldots, N-1\}$, $x_b^n < \infty$*. If* $\alpha_{n-1} > 0$*, then*

$$x_b^{n-1} = x_b^n + \frac{1}{\mu_n}\left[-\frac{n(n+1)}{1+S^{(n-1)}}(s_n - s_{n-1}) - \log\left(\frac{\alpha_{n-1}}{\alpha_n}\right)\right],$$

otherwise $x_b^{n-1} = \infty$*.*

Assume hereon s such that all $\alpha_n > 0 \Rightarrow x_b^n < \infty$.

Value Function Properties

For $x \in [x_b^n, x_b^{n-1})$, denote the value function by $v_0(x) = v^{(n)}(x - x_b^n)$(known explicitly).

Proposition

For $n \geq 2$, the first derivative of v_0 is continuous at x_b^{n-1}:

$$v^{(n)'}(x_b^{n-1} - x_b^n) = v^{(n-1)'}(0).$$

But there is a downward jump when moving in the direction of larger x in the second derivative of v_0 at the point x_b^{n-1}:

$$v^{(n)''}(x_b^{n-1} - x_b^n) > v^{(n-1)''}(0).$$

Hotelling's Rule

- A modified version of Hotelling's rule for exhaustible resources holds:

Proposition

For $n \in \{1, \ldots, N\}$, for $x \in (x_b^n, x_b^{n-1})$, (we identify $x_b^N = 0$ and $x_b^0 = \infty$),

$$\frac{d}{dt} v^{(n)\prime}(x(t) - x_b^n) = \left(\frac{1}{2} + \frac{1}{2n}\right) r\, v^{(n)\prime}(x(t) - x_b^n).$$

- Coincides with the classical Hotelling rule (1931) for $n = 1$: the marginal value grows (exponentially) at the discount rate.

Market Price

- Price is $P(t) = P^{(n)}(x(t) - x_b^n)$ where for $x \in (x_b^n, x_b^{n-1})$,

$$
\begin{aligned}
P^{(n)}(x(t) - x_b^n) &= 1 - \bar{q}_0^\star(x(t)) - \sum_{i=1}^{n-1} \bar{q}_i^\star(x(t)) \\
&= \frac{1}{n+1}\left(1 + v^{(n)\prime} + S^{(n-1)}\right).
\end{aligned}
$$

- It can be shown that $P^{(n)}(x_b^{n-1} - x_b^n) = s_{n-1}$, *i.e.* the blockading point x_b^{n-1} is exactly the point at which the market price equals the cost of Firm $n-1$.
- Turns out there is an autonomous linear ODE for the price:

$$
\frac{d}{dt}P(t) = \left(\frac{1}{2} + \frac{1}{2n}\right) r \left(P(t) - \frac{1 + S^{(n-1)}}{n+1}\right).
$$

Blockading Times

Proposition

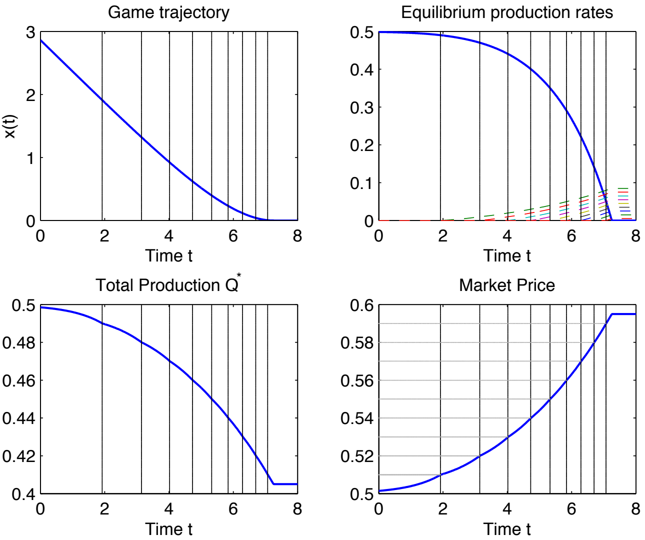
For $n \in \{2, \ldots, N-1\}$, the time at which Firm n enters the game is

$$t_b^n = t_b^{n-1} + \frac{2n}{(n+1)r} \log\left(\frac{\alpha_n}{\alpha_{n-1}}\right),$$

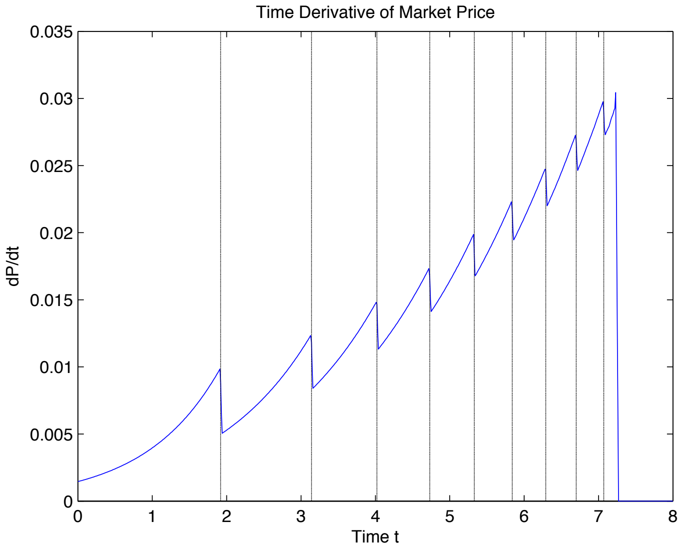
and for $n = 1$ by

$$t_b^1 = \frac{1}{r} \log\left(\frac{s_1 - \frac{1}{2}}{P(0) - \frac{1}{2}}\right).$$

Example: $N = 10$, $s = (0.51, 0.52, \ldots, 0.59)$



dP/dt



Time Derivative of Market Price
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
dP/dt
0
1
2
3
4
5
6
7
8
Time t

Summary

- Exhaustibility wins over increased competition: oil runs low, competing energy sources enter the market, but price rises. However, exponential rate of price increase decreases like $(\frac{1}{2} + \frac{1}{2n})r$.
- Remains to understand the blockading issue with multiple exhaustible suppliers: involves strongly coupled systems of nonlinear PDEs with nonsmooth coefficients.
- Those PDEs require subtle regularization in the form of trembling: bounding below $\bar{q}_i \geq \varepsilon$ and passing $\varepsilon \downarrow 0$.
- Next: incorporate exploration.

Exploration and Random Discoveries

- So far: exhaustibility or scarcity leads to price increases/shocks.
- However there were over 30 new discoveries in 2009. Proved reserves of crude oil rose 13% to 25.2 billion barrels in 2010, the largest annual increase since 1977, and the highest total level since 1991.
- We analyze effect of exploration and random discoveries in a dynamic Cournot game . This was studied in the monopoly context: Pindyck '78, Arrow & Chang '82, Deshmukh & Pliska '80-'85, Soner '85, Hagan *et al.* '94.
- Concentrate on two-player game: player 2 is clean (solar) with fixed cost $c > 0$; player 1 produces oil at zero cost, but can explore for new reserves.

Axis Game with Exploration

The remaining reserves X of Player 1 follows

$$dX_t = -q_1(X_t)\,\mathbb{1}_{\{X_t>0\}}\,dt + \delta\,dN_t,$$

where (N_t) is a controlled point process with intensity λa_t, penalized by cost $\mathcal{C}(a_t)$. Market price:
$P(t) = (1 - q_1(X_t) - q_2(X_t))$.
Value functions of each player:

$$v(x) = \sup_{q_1, a} \mathbb{E}\left[\int_0^\infty e^{-rt}\,(q_1(X_t)P(t) - \mathcal{C}(a_t))\,dt \mid X_0 = x\right],$$

$$w(x) = \sup_{q_2 \geq 0} \mathbb{E}\left[\int_0^\infty e^{-rt} q_2(X_t)(P(t) - c)\,\mathbb{1}_{\{X_t>0\}}\,dt + \int_0^\infty e^{-rt}\frac{1}{4}(1-c)^2\,\mathbb{1}_{\{X_t=0\}}\,dt \mid X_0 = x\right].$$

Axis Game HJB System

The ODEs for v and w are

$$\begin{aligned} \sup_{q_1,a}\left\{(1-q_1-q_2^*)q_1-q_1v'(x)-\mathcal{C}(a)+a\lambda\Delta v(x)\right\}-rv(x) &= 0, \\ \sup_{q_2\geq 0}\left\{(1-q_1^*-q_2-c)q_2\right\}-q_1^*w'(x)+a^*(x)\lambda\Delta w(x)-rw(x) &= 0, \end{aligned}$$

where $\Delta v(x) = v(x+\delta) - v(x)$ is the non-local or jump term, and

$$a^*(x) = \operatorname*{argsup}_{a\geq 0}\{-\mathcal{C}(a)+a\lambda\Delta v(x)\}$$

is the optimal exploration effort.
Boundary conditions:

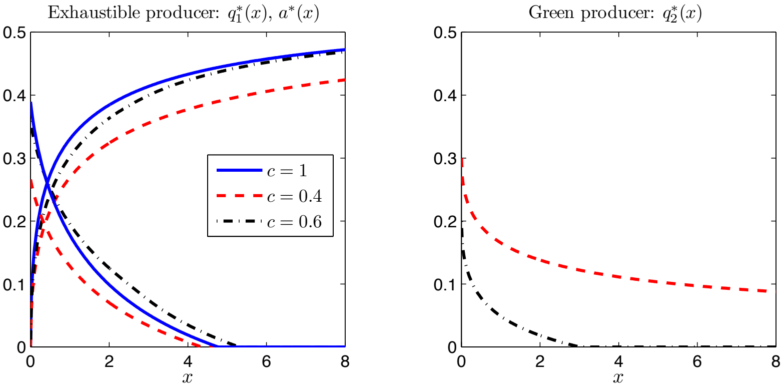
$$v(0) = \sup_a \frac{a\lambda v(\delta)-\mathcal{C}(a)}{\lambda a + r}, \qquad w(0) = \frac{(1-c)^2/4+\lambda a^*(0)w(\delta)}{\lambda a^*(0)+r}.$$

Power Function Costs

- If $a^* > 0$ for all x then X^* is recurrent on its full state space. Therefore $\sup_t X_t^* = +\infty$ and reserves will become arbitrarily large infinitely often.
- Unrealistic for describing non-renewable resources, and suggests that we should take $\mathcal{C}'(0) > 0$.
- Then there exists a *saturation level* x_{sat} such that $a^*(x) = 0$ for $x > x_{\text{sat}}$ and X^* would be positive recurrent on $[0, x_{\text{sat}} + \delta)$ only.
- Take $\mathcal{C}(a) = \frac{1}{\beta}a^\beta + \kappa a$, with $\beta > 1, \kappa \geq 0$. Note that $\mathcal{C}'(0) = \kappa$. Then $a^*(x) = \left[(\lambda\Delta v(x) - \kappa)^+\right]^{\gamma-1}$, where $\beta^{-1} + \gamma^{-1} = 1$, and

$$\frac{1}{9}(1 - 2v' + c)^2 + \frac{1}{\gamma}\left[(\lambda\Delta v(x) - \kappa)^+\right]^\gamma - rv = 0.$$

Effect of Competition on Exploration Effort

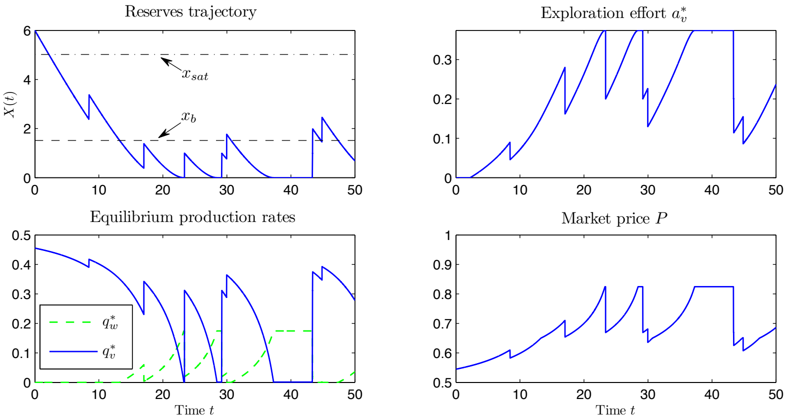


The parameters are $\delta = 1$, $\lambda = 1$, $r = 0.1$, $\mathcal{C}(a) = 0.1a + a^2/2$.

Comments & Observations

- For small c, the green producer is the effective leader in the market and leads to significant losses for the exhaustible producer, who gives up and reduces efforts.
- For moderate c, the exhaustible (respectively green) producer is the leader for large (resp. small) reserves levels. For $x \sim 0$, the exhaustible producer is discouraged and lowers exploration; when x is moderate, he puts in extra effort to stay in front.
- For large c, the exhaustible producer is the effective leader and the green producer only has a small marginal negative impact.

Sample Game Dynamics



Hotelling's Rule Updated

Monopoly exhaustible resources, Hotelling 1931:

$$\frac{d}{dt} v'(X_t^*) = r v'(X_t^*).$$

See Guéant-Lasry-Lions (2010) for Mean-Field Games version. Here we have

$$\frac{d}{dt} v'(X_t^*) \mid_{X_t^*=x} = \mathcal{D} v'(x) = \lambda a^*(x) \Delta v'(x) - q_1^*(x) v''(x),$$

and we find:

$$\mathcal{D} v'(x) = \begin{cases} r v'(x) + q_1^*(x) \dfrac{\partial}{\partial x} q_2^*(x) & \text{if} \quad x < x_{\mathrm{b}} \wedge x_{\mathrm{sat}} \\ \dfrac{3}{4} r v'(x) & x_{\mathrm{sat}} < x < x_{\mathrm{b}} \\ r v'(x) & x > x_{\mathrm{b}}. \end{cases}$$

With competition, shadow prices grow *slower* than r.

Concluding Remarks

Energy/fuels markets have seen dramatic changes in just the past few years:

- natural gas discoveries and drop in price due to fracking technology; reserves up 12% in 2010; (bumping coal as marginal fuel in electricity production);
- oil plateauing above $100/barrel since 2005;
- rapid drop in cost of solar panel production (Solyndra 'scandal');
- shale oil technology and discoveries in Canada (+expensive, +dirty);
- increased speculative participation in commodities markets via ETFs, commodities index funds, etc.

However oil has virtual monopoly over petroleum-driven transportation sector. (See "Petropoly" by Korin & Luft 2013 on the case for fuel competition).

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- Passage to exhaustibility is through increased costs: $s_0(x)$, increasing as $x \downarrow 0$.
- On the other hand, improved renewable technologies: $s_i(x)$, decreasing as $x \downarrow 0$.
- Leads to games in which the cost-ordering may change over time.
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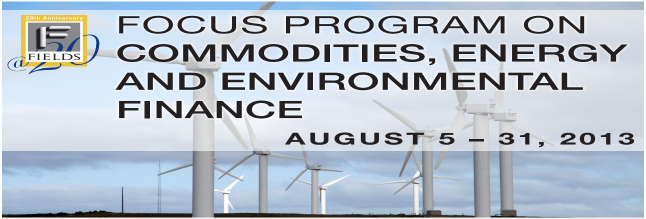
Tilting at Windmills



ACTIVITIES

Short Course on Stochastic Models of Electricity Markets
AUGUST 19–23, 2013
Instructor: Fred Benth

Short Course on Financialization of the Commodity Markets and Mean Field Games
AUGUST 12–13 and 26–27, 2013
Instructor: René Carmona

Short Course on Valuing and Trading Correlation Structures in Commodities
AUGUST 6–8 and 12–13, 2013
Instructor: Glen Swindle

Workshop on Electricity, Energy and Commodities Risk Management
AUGUST 14–16, 2013

Workshop on Stochastic Games, Equilibrium, and Applications to Energy and Commodities Markets
AUGUST 27–29, 2013

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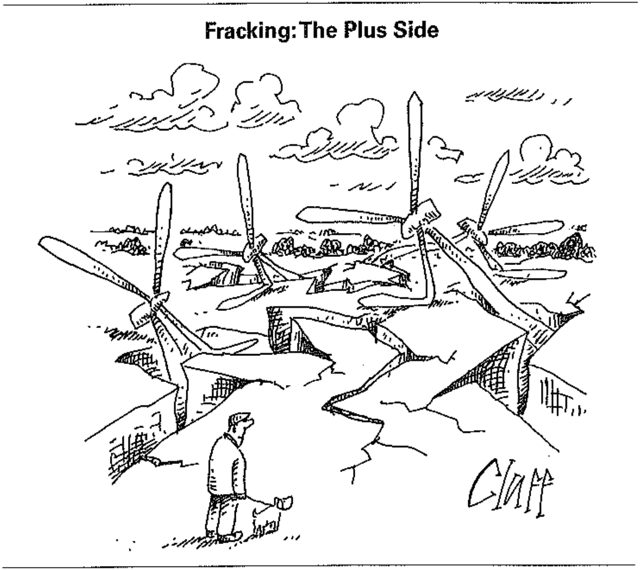

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